Mathematical
And
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Recreations
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PART IV.

MATHEMATICAL AND PHILOSOPHICAL RECREATIONS.

PART THIRD.

Containing various Problems in Mechanics.

AFTER arithmetic and geometry, mechanics is the next of the physico-mathematical sciences having their certainty resting on the simplest foundations. It is a science also the principles of which, when combined with geometry, are the most fertile and of the most general use in the other parts of the mixed mathematics. All those mathematicians therefore who have traced out the development of mathematical knowledge, place mechanics immediately after the pure mathematics, and this method we shall here adopt also.

We suppose, as in every other part of the mathematics introduced into this work, that the reader is acquainted with the first principles of the science of which we treat. Thus, in regard to mechanics, we suppose him acquainted with the principles of equilibrium and of hydrostatics; with the chief laws of motion, &c. For it is not our intention to teach these principles; but only to present a few of the

most curious and remarkable problems, which arise from them.

PROBLEM I.

To cause a ball to proceed in a retrograde direction, though it meets with no apparent obstacle.

Place an ivory ball on a billiard table, and give it a stroke on the side or back part, with the edge of the open hand, in a direction perpendicular to the table, or downward.—It will then be seen to proceed a few inches forward, or towards the side where the blow ought to carry it; after which it will roll in a retrograde direction, as it were of itself, and without having met with any obstacle.

Remark.—This effect is not contrary to the well known principle in mechanics, that a body once put in motion, in any direction, will continue to move in that direction until some foreign cause oppose and prevent or turn it. For, in the present case, the blow given to the ball, communicates to it two kinds of motion; one of rotation about its own centre, and the other direct, by which its centre moves parallel to the table, as impelled by the blow. The latter motion, on account of the friction of the ball on the table, is soon annihilated; but the rotary motion about the centre continues, and when the former has ceased, the latter makes the ball roll on the retrograde direction. In this effect, therefore, there is nothing contrary to the well known laws of mechanics.

PROBLEM II.

To make a false ball, for playing at nine pins.

Make a hole in a common ball used for playing at the above game; but in such a manner as not to proceed entirely to the centre; then put some lead into it, and close it with a piece of wood, so that the joining may not be easily perceived. When this ball is rolled towards the pins, it will not fail to turn aside from the proper direction, unless thrown by chance or dexterity in such a man-

ner; that the lead shall turn exactly at the top and bottom while the ball is rolling.

REMARK.—The fault of all balls used for billiards depends on this principle. For, as they are all made of ivory, and as, in every mass of that substance, there are always some parts more solid than others, there is not a single ball perhaps which has the centre of gravity exactly in the centre of the figure. On this account every ball deviates more or less from the line in which it is impelled, when a slight motion is communicated to it, in order to make it proceed towards the other side of the billiard table, unless the heaviest part be placed at the top or bottom. We have heard an eminent maker of these balls declare, that he would give two guineas for a ball that should be uniform throughout; but that he had never been able to find one perfectly free from the above mentioned fault.

Hence it happens, that when a player strikes the ball gently, he often imagines that he has struck it unskilfully, or played badly; while his want of success is entirely the consequence of a fault in the ball. A good billiard player, before he engages to play for a large sum, ought carefully to try the ball, in order to discover the heaviest and lightest parts. This precaution was communicated to us by a first rate player.

PROBLEM III.

How to construct a balance, which shall appear just when not loaded, as well as when loaded with unequal weights.

We certainly do not here intend to teach people how to commit a fraud, which ought always to be condemned; but merely to show that they should be on their guard against false balances, which often appear to be exact; and that in purchasing valuable articles, if they are not well acquainted with the vender, it is necessary to examine the balance, and to subject it to trial. It is possible indeed to make one, which when unloaded shall be in per-

fect equilibrium, but which shall nevertheless be false. The method is as follows:

Let A and B be the two scales of a balance, and let A be heavier than B: if the arms of the balance be made of unequal lengths, in the same ratio as the weights of the two scales, and if the heavier scale A be suspended from the shorter arm, and the lighter scale B from the longer, these scales when empty will be in equilibrium. They will be in equilibrium also when they contain weights which are to each other in the same ratio as the scales. A person therefore unacquainted with this artifice will imagine the weights to be equal; and by these means may be imposed on.

Thus, for example, if one of the scales weighs 15, and the other 16; and if the arms of the balance from which they are suspended be, the one 16 and the other 15 inches in length; the scales when empty will be in equilibrium, and they will remain so when loaded with weights which are to each other in the ratio of 15 to 16, the heaviest being put into the heaviest scale. It will even be difficult to observe this inequality in the arms of the balance. Every time therefore that goods are weighed with such a balance, by putting the weight into the heavier scale and the merchandise into the other, the purchaser would be cheated of a 16th part, or an ounce in every pound.

But, this deception may be easily detected by transposing the weights; for if they are not then in equilibrium, it is a proof that the balance is not just.

And indeed in this way the true weight of any thing may be discovered, even by such a false balance, namely, by first weighing the thing in the one scale, and then in the other scale; for a mean proportional between the two weights, will be the true quantity; that is, multiply the numbers of these two weights together, and take the square root of the product. Thus, if the thing weigh 16 ounces in the one scale, and only 14 in the other: then the pro-

duct of 16 multiplied by 14 is 224, the square root of which gives $14\frac{29}{30}$ for the true weight, or nearly 15 ounces. Or indeed the just weight is found nearly by barely adding the two numbers together, and dividing the sum by 2. Thus 16 and 14 make 30, the half of which, or 15, is the true weight very nearly.

PROBLEM IV.

To find the centre of gravity of several weights.

As the solution of various problems in mechanics depends on a knowledge of the nature and place of the centre of gravity, we shall here explain the principles of its theory.

The centre of gravity of a body, is that point around which all its parts are balanced, in such a manner, that if it were suspended by that point, the body would remain at rest in every position, in which it might be placed around that point.

It may be readily seen that, in regular and homogeneous bodies, this point can be no other than the centre of magnitude of the figure. Thus, the centre of gravity in the globe and spheroid, is the centre of these bodies; in the cylinder it is in the middle of the axis.

The centre of gravity between two weights, or bodies of different gravities, is found by dividing the distance between their points of suspension into two parts, which shall be inversely proportional to the weights; so that the shorter part shall be next to the heavier body, and the longer part towards the lighter. This is the principle of balances with unequal arms, by means of which any bodies of different weights may be weighed with the same weight, as in the steel yard.

When there are several bodies, the centre of gravity of two of them must be found by the above rule; these two are then supposed to be united in that point, and the com-

mon centre of gravity between them and the third is to be found in the same manner, and so of the rest.

Let the weights A, B and C, for example, be suspended from three points of the line or balance DF (pl. 1 fig. 1), which we shall suppose to have no weight. Let the body A weigh 108 pounds; B 144, and C 180; and let the distance DE be 11 inches, and EF 9.

First find the common centre of gravity of the bodies B and C, by dividing the distance EF, or 9 inches, into two parts, which are to each other as 144 to 180, or as 5 to 4. These two parts will be 5 and 4 inches; the greater of which must be placed towards the smaller weight: the body B being here the smaller, we shall have EG equal to 5 inches, and FG to 4; consequently DG will be 16.

If we now suppose the two weights B and C, united into one in the point G, and consequently equal in that point to 324 pounds; the distance DG, or 16 inches, must be divided in the ratio of 108 to 324, or of 1 to 3. One of these parts will be 12 and the other 4; and as A is the less weight, DH must be made equal to 12 inches, and the point H will be the common centre of gravity of all the three bodies, as required.

The result would have been the same, had the bodies A and B been first united. In short, the rule is the same whatever be the number of the bodies, and whatever be their position in the same straight line, or in the same plane.

This may suffice here in regard to the centre of gravity. But for many curious truths, deduced from this consideration, recourse may be had to books which treat on mechanics. We shall however mention one beautiful principle in this science deduced from it, which is as follows:

If several bodies or weights be so disposed, that by communicating motion to each other, their common centre of gravity remains at rest, or does not deviate from the hori-

zontal line, that is to say neither rises nor falls, there will then be an equilibrium.

The demonstration of this principle is almost evident from its enunciation; and it may be employed to demonstrate all the properties of machines. But we shall leave the application of it to the reader.

REMARK.—As this is the proper place, we shall here discharge a promise made in the preceding volume, prob. 72 Geom. viz, to resolve a geometrical problem, the solution of which, as we said, seems to be only deducible from the property of the centre of gravity.

Let the proposed irregular polygon then be ABCDE (pl. 1 fig. 2 No. 1); the sides of which are each divided into two equal parts, in *a*, *b*, *c*, *d* and *e*, from which results a new polygon *a b c d e a*; let the sides of the latter be each divided also into two equal parts, by the points a', b', c', d', e', which when joined will give a third polygon $a'\ b'\ c'\ d'\ e'\ a'$; and so on. In what point will this division terminate?

To solve this problem, if we suppose equal weights placed at *a*, *b*, *c*, *d*, *e*, their common centre of gravity will be the point required. But, to find this centre of gravity, we must proceed in the following manner, which is exceedingly simple. First draw *a b* (fig. 2 No. 2), and let the middle of it be the point *f*; then draw *f e*, and divide it in *g*, in such a manner that *f g* shall be one third of it; draw also *g d*, and let *g h* be the fourth of it; in the last place, draw *h c* and let *h i* be the fifth of it: the weight *e* being the last, the point *i*, as may be demonstrated from what has been before said, will be the centre of gravity of the five equal weights placed at *a*, *b*, *c*, *d* and *e*; and will solve the proposed problem.

PROBLEM V.

When two persons carry a burthen, by means of a lever or

pole, which they support at the extremities; to find how much of the weight is borne by each person.

It may be readily seen that, if the weight C were exactly in the middle of the lever AB (pl. 1 fig. 3), the two persons would each bear one half. But if the weight is not in the middle, it can be easily demonstrated, that the parts of the weight borne by the two persons, are in the reciprocal ratio of their distance from the weight. Nothing then is necessary but to divide the weight according to this ratio; and the greater portion will be that supported by the person nearest the weight, and the least that supported by the person farthest distant. The calculation may be made by the following proportion:

As the whole length of the lever AB, is to the length AE, so is the whole weight, to the weight supported by the power or person at the other extremity B; or as AB is to BE, so is the whole weight, to the part supported by the power or person placed at A.

If AB, for example, be 6 feet, the weight C 150 pounds, AE 4 feet, and BE 2, we shall have this proportion: as 6 is to 4, so is 150 to a fourth term, which will be 100. The person placed at the extremity B, will therefore support 100 pounds, and consequently the one placed at A will have to support only 50.

REMARK.—The solution of this problem affords the means of dividing a burthen or weight proportionally to the strength of the agents employed to raise it. Thus, for example, if the one has only half the strength of the other, nothing is necessary but to place him at a distance from the weight double to that of the other.

PROBLEM VI.

How 4, 8, 16 or 32 men may be distributed in such a manner, as to carry a considerable burthen with ease.

If the burthen can be carried by 4 men, after having

made it fast to the middle of a large lever AB (pl. 1 fig. 4) cause the extremities of this lever to rest on two shorter ones CD and EF, and place a man at each of the points C, D, E and F: it is evident that the weight will then be equally distributed among these four persons.

If 8 men are required, pursue the same method with the levers CD and EF, as was employed in regard to the first; that is, let the extremities of CD be supported by the two shorter ones *a b* and *c d;* and those of EF by the levers *e f* and *g h:* if a man be then stationed at each of the points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, they will be all equally loaded.

The extremities of the levers or poles *a b*, *c d*, *e f*, and *g h*, might, in like manner, be made to rest on others placed at right angles to them: by means of this artifice the weight would be equally distributed among 16 men, and so of any other number.

We have heard that this artifice is employed at Constantinople, to raise and carry the heaviest burthens, such as cannons, mortars, enormous stones, &c. The velocity, it is added, with which burthens are transported from one place to another by this method is truly astonishing.

PROBLEM VII.

A rope ACB (pl. 1 fig. 5), *of a determinate length, being made fast by both ends, but not stretched, to two points of unequal height,* A *and* B; *what position will be assumed by the weight* P, *suspended from a pulley, which rolls freely on that rope?*

From the points A and B, let fall the indefinite vertical lines AD and BG; then from the point A, with an opening of the compasses equal to the length of the rope, describe an arc of a circle, intersecting the vertical line BG, in E; and from the point B describe a similar arc of a circle, intersecting the vertical line AD in D: if the lines AE and BD be then drawn, the point C, where they cut each other, will give the position of the rope ACB, when the weight

has assumed that position in which it must rest; and the point C will be that in which the pulley will settle. For it may be easily demonstrated, that in this situation the weight P will be in the lowest position possible, which is an invariable principle of the centre of gravity.

PROBLEM VIII.

To cause a pail full of water to be supported by a stick, one half of which only, or less, rests on the edge of a table.

To make the reader comprehend properly the method of performing this trick, in regard to equilibrium; which is but ill explained in the old books of Mathematical Recreations, both in the text and in the engraving; we have given, in the 6th figure of the 1st plate, a section of the table and the bucket.

In this figure, let AB be the top of the table, on which is placed the stick CD. Convey the handle of the bucket over this stick, in such a manner that it may rest on it in an inclined position; and let the middle of the bucket be within the edge of the table. That the whole apparatus may be fixed in this situation, place another stick GFE, with one of its ends resting against the corner G of the bucket, while the middle part rests against the edge F, of the bucket, and its other extremity against the first stick CD, in E, where there ought to be a notch to retain it. By these means the bucket will remain fixed in that situation, without being able to incline to either side; and if not already full of water, it may be filled with safety; for its centre of gravity being in the vertical line passing through the point H, which itself meets with the table, it is evident that the case is the same as if the pail were suspended from the point of the table where it is met by that vertical. It is also evident that the stick cannot slide along the table, nor move on its edge, without raising the centre of gravity of the bucket, and of the water it contains. The heavier therefore it is, the greater will be the stability.

REMARK.—According to this principle, various other tricks of the same kind, which are generally proposed in books on mechanics, may be performed. For example, provide a bent hook DGF, as seen at the opposite end of the same figure, and insert the part, FD, in the pipe of a key at D, which must be placed on the edge of a table; from the lower part of the hook suspend a weight G, and dispose the whole in such a manner that the vertical line GD may be a little within the edge of the table. When this arrangement has been made, the weight will not fall, and the case will be the same with the key, which had it been placed alone in that situation would perhaps have fallen; and this resolves the following mechanical problem, proposed in the form of a paradox: *A body having a tendency to fall by its own weight, how to prevent it from falling, by adding to it a weight on the same side on which it tends to fall.*

The weight indeed appears to be added on that side, but in reality it is on the opposite side.

PROBLEM IX.

To hold a stick upright on the tip of the finger, without its being able to fall.

Affix two knives, or other bodies, to the extremity of the stick, in such a manner that one of them may incline to one side, and the second to the other, as seen in the figure (pl. 2 fig. 7): if this extremity be placed on the tip of the finger, the stick will keep itself upright, without falling; and if it be made to incline, it will raise itself again, and recover its former situation.

For this purpose, the centre of gravity of the two weights added, and of the stick, must be below the point of suspension, or the extremity of the stick, and not at the extremity, as asserted by Ozanam; for in that case there would be no stability.

It is the same principle that keeps in an upright position

those small figures furnished with two weights, to counter-balance them; and which are made to turn and balance, while the point of the foot rests on a small ball, loosely placed on a sort of stand. Of this kind is the small figure D (fig. 8 pl. 2), supported on the stand I, by a ball E, through which passes a bent wire, having affixed to its extremities two balls of lead C and F. The centre of gravity of the whole, which is at a considerable distance below the point of support, maintains the figure upright, and makes it resume its perpendicular position, after it has been inclined to either side; for this centre tends to place itself as low as possible, which it cannot do without making the figure stand upright.

By the same mechanism, three knives may be disposed in such a manner as to turn on the point of a needle; for being disposed as seen in the figure (fig. 9 pl. 2) and placed in equilibrio on the point of a needle held in the hand, they cannot fall, because their common centre of gravity is far below the point of the needle, which is above the point of support.

PROBLEM X.

To construct a figure, which, without any counterpoise, shall always raise itself upright, and keep in that position, or regain it, however it may be disturbed.

Make a figure resembling a man of any substance exceedingly light, such as the pith of the elder tree, which is soft and can be easily cut into any form at pleasure. Then provide for it an hemispherical base of some very heavy substance, such as lead. The half of a leaden bullet, made very smooth on the convex part, will be proper for this purpose. If the figure be cemented to the plane part of this hemisphere; then, in whatever position it may be placed, as soon as it is left to itself, it will rise upright (fig. 10 pl. 2); because the centre of gravity of its hemispherical base being in the axis, tends to approach the

horizontal plane as much as possible, and this can never be the case till the axis becomes perpendicular to the horizon; *for the small figure above scarcely deranges it from its place, on account of the disproportion between its weight and that of its base.*

In this manner, were constructed those small figures called Prussians, sold at Paris some years ago. They were formed into battalions, and being made to fall down by drawing a rod over them, they immediately started up again as soon as it was removed.

Screens of the same form have been since invented, which always rise up of themselves, when they happen to be pressed down.

PROBLEM XI.

If a rope ACB, *to the extremities of which are affixed the given weights* P *and* Q, *be made to pass over two pulleys* A *and* B; *and if a weight* R *be suspended from the point* C, *by the cord* RC; *what position will be assumed by the three weights and the rope* ACB? (fig. 11 pl. 2).

In the line *a b*, perpendicular to the horizon, assume any part *a c*, and on that part as a base, describe the triangle *a d c*, in such a manner, that *a c* shall be to *c d*, as the weight R, to the weight P; and that *a c* shall be to *a d*, as R to Q; then through A, draw the indefinite line AC parallel to *c d*; and through B, draw BC, parallel to *a d*: the point C, where these two lines intersect each other, will be the point required, and will give the position ACB of the rope.

For, if in RC continued we assume CD, equal to *a c*, and describe the parallelogram EDFC; it is evident that we shall have CF and CE, equal to *c d* and *a d*; and therefore the three lines EC, CD, and CF will be as the weights P, R and Q; consequently the two forces acting from C to P, and from C to R, or in the direction of the lines CA and CB, will be in equilibrio with the force which acts from C towards R.

REMARKS.—1st. If the ratio of the weights were such, that the point of intersection C should fall on the line AB, or above it, the problem in this case would be impossible. The weight Q, or the weight P, would overcome the other two in such a manner, that the point C would fall in B or A; so that the rope would form no angle.

These weights also might be such that it would be impossible to construct the triangle *a c d*, as if one of them were equal to or greater than the other two taken together; for, to make a triangle of three lines, each of them must be less than the other two. In that case we ought to conclude that the weight equal or superior to the other two would overcome them both, so that no equilibrium could take place.

2d. If instead of a knot at C, we should suppose the weight R suspended from a pulley capable of rolling on the rope ACB, the solution would be still the same; for it is evident that, things being in the same state as in the first case, if a pulley were substituted for the knot C, the equilibrium would not be destroyed. But there would be one limitation more than in the preceding case. It would be necessary that the point of intersection, C, determined as above, should fall below the horizontal line, drawn through the point B; otherwise the pulley would roll to the point B, as if on an inclined plane.

PROBLEM XII.

Calculation of the time which Archimedes would have required to move the earth, with the machine of which he spoke to Hiero.

The expression which Archimedes made use of to Hiero, king of Sicily, is well known, and particularly to mathematicians. "Give me a fixed point," said the philosopher, "and I will move the earth from its place." This affords matter for a very curious calculation, viz. to determine

how much time Archimedes would have required to move the earth only one inch, supposing his machine constructed and mathematically perfect; that is to say, without friction, without gravity, and in complete equilibrium.

For this purpose, we shall suppose the matter of which the earth is composed to weigh 300 pounds the cubic foot; being the mean weight nearly of stones mixed with metallic substances, such in all probability as those contained in the bowels of the earth. If the diameter of the earth be 7930 miles, the whole globe will be found to contain 261107411765 cubic miles, which make 1423499120882544640000 cubic yards, or 38434476263828705280000 cubic feet; and allowing 300 pounds to each cubic foot, we shall have 11530342879148611584000000 for the weight of the earth in pounds.

Now, we know by the laws of mechanics that, whatever be the construction of a machine, the space passed over by the weight, is to that passed over by the moving power, in the reciprocal ratio of the latter to the former. It is known also, that a man can act with an effort equal only to about 30 pounds for eight or ten hours, without intermission, and with a velocity of about 10000 feet per hour. If we suppose the machine of Archimedes then to be put in motion by means of a crank, and that the force continually applied to it is equal to 30 pounds, then with the velocity of 10000 feet per hour, to raise the earth one inch, the moving power must pass over the space of 384344762638287052800000 inches; and if this space be divided by 10000 feet, or 120000 inches, we shall have for quotient 3202873021985725440, which will be the number of hours required for this motion. But as a year contains 8766 hours, a century will contain 876600; and if we divide the above number of hours by the latter, the quotient, 3653745176803, will be the number of centuries during which it would be necessary to make the crank of the machine continually turn, in order to move the earth only

one inch. We have omitted the fraction of a century, as being of little consequence in a calculation of this kind *.

PROBLEM XIII.

With a very small quantity of water, such as a few pounds, to produce the effect of several thousands. (Plate 3 fig. 12).

Place a cask on one of its ends, and make a hole in the other end, capable of admitting a tube, an inch in diameter and from 12 to 15 feet in length; which must be fitted closely into the aperture by means of pitch or tow. Then load the upper end of the cask with several weights, so that it shall be sensibly bent downwards; and having filled the cask with water, continue to pour some in through the tube. The effort of this small cylinder of water will be so great, that not only the weights which pressed the upper end of the cask downwards will be raised up, but very often the end itself will be bent upwards, and form an arch in a contrary direction.

Care however must be taken that the lower end of the cask rest on the ground; otherwise the first effort of the water would be directed downwards, and the experiment might seem to fail.

By employing a longer tube, the upper end of the cask might certainly be made to burst.

The reason of this phenomenon may be easily deduced from a property peculiar to fluids, of which it is an ocular demonstration, viz, that when they press upon a base they exercise on it an effort proportioned to the breadth of that base multiplied by the height. Thus, though the tube used in this experiment contains only about 150 or 180 cylindric inches of water, the effort is the same as if the tube were equal in breadth to the cask, and at the same time 12 or 15 feet in height.

* The machine is here supposed to be constantly in action; but if it should be worked only 8 hours each day, the time required would be three times as long.

Another Method. (Plate 3 fig. 16).

Suspend from a hook, well fixed in a wall, or any other firm support, a body weighing 100 pounds or more; then provide a vessel of such dimensions, that between that body and its sides, there shall be room for only one pound of water; and let the vessel be suspended to one of the arms of a balance, the other arm of which has suspended from it a scale, containing a weight of 100 pounds. Pour a pound of water into the vessel suspended from the one arm of the balance, and it will raise the scale containing the 100 pounds.

Those who have properly comprehended the preceding explanation, will find no difficulty in conceiving the cause and necessity of this effect; for they are both the same, with this difference only, that the water, instead of being collected in a cylindric tube, is in the narrow interval between the body L and the vessel, which surrounds it; but this water exercises on the bottom of the vessel the same pressure that it would experience if entirely full of water.

Another Method.

Provide a cubic foot of very dry oak, weighing about 60 pounds, and a cubical vessel about a line or two larger every way. If the cubic foot of wood be put into the vessel, and water be poured into it, when the latter has risen to nearly two thirds of its height, the cube will be detached from the bottom, and float. Thus we see a weight of about 60 pounds overcome by half-a-pound of water and even less.

Remark.—Hence it appears that the vulgar are in an error, when they imagine that a body floats more readily in a large quantity of water than in a small one. It will always float, provided there be a sufficiency to prevent it from touching the bottom. If vessels are lost at the mouths of rivers, it is not because the water is too shallow; but because the vessels are loaded so much, as to be almost ready to sink, even in salt water. But as the water of the

sea is nearly a 30th part heavier than fresh water, when a ship passes from the one into the other, it must sink more and go to the bottom. Thus, an egg, which sinks in fresh water, will float in water which holds in solution a great deal of salt.

The principle on which the foregoing experiments are performed, is no other than the famous hydrostatical paradox, and on which principle Mr. Bramah, an ingenious engine maker, has invented a new power, in mechanics, of such efficacy as to raise, with great ease, the heaviest loads, or crush the hardest bodies.

PROBLEM XIV.

To find the weight of a cubic foot of water.

To know the weight of a cubic foot of water is one of the most essential elements of hydrostatics and hydraulics; and for that reason we shall here show how it may be accurately determined.

Provide a vessel, capable of containing exactly a cubic foot, and having first weighed it empty, weigh it again when filled with water. But as liquids always rise considerably above the edges of the vessel that contains them, the result in this case will not be very correct. There are means indeed to remedy this defect; but we are furnished with a very accurate method of doing it by hydrostatics.

Provide a cube of some very homogeneous matter, such as metal, each side of which is exactly 4 inches; weigh it by a good balance, in order to ascertain its weight within a few grains; then suspend it by a hair, or strong silk thread, from one of the scales of the same balance, and again find its weight when immersed in water. We are taught by hydrostatics that it will lose exactly as much in weight as the weight of an equal volume of water. The difference of these two weights therefore will be the weight of a cube of water, each side of which is 4 inches, or of the 27th part of a cubic foot.

If very great precision is not required, provide a cube or rectangular parallelopipedon, of any homogeneous matter, lighter than water, such, for example, as wood; and, having weighed it as accurately as possible, immerse it gently in water, in such a manner that the water may not wet it above that point at which it ought to float above the liquid. We shall here suppose that IMD (fig. 14 pl. 3) is the line, which exactly marks how much of it is immersed. Find the content of the solid ABCDMI, by multiplying its base by the height; the product will be the volume of water displaced by the body; and this volume, according to the principles of hydrostatics, must weigh as much as the body itself. If this volume of water be 720 cubic inches, for example, and if the weight of the body be 26·0416 pounds, we consequently know that 720 cubic inches of water weigh 26·0416 pounds. Hence it will be easy to determine the weight of a cubic foot, which contains 1728 cubic inches. Nothing is necessary but to make this proportion: as 720 cubic inches are to 1728, so are 26·0416 pounds to a fourth term, which will be 62·5 pounds, or 62 pounds and a half; which therefore is the weight of the cubic foot of water.

Two liquors being given; to determine which of them is the lightest.

This problem is generally solved by means of a well known instrument called the Areometer, or Hydrometer. This instrument is nothing else than a small hollow ball, joined to a tube 4 or 5 inches in length (fig. 15 pl. 3); a few grains of shot, or a little mercury, being put into the ball, the whole is so combined, that in water of mean gravity, the small ball and part of the tube are immersed.

It may now be readily conceived that when the instrument is put into any fluid, for example river water, care must be taken to observe how far it sinks in it; if it be

then placed in another kind of water, such as sea water, for instance, it will sink less; and if immersed in any liquor lighter than the first, such as oil for example, it will sink farther.* Thus it can be easily determined, without a balance, which of two liquors is the heavier or lighter. This instrument has commonly on the tube a graduated scale, in order to show how far it sinks in the fluid.

But this instrument is far inferior to that presented, in 1766, by M. de Parcieux, to the academy of sciences; and yet nothing is simpler. This instrument consists of a small glass bottle, 2 inches or 2 inches and a half, at most, in diameter, and from 6 to 8 inches in length. The bottom must not be bent inwards, lest air should be lodged in the cavity when it is immersed in any liquid. The mouth is closed with a very tight cork stopper, into which is fixed, without passing through it, a very straight iron wire, 25 or 30 inches in length, and about a line in diameter. The bottle is then loaded in such a manner, by introducing into it grains of small shot, that the instrument, when immersed in the lightest of the liquors to be compared, sinks so as to leave only the end of the iron wire above its surface, and that in the heaviest the wire is immersed some inches. This may be properly regulated by augmenting or diminishing either the weight with which the bottle is loaded, or the diameter of the wire, or both these at the same time. The instrument, when thus constructed, will exhibit, in a very sensible manner, the least difference in the specific gravities of different liquors, or the changes which the same liquor may experience, in this respect, under different circumstances; as by the effect of heat, or by the mixture of various salts, &c.

It may be readily conceived, that to perform experiments of this kind, it will be necessary to have a vessel of a sufficient depth, such as a cylinder of tin-plate, 3 or 4 inches in diameter, and 3 or 4 feet in length.

We have seen an instrument of this kind the movement

of which was so sensible, that when immersed in water, cooled to the usual temperature, it sunk several inches, while the rays of the sun fell upon the water; and immediately rose on the rays of that luminary being intercepted. A very small quantity of salt or sugar, thrown into the water, made it also rise some inches.

By means of this instrument, M. de Parcieux examined the gravity of different kinds of the most celebrated waters; among which was that drank at Paris; and he found that the lightest of all was distilled water. The next in succession, according to their lightness, were as in the following order; viz, the water of the Seine, that of the Loire, that of Yvette, that of Arcueil, that of Sainte-Reine, that of Ville d'Avray, the Bristol water, and well water.

We hence see the error of the vulgar, who imagine that the water of Ville d'Avray, that of Sainte-Reine, and that of Bristol, particularly the last, brought to France, at so great expence, are better than common river water; for they are, on the contrary, worse, since they are heavier.

If different kinds of water differ in their gravity, the case is the same with wines also. The lightest of all the known wines, at least in France, is the Rhenish. The next in succession are Burgundy, red Champagne, the wines of Bourdeaux, Languedoc, Spain, the Canaries, Cyprus, &c.

Some years ago we saw, exposed for sale, an *Oinometer*, or instrument for measuring the different degrees of the gravity of wines. It consisted of a hollow silver ball, joined to a small plate of the same metal, 3 or 4 inches in length, and a line or a line and a half in breadth, on which were marked the divisions that indicated how far the instrument ought to sink into the different kinds of wine. It may be readily seen that this was only the common areometer, constructed of silver.

The lightest of all the known liquors is ether. The

others, which follow in the order of gravity, are, alcohol, oil of turpentine, distilled water, rain water, river water, spring water, well water, mineral waters. Among the tables, annexed to this part of the work, the reader will find one containing the specific gravity of various liquors, compared with that of rain water; which, being the easiest procured, may serve as a common standard, and also the specific gravity of the different solid bodies, whether belonging to the mineral, vegetable, or animal kingdom; which will doubtless be found very useful, as it is often necessary to have recourse to tables of this kind.

As the following rules, for calculating the absolute gravity, in English troy weight, of a cubic foot and inch, English measure, of any substance, whose specific gravity is known, may be of use to the reader, the Translator has thought proper to subjoin them to this article of the original.

In 1696, Mr. Everard, balance maker to the Exchequer, weighed before the commissioners of the house of commons, 2145·6 cubical inches, by the Exchequer standard foot, of distilled water, at the temperature of 55°, of Fahrenheit, and found that it weighed 1131 oz. 14 drs. Troy, of the Exchequer standard. The beam turned with 6 grains, when loaded with 30 pounds in each scale. Hence, supposing the pound averdupois to weigh 7000 grs. Troy, a cubic foot of water weighs $62\frac{1}{2}$ pounds averdupois, or 1000 ounces averdupois, wanting 106 grs. Troy. If the specific gravity of water therefore be called 1000, the proportional specific gravities of all other bodies will express nearly the number of averdupois ounces in a cubic foot. Or, more accurately, supposing the specific gravity of water expressed by 1, and that of all other bodies in proportional numbers, as the cubic foot of water weighs, at the above temperature, exactly 437489·4 grains Troy, and the cubic inch of water 253·175 grains, the absolute

weight of a cubical foot or inch of any body, in Troy grains, may be found by multiplying its specific gravity by either of the above numbers respectively.

By Everard's experiment, and the proportions of the English and French foot, as established by the Royal Society and French Academy of Sciences, the following numbers have been ascertained:

Paris grains, in a Paris cube foot of water . . .	645511
English grains, in a Paris cube foot of water .	529922
Paris grains, in an English cube foot of water .	533247
English grains in an English cube foot of water	437489·4
English grains in an English cube inch of water	253·175
By an experiment of Picard, with the measure and weight of the Chatelet, the Paris cube foot of water contains of Paris grains	641326
By one of Du Hamel, made with great care . . .	641376
By Homberg	641666

These results show some uncertainty in measure or in weights; but the above computation from Everard's experiment may be relied on; because the comparison of the English foot with that of France, was made by the joint labour of the Royal Society of London, and the French Academy of Sciences. It agrees likewise, very nearly, with the weight assigned by Lavoisier, which is 70 Paris pounds to the cubical foot of water.

PROBLEM XVI.

To determine whether a mass of gold or silver, suspected to be mixed, is pure or not.

If the mass or piece, the fineness of which is doubtful, be silver for example, provide another mass of good silver equally heavy, so that the two pieces when put into the scales of a very accurate balance may remain in equilibrio in the air. Then suspend these two masses of silver from the scales of the balance, by two threads or two horse-

hairs, to prevent the scales from being wetted when the two masses are immersed in the water: if the masses are of equal fineness, they will remain in equilibrio in the water, as they did when in the air; but if the proposed mass weighs less in water, it is adulterated; that is to say, is mixed with some other metal, of less specific gravity than that of silver, such as copper for example; and if it weighs more, it is mixed with some metal of greater specific gravity, such as lead.

Remarks.—I. This problem is evidently the same as that whose solution gave so much pleasure to Archimedes. Hiero, king of Syracuse, had delivered to a goldsmith a certain quantity of gold, for the purpose of making a crown. When the crown was finished, the king entertained some suspicion in regard to the fidelity of the goldsmith, and Archimedes was consulted respecting the best means of detecting the fraud, in case one had been committed. The philosopher, having employed the above process, discovered that the gold, of which the crown consisted, was not pure.

If a large mass of metal were to be examined, as in the case of Archimedes, it would be sufficient to immerse the mass of gold or silver, known to be pure, in a vessel of water, and then the suspected mass. If the latter expelled more water from the vessel it would be a proof of the metal being adulterated by another lighter, and of less value.

But notwithstanding what Ozanam says, the difference between the weight in air and that in water will indicate the mixture with more certainty; for every body knows that it is not so easy, as it may at first appear, to measure the quantity of water expelled from any vessel.

II. According to mathematical rigour, the two masses ought first to be weighed in vacuo; for since air is a fluid, it lessens the real gravity of bodies by a quantity equal to the weight of a similar volume of itself. Since the two

masses then, the one pure and the other adulterated, are unequal in volume, they ought to lose unequal quantities of their weight in the air. But the great tenuity of air, in regard to that of water, renders this small error insensible.

PROBLEM XVII.

The same supposition made; to determine the quantity of mixture in the gold.

The ingenious artifice employed by Archimedes, is contained in the solution of this problem, and is as follows.

Suspecting that the goldsmith had substituted silver or copper for an equal quantity of gold, he weighed the crown in water, and found that it lost a weight, which we shall call A; he then weighed in the same fluid a mass of pure gold, which in air was in equilibrio with the crown, and found that it lost a weight, which we shall call B; he next took a mass of silver, which in air was equal in weight to the crown, and weighing it in water, found that it lost a quantity C. He then employed this proportion: as the difference of the weights B and C, is to that of the weights A and B, so is the whole weight of the crown, to that of the silver mixed in it. The answer, in this case, may be obtained by a very short algebraical calculation, though the reasoning is rather too prolix; we shall however explain it after having illustrated this rule by an example.

Let us suppose that Hiero's crown weighed 20 pounds in the air, and that when weighed in water it lost a pound and a half. Archimedes, by weighing in air and in water, a mass of gold containing 20 pounds, must have found a difference of $1\frac{1}{19}$ pound; and by weighing in like manner a mass of silver of 20 pounds, he must have found a difference of $1\frac{9}{11}$ pound. As A, in this case, is equal to $\frac{3}{2}$, B to $\frac{20}{19}$, and C to $\frac{20}{11}$; hence the difference of A and B is $\frac{17}{38}$, and that of B and C is $\frac{160}{209}$: we must therefore use the fol-

lowing proportion: as $\frac{160}{209}$ are to $\frac{17}{38}$, so is 20 to a fourth term, which will be $\frac{187}{16}$ = 11 lbs. 8 oz. 5 dwts.

The reasoning which conducted, or might have conducted, the Syracusan philosopher to this solution, is as follows. If the whole mass were of pure gold, it would lose, when weighed in water, $\frac{1}{19}$ of its weight; and if it were of pure silver, it would lose, when weighed in water, $\frac{1}{11}$ of its weight: consequently, if it loses less than the latter quantity, and more than the former, it must be a mixture of gold and silver; and the quantity of silver substituted for gold will be greater as the quantity of weight which the crown loses in water approaches nearer to $\frac{1}{11}$, and vice versa. This mass of 20 pounds then must be divided into two parts, in the ratio of the following differences: viz, the difference between the loss which the crown experiences and that experienced by the pure gold; and the difference between the loss experienced by the crown and that experienced by the pure silver; these will be the proportions of the gold and silver mixed together in the crown: and from this reasoning is deduced the preceding rule.

We must here observe that it is not necessary to take two masses, one of gold and another of silver, each equal in weight to the crown. It will be sufficient to ascertain that gold loses a 19th of its weight, when weighed in water; and silver one 11th, and perhaps this was really the method employed by Archimedes.

PROBLEM XVIII.

Suppose there are two boxes exactly of the same size, similar and of equal weight, the one containing gold and the other silver: is it possible, by any mathematical means, to determine which contains the gold, and which the silver? Or, if we suppose two balls, the one made of gold and hollow, the other of solid silver gilt, is it possible to distinguish the gold from the silver?

In the first case, if the masses of gold and silver are

each placed exactly in the middle of the box which contains it, so that their centres of gravity coincide, whatever may be said in the old books on Mathematical Recreations, we will assert that there are no means of distinguishing them, or at least that the methods proposed are defective.

The case is the same in regard to the two similar globes of equal size and weight.

If we were however under the necessity of making a choice, we would endeavour to distinguish the one from the other in the following manner.

We would suspend both balls by as delicate a thread as possible to the arms of a very accurate balance, such as those which, when loaded with a considerable weight, are sensibly affected by the difference of a grain. We would then immerse the two balls in a large vessel filled with water, heated to the degree of ebullition, and that which should preponderate we would consider as gold. For, according to the experiments made on the dilatation of metals, the silver, passing from a mean temperature, to that of boiling water, would probably increase more in volume than the gold; in that case the two masses, which in air and in temperate water, were in equilibrio, would not be so in boiling water.

Or, we might make a round hole in a plate of copper, of such a size, that both balls should pass exactly through it with ease; we might then bring them to a strong degree of heat, superior even to that of boiling water. Now, if we admit that silver expands more than gold, as above supposed, we might apply each of them to the hole in question, and the one which experienced the greater difficulty in passing, ought to be accounted silver.

PROBLEM XIX.

Two inclined planes AB *and* AD *being given, and two unequal spheres* P *and* p; *to bring them to an equilibrium in the angle, as seen in the figure* (pl. 3 fig. 16).

The globes P and p, will be in equilibrio if the powers with which they repel each other, in the direction of the line C c, which joins their centres, are equal.

But, the force with which the globe P tends to descend along the inclined plane BA, which is known, the inclination of the plane being given, is to the force with which it acts in the direction C c, as radius is to the cosine of the angle c C F; and, in like manner, the force with which the weight p descends along DA, is to that with which it tends to move in the direction c C, as radius is to the cosine of the angle C cf: hence it follows, that as these second forces must be equal, the cosine of the angle c must have the same ratio to the cosine of the angle C, as the force with which the globe P tends to roll along BA, has to that with which p tends to roll along DA. The ratio of these cosines therefore is known; and as in the triangle C G c the angle G is known, since it is equal to the angle DAB, it thence follows that its supplement, or the sum of the two angles C and c, is also known; and hence the problem is reduced to this, viz, to dividing a known angle into two such parts, that their cosines shall be in a given ratio; which is a problem purely geometrical.

But, that we may confine ourselves to the simplest case, we shall suppose the angle A to be a right-angle. Nothing then will be necessary but to divide the quadrant into two arcs, the cosines of which shall be in the given ratio, which may be done with great ease.

Let the force then with which P tends to move along its inclined plane be equal to M; and that of p to roll along its plane equal to m. Draw a line parallel to the plane AB, at a distance from it equal to the radius of the globe P, and another parallel to the plane DA, at a distance from it equal to the radius of p, which will intersect each other in G; having then made GL to G l, as m to M, employ the following proportion: as L l is to LG, so is the sum of the radii of the two globes to GC; and from the point C, draw

cc parallel to Ll: the points c and c will be the places of the centres of the two globes, and in this situation alone they will be in equilibrio.

PROBLEM XX.

Two bodies, P *and* Q, *depart at the same time from two points* A *and* B, *of two lines given in position, and move towards* a *and* b, *with given velocities: required their position when they are the nearest to each other possible?* (Pl. 3 fig. 17.)

If their velocities were to each other in the ratio of the lines BD and AD, it is evident that the two bodies would meet in D. But supposing their velocities different from that, there will be a certain point where, without meeting, they will be at the least distance from each other possible; and after that they will continually recede from each other. Here, for example, the lines BD and AD are nearly equal. If we suppose then that the velocity of P is to that of Q, in the ratio of 2 to 1, required the point of the nearest approach.

Through any point R, in AD, draw the line RS parallel to BD, and in such a manner, that AR shall be to RS, as the velocity of P is to that of Q; that is to say, in the present case, as 2 to 1; produce indefinitely the line AST, and from the point B draw BC perpendicular to AT; through the point C draw CE parallel to BD, till it meet AD in E; and having drawn EF parallel to CB, meeting BD in F, the points F and E, will be those required.

PROBLEM XXI.

To cause a cylinder to support itself on a plane, inclined to the horizon, without rolling down; and even to ascend a little along that plane. (Pl. 4 fig. 18).

If a cylinder be homogeneous, and placed on an inclined plane, its axis being in a horizontal situation, it is evident that it will roll down; because its centre of gravity being the same as that of the figure, the vertical line, drawn

from this centre, will always fall beyond the point of contact of the lowest side; consequently the body must of necessity roll down towards that side.

But, if the cylinder be heterogenous, so that its centre of gravity is not that of the figure, it may support itself on an inclined plane, provided the angle which the plane makes with the horizon does not exceed certain limits.

Let there be a cylinder, for example, of which HFD is a section perpendicular to the axis. To remove its centre of gravity from the centre of the figure, make a grove in it parallel to its axis, of a semicircular form, and fill it with some substance F much heavier, so that the centre of gravity of the cylinder shall be removed from C to E. Let the inclined plane be AB, and let BG be to GA in a less ratio than CF to CE. The cylinder may then support itself on the inclined plane, without rolling down; and if it be moved from that position, in a certain direction, it will even resume it by rolling a little towards the summit of the plane.

For, let us suppose the cylinder placed on the inclined plane with its axis horizontal, and its centre of gravity in a line parallel to the plane, and passing through the centre, in such a manner that the centre of gravity shall be towards the upper part of the plane, fig. 19. Through the point of contact, D, draw CDH, perpendicular to the inclined plane, and IDe perpendicular to the horizon. We shall then have BG to GA, or BI to ID, as DI to IH, or as DC to Ce; and since the ratio of BG to GA is less than that of CF or CD to CE, it follows that Ce is less than CE, consequently the vertical line drawn from the point E will fall without the point of contact towards A; the body therefore will have a tendency to fall on that side, and will roll towards it ascending a very little till its centre of gravity E has assumed a position as seen fig. 18, where it coincides with the vertical line passing through the point of contact. When the cylinder arrives at this situation, it will main-

tain itself in it, provided neither its surface nor that of the plane be so smooth as to admit of its sliding parallel to itself. In this situation it will even have greater stability, according as the ratio of BG to GA is less than that of CF or CD to CE, or as the angle ABG or CDe is less than CDE.

This is also a truth which we must demonstrate. For this purpose, it is to be remarked that E, the centre of gravity of the cylinder, in rolling along the inclined plane, describes a curve, such as is seen in fig. 20; this is what geometricians call an elongated cycloid, which rises and descends alternately below the line drawn parallel to the inclined plane, through the centre of the cylinder. But, the cylinder being in the position represented in fig. 20, if the line ED be drawn from the centre of gravity to the point of contact, it may be demonstrated that the tangent to the point E of that curve, is perpendicular to DE: if the inclination of the plane therefore is less than the angle CDE, that tangent will meet the horizontal line towards the ascending side of the plane; and the centre of gravity of the cylinder will then be as on an inclined plane IK; consequently it must descend to the point L of the hollow of the curve, which it describes, where that curve is touched by the horizontal line.

When it reaches this point it cannot deviate from it, without ascending on the one side or the other: if it be then removed a little from this point, it will return to its former position.

PROBLEM XXII.

To construct a clock which shall point out the hours, by rolling down an inclined plane.

This small machine, invented by an Englishman named Wheeler, is exceedingly ingenious, and is founded on the principle contained in the solution of the preceding problem.

It consists of a cylindrical box, made of brass, 4 or 5 inches in diameter, and having on one side a dial plate, divided into 12 or 24 hours. In the inside, represented by fig. 21, is a central wheel, which by means of a pinion moves a second wheel, and the latter moves a third, &c, while a scapement, furnished with a balance or spiral spring, acts the part of a moderator, as in common watches. To the central wheel is affixed a weight P, which must be sufficient, with a moderate inclination, as 20 or 30 degrees, to move that wheel, and those which receive motion from it. But, as the machine ought to be perfectly in equilibrio around its central axis, a counteracting weight of such a nature, that the machine shall be absolutely indifferent to every position around this axis, must be placed diametrically opposite to the small system of wheels 2, 3, 4, &c. When this condition has been obtained, the moving weight P must be applied; the effect of which will be, to make the central wheel, 1, revolve, and by its means the clock movement 2, 3, 4, &c; but, at the same time that this motion takes place, the cylinder will roll down the plane a little, which will bring the weight P to its primitive position, so that the effect of this continual pressure will make the cylinder roll while the weight P changes its place relatively, in regard to the cylinder, but not in regard to the vertical line. The weight P, or the inclination of the plane, must be regulated in such a manner, that the machine shall perform a whole revolution in 24 or 12 hours. The handle must be affixed to the common axis of the central wheel and weight P; so that it shall always look towards the zenith or the nadir. If more ornaments are required, the axis may support a small globe with a figure placed on it, to point out the hours with its finger raised in a vertical position. It may be readily conceived, that when the machine has got to the lowest part of the inclined plane, to make it continue going, nothing will be necessary but to cause it to ascend to the highest.

If it goes rather too slow, its movement may be accelerated by raising up the inclined plane, and vice versa.

PROBLEM XXIII.

To construct a dress, by means of which it will be impossible to sink in the water, and which shall leave the person, who wears it, at full freedom to make every kind of movement.

As a man weighs very nearly the same as an equal volume of water, it is evident that a mass of some substance much lighter than that fluid may be added to his body, by which means both together will be lighter than water, and of course must float. It is in consequence of this principle that, in order to learn to swim, some people tie to their breast and back two pieces of cork, or affix full blown bladders below their arms. But these methods are attended with inconveniences, which may be remedied in the following manner.

Between the cloth and lining of a jacket, without arms, place small pieces of cork, an inch and a half square, and about half or three quarters of an inch in thickness. They must be arranged very near to each other, that as little space as possible may be lost; but yet not so close as to affect in any great degree the flexibility of the jacket, which must be quilted to prevent their moving from their places. The jacket must be made to button round the body, by means of strong buttons, well sewed on; and to prevent its slipping off it ought to be furnished behind with a kind of girdle, so as to pass between the thighs and fasten before.

By means of such a jacket, which will occasion as little embarrassment as a common dress, people may throw themselves into the water with the greatest safety; for if it be properly made the water will not rise over their shoulders. They will sink so little, that even a dead body in that situation will infallibly float. The wearers there-

fore need make no effort to support themselves; and while in the water they may read or write, and even load a pistol and fire it. In the year 1767 an experiment was made of all these things, by the abbé de la Chapelle, fellow of the royal society of London, by whom this jacket was invented.

It is almost needless to observe how useful this invention might be on land as well as at sea. A sufficient number of soldiers, provided with these jackets, might pass a deep and rapid river in the night time, armed with pistols and sabres, and surprise a corps of the enemy. If repulsed, they could throw themselves into the water, and escape without any fear of being pursued.

During sea voyages, the sailors, while employed in dangerous manœuvres, often fall overboard and are lost; others perish in ports and harbours by boats oversetting in consequence of a heavy swell, or some other accident; in short, some vessel or other is daily wrecked on the coasts, and it is not without difficulty that only a part of the crew are saved. If every man, who trusts himself to this perfidious element, were furnished with such a cork jacket, to put on during the moments of danger, it is evident that many of them might escape death.

PROBLEM XXIV.

To construct a boat which cannot be sunk, even if the water should enter it on all sides.

Cause a boat to be made with a false bottom, placed at such a distance from the real one, as may be proportioned to the length of the boat, and to its burthen and the number of persons it is intended to carry. According to the most accurate calculation, this distance, in our opinion, ought to be one foot, for a boat 18 feet in length, and 5 or 6 in breadth. The vacuity between this false bottom and the real one ought to be filled up with pieces of cork, placed as near to each other as possible; and as the false bottom will

lessen the sides of the boat, they may be raised proportionally; leaving large apertures, that the water thrown into the vessel may be able to run off. It may be proper also to make the stern higher, and to furnish it with a deck, that the people may take shelter under it, in case the boat should be thrown on its side by the violence of the waves.

Boats constructed in this manner might be of great utility for going on board a vessel lying in a harbour, perhaps several miles from the shore; or for going on shore from a ship anchored at a distance from the land. Unfortunate accidents too often happen on such occasions, when there is a heavy surf, or in consequence of some sudden gust of wind; and it even appears that sometimes the greatest danger of a voyage is to be apprehended under circumstances of this kind. But boats constructed on the above principle would prevent such accidents.

Much we confess is to be added to this idea, presented here in all its simplicity; for some changes perhaps ought to be made in the form of the vessel; or heavy bodies ought to be added in certain places to increase its stability. This is a subject of research well worth attention, as the result of it might be the preservation of thousands of lives every year.

For this invention we are indebted to M. de Bernieres, one of the four controllers general of bridges and causways; who in 1769 constructed a boat of this kind for the king. He afterwards constructed another with improvements for the duke de Chartres; and a third for the marquis de Marigny. The latter was tried by filling it with water, or endeavouring to make it overset; but it righted as soon as left to itself; and though filled with water, was still able to carry six persons.

By this invention the number of accidents which befall those who lead a sea-faring life, may in future be diminished; but the indifference with which the invention of M. de Bernieres was received, shows how regardless men are of

the most useful discoveries, when the general interests of humanity only are concerned, and when trouble and expence are required to render them practically useful*.

PROBLEM XXV.

How to raise from the bottom of the sea a vessel which has sunk.

This difficult enterprise has been several times accomplished by means of a very simple hydrostatical principle, viz, that if a boat be loaded as much as possible, and then unloaded, it tends to raise itself with a force equal to that of the weight of the volume of water which it displaced when loaded. And hence we are furnished with the means of employing very powerful forces to raise a vessel that has been sunk.

The number of boats employed for this purpose, must be estimated according to the size of the vessel, and by considering that the vessel weighs in water no more than the excess of its weight over an equal volume of that fluid; unless the vessel is firmly bedded in the mud; for then she must be accounted of her full weight. The boats being arranged in two rows, one on each side of the sunk vessel, the ends of cables, by means of divers, must be made fast to different parts of the vessel, so that there shall be four on each side, for each boat. The ends of these cables, which remain above water, are to be fastened to the head and stern of the boat for which they are intended. Thus, if there are four boats on each side, there must be 32 cables, being 4 for each boat.

When every thing is thus arranged, the boats are to be loaded as much as they will bear without sinking, and the cables must be stretched as much as possible. The boats are then to be unloaded, two and two, and if they raise the

* Vessels constructed on this principle, and known under the name of life-boats, are now used we believe on different parts of the British coasts; particularly at Shields, &c.

vessel, it is a sign that there is a sufficient number of them; but, in raising the vessel, the cables affixed to the boats which remain loaded will become slack, and for this reason they must be again stretched as much as possible. The rest of the boats are then to be unloaded, by shifting their lading into the former. The vessel will thus be raised a little more, and the cables of the loaded boats will become slack; these cables being again stretched, the lading of the latter boats must be shifted back into the others, which will raise the vessel still a little higher; and if this operation be repeated as long as necessary, she may be brought to the surface of the water, and conveyed into port, or into dock.

An account of the manœuvres employed to raise, in this manner, the Tojo, a Spanish ship belonging to the Indian fleet, sunk in the harbour of Vigo, during the battle on the 10th of October 1702, may be seen in the Mémoires des Academiciens étrangers, vol. 2. But as this vessel had remained more than 36 years in that state, it was imbedded in a bank of tenacious clay, so that it required incredible labour to detach it; and when brought to the surface of the water, it contained none of the valuable articles expected. It had been one of those unloaded by the Spaniards themselves, before they were sunk, to prevent them from falling into the hands of the English.

Additions.

On the same principle is constructed the camel, a machine employed by the Dutch for carrying vessels heavily laden over the sand banks in the Zuyder-Zee. In that sea, opposite to the mouth of the river Y, about six miles from the city of Amsterdam, there are two sand banks, between which is a passage, called the Pampus, sufficiently deep for small vessels, but not for those which are large and heavily laden. On this account ships which are outward

bound, take in before the city only a small part of their cargo, receiving the rest when they have got through the Pampus. And those that are homeward bound must in a great measure unload before they enter it. For this reason the goods are put into lighters, and in these transported to the warehouses of the merchants in the city; and the large vessels are then made fast to boats, by means of ropes, and in that manner towed through the passage to their stations.

Though measures were adopted, so early as the middle of the sixteenth century, by forbidding ballast to be thrown into the Pampus, to prevent the farther accumulation of sand in this passage, that inconvenience increased so much, from other causes, as to occasion still greater obstruction to trade; and it at length became impossible for ships of war and others heavily laden to get through it. About the year 1672, no other remedy was known, than that of making fast to the bottoms of ships large chests filled with water, which was afterwards pumped out, so that the ships were buoyed up and rendered sufficiently light to pass the shallow. By this method, which was attended with the utmost difficulty, the Dutch carried out their numerous fleet to sea in the above-mentioned year. This plan however gave rise soon after to the invention of the camel, by which the labour was rendered easier. The camel consists of two half ships, constructed in such a manner that they can be applied, below water, on each side of the hull of a large vessel. On the deck of each part of the camel are a great many horizontal windlasses; from which ropes proceed through apertures in the one half, and, being carried under the keel of the vessel, enter similar apertures in the other, from which they are conveyed to the windlasses on its deck. When they are to be used, as much water as may be necessary is suffered to run into them; all the ropes are cast loose, the vessel is conducted between

them, and large beams are placed horizontally through the port holes of the vessel, with their ends resting on the camel, on each side. When the ropes are made fast, so that the ship is secured between the two parts of the camel, the water is pumped from them, by which means they rise, and raise the ship along with them. Each half of the camel is generally 127 feet in length; the breadth at one end is 22, and at the other 13. The hold is divided into several compartments, that the machine may be kept in equilibrio, while the water is flowing into it. An East-India ship that draws 15 feet of water, can by the help of the camel be made to draw only 11; and the heaviest ships of war, of 90 or 100 guns, can be so lightened as to pass without obstruction all the sand banks of the Zuyder-Zee.

Leopold, in his Theatrum Machinarum, says that the camel was invented by Cornelius Meyer, a Dutch engineer. But the Dutch writers, almost unanimously, ascribe this invention to a citizen of Amsterdam, called Meeuves Meindertszoon Bakker. Some make the year of the invention to have been 1688, and others 1690. However this may be, we are assured on the testimony of Bakker himself, written in 1692, and still preserved, that in the month of June, when the water was at its usual height, he conveyed in the course of 24 hours, by the help of the camel, a ship of war called the Maagt van Enkhuysen, which was 156 feet in length, from Enkhuysen Hooft, to a place where there was sufficient depth; and that this could have been done much sooner had not a perfect calm prevailed at the time. In the year 1693, he raised a ship called the Unie, 6 feet, by the help of this machine, and conducted her to a place of safety.

As ships built in the Newa cannot be conveyed into harbour, on account of the sand banks formed by the current of that river, camels are employed also by the Russians, to carry ships over these shoals: and they have them of various sizes. Bernoulli saw one, each half of which

was 217 feet in length, and 36 in breadth. Camels are used likewise at Venice *.

PROBLEM XXVI.

To make a body ascend as if of itself along an inclined plane, in consequence of its own gravity.

Provide a double cone (fig. 22 pl. 5), that is, two right cones united at their bases, so as to have a common axis. Then make a supporter, consisting of two branches, forming an angle at the point C (fig. 23), which must be placed in such a manner, that the summit C shall be below the horizontal line, and that the two branches or legs shall be equally inclined to the horizon. The line AB must be equal to the distance between the summits of the double cone, and the height AD a little less than the radius of the base. These conditions being supposed, if the double cone be placed between the legs of this angle, it will be seen to roll towards the top; so that the body, instead of descending, will seem to ascend, contrary to the nature of gravity: this however is not the case; for its centre of gravity really descends, as we shall here show.

Let *ac* (fig. 24) be the inclined plane, containing the angle ACB; *ce* the horizontal line, passing through the summit *c*, and consequently *ea* will be the elevation of the plane above the horizontal line, which is less than the radius of the circle forming the base of the double cone. It is evident that when this double cone is at the summit of the angle, it will be as seen at *cd*; and when it reaches the highest part of the plane, it will have the position seen at *af*; its centre then will have passed from *d* to *a*, and since *dc* is equal to *af*, and *ce* is the horizontal line, *cf* will be a line declining below the horizon; and consequently *da*, which is parallel to it, will be so also. The centre of gra-

* An engraving of the camel may be seen in, L'Art de batir les Vaisseaux, Amsterdam 1719, 4to. vol. 2, p. 93. See also the Encyclopédie, Paris edition, vol. 3, p. 67.

vity of the cone will therefore have descended, while the cône appeared to ascend. But, as has been already seen, it is the descent or ascent of the centre of gravity that determines the real descent or ascent of a body. As long as the centre of gravity can descend, the body therefore really moves in that direction, &c.

It will be found, in the present case, that the course of the centre of gravity, in its whole descent, is a straight line. But a parabola or hyperbola might be situated in the same manner, with its summit downwards, and in that case the course of the centre of gravity of the double cone would be a curve. This may furnish a subject of exercise for young geometricians.

PROBLEM XXVII.

To construct a clock with water. (Fig. 25 pl. 5.)

If the water which issues from a cylindric vessel, through a hole formed in its bottom, flowed in a uniform manner, nothing would be easier than to construct a clock to indicate the hours by means of water. But it is well known that the greater the height of the water above the orifice, through which it issues, the greater is the rapidity with which it flows; so that the vertical divisions ought not to be equal: the solution of the problem therefore consists in determining their ratio.

It is demonstrated in hydraulics, that the velocity with which water flows from a vessel, through a very small orifice, is proportional to the square root of the height of the water above the aperture. And hence the following rule, for dividing the height of the vessel, which we suppose to be cylindrical, has been deduced.

If we suppose that the whole water can flow out in 12 hours; divide the whole height into 144 parts; then 23 of these will be emptied in the first hour; so that there will remain 121 for the other eleven. Of these 121 parts, 21 will be emptied during the second hour; then 19 will

be emptied in the third, 17 in the fourth, and so on. As the 144th division therefore corresponds to 12 hours, the 121st will correspond to 11; the 100th to 10; the 81st to 9, &c, till the last hour, during which only one division will be emptied. These divisions will comprehend, in the retrograde order, beginning at the lowest, the first, 1 part; the second, 3; the third, 5; the fourth, 7; &c; which is exactly the ratio of the spaces passed over in equal times by a body falling freely in consequence of its gravity.

But, if it were required that the divisions in the vertical direction, should be equal in equal times, what figure ought to be given to the vessel?

The vase, in this case, ought to be a paraboloid, formed by the circumvolution of a parabola of the 4th degree; or the biquadrates of the ordinates ought to be as the abscissas. If an orifice of a proper size were made in the summit of this paraboloid; and if it were then inverted; the water would flow from it in such a manner, that equal spaces of the vertical height would be emptied in equal times.

The method of describing this parabola is as follows. Let ABS (fig. 26 pl. 5), be a common parabola, the axis of which is PS, and the summit S. Draw, in any manner, the line RRT, parallel to that axis, and then draw any ordinate of the parabola AP, intersecting RT in R; make PQ a mean proportional between PR and PA; and let pq be a mean proportional also between pr and pa; and so on. The curve passing through all the points Q, q, &c, will be the one required; and it may be employed to form a mould for constructing a vessel of the required concavity. To whatever height it shall be filled with water, equal heights will always be emptied in equal times.

In another part of this work, we shall give a method of making equal quantities of water flow from a vessel of any form in equal times. As this depends on the property of the syphon, it belongs to a different head.

PROBLEM XXVIII.

A point being given, and a line not horizontal, to find the position of the inclined plane along which, if a body descend, setting out from the given point, it shall reach that line in the least time. (Fig. 27 pl. 5).

This mechanical problem is exceedingly curious, and admits of a very elegant solution. Let A be the given point, and BC the given line. From the point A, draw the vertical line AD, and AE perpendicular to the given line; then from the point D, where the vertical line meets BC, draw DG parallel to AE, and equal to AD: if AG be then drawn intersecting BC in F, the line AF will be the position of the plane along which a body, setting out from A, and descending by the effect of its own gravity, will arrive at the line BC in less time than by any other plane differently inclined.

To demonstrate this problem, draw FH parallel to AE or DG, till it meet the vertical line AD in H. On account of the similar triangles then, we shall have AD to DG, as AH to HF; consequently, DG being equal to AD, AH will be equal to HF, which is also perpendicular to BC, because it is parallel to AE. The circle therefore described from the point H, as a centre, through the point A, will pass through F, and touch the line BC.

But it is well known, that if a vertical diameter, as AHI, be drawn in a circle, and any cords AF and AK, a body left to descend by the effect of its own gravity will pass over the spaces represented by these lines, in the same time. Since the time then employed to fall along AK or AI, is equal to that employed to fall along AF, the time required to fall along AD or AE will be greater than that employed to descend along AF. And the same reasoning being applicable to all the other lines that can be drawn from the point A to BC, it follows that AF is the line along which the body will arrive, in the least time, at the line BC.

If the line BC were vertical, AE would then be horizontal, as well as DG; AD and DG would both be infinite, and equal; which would give the angle FAD equal to 45°. Hence it follows, that in this case it would be along a plane inclined at an angle of 45° that the body, left to itself, would arrive at the vertical line in the least time possible.

PROBLEM XXIX.

Two points A *and* B *being given in the same horizontal line; required the position of two planes* AC *and* CB, *of such an inclination, that two bodies descending with accelerated velocity from* A *to* C, *and then ascending along* CB *with the acquired velocity, shall do so in the least time possible.* (Fig. 28 pl. 5.)

It is evident that a body placed at A, on the horizontal line AB, would remain there eternally without moving towards B. To make it proceed therefore by the effect of its own gravity from A to B, it must fall along an inclined plane or a curve; so that, after having descended a certain space, it shall ascend along a second plane, or the remainder of the curve, as far as B. But we shall suppose that this is done by means of two inclined planes. It is here to be observed, that the time employed to descend and ascend, must be longer or shorter according to the inclination and the length of these planes. The question then is, to determine what position of them is most advantageous, in order that the time may be the least. Now it will be found that to obtain the required position, the two planes must be equal and inclined to the horizon at an angle of 45°; that is, the triangle ACB ought to be isosceles and right-angled at C.

This solution is deduced from that of the preceding problem; for if we conceive a vertical line drawn through the point C, it has been shown that the plane AC, inclined at an angle of 45 degrees, is the most favourably disposed

to make the body, sliding along it, arrive at the vertical line in the least time possible; but the time of the ascent along CB, is equal to that of the descent; whence it follows that their sum, or the double of the former, is also the shortest possible.

PROBLEM XXX.

If a chain and two buckets be employed to draw up water from a well of very great depth; it is required to arrange the apparatus in such a manner, that in every position of the buckets, the weight of the chain shall be destroyed; so that the weight to be raised shall be that only of the water contained in the ascending bucket. (Fig. 29 pl. 6.)

If two buckets be suspended from the two ends of a rope or chain, so as to ascend and descend alternately, while the rope rolls round the axis or wheel of the windlass, which serves to raise them, it is evident that when one of the buckets is at the bottom, the person who begins to raise it has not only the weight of the bucket to support, but that also of the whole chain or rope from the top to the bottom of the well; and there are some cases, as in mines of three or four hundred feet in depth, where the weight of several quintals must be overcome to raise only two or three hundred pounds to the mouth of the mine. Such were the mines of Pontpean, until M. Loriot suggested a remedy for this inconvenience.

This remedy is so simple, that it is astonishing no one ever thought of it before. Nothing indeed is necessary but to convert the rope or chain into a complete ring, one of the ends of which descends to the depth where the water or the ore is to be drawn up, and to affix the buckets to two points of the rope in such a manner that when one of them is at the highest part, the other shall be at the lowest. For it is evident that, as equal parts of the chain ascend and descend, these parts will counter-balance each other; and the weight to be raised, were the pit several thousand

feet in depth, will be that only of the ore or other substances drawn up.

The case would evidently be the same if there were only one bucket: in every position, the only weight to be raised would be that of the bucket, and the matter it contained; but the machine would be attended with only one half of its advantage; for, by having no more than one bucket, the time which the bucket when emptied would employ in descending would be lost.

Remark.—In the Memoirs of the Academy of Sciences for 1731, M. Camus gave another method of remedying the above inconvenience. It consists, when there is only one bucket, in employing an axis nearly in the form of a truncated cone; so that when the bucket is at the lowest depth, the rope is rolled round the part which has the least diameter; and when the bucket is at the top, it is rolled round that which has the greatest. By these means, the same force is always required. But it is evident that, in every case, more must be applied than is necessary.

When there are two buckets, M. Camus proposes that one half of the rope should be rolled round one half of the axis, which he divides into two equal parts; so that one half is covered by the rope belonging to the bucket raised up, while the other is uncovered, the bucket which corresponds to it being at the bottom. By these means the two efforts are combined in such a manner, that nearly the same force is always required to overcome them. But these inventions, though ingenious, are inferior to that of M. Loriot.

PROBLEM XXXI.

Method of constructing a jack which moves by means of the smoke of the chimney. (Fig. 30 pl. 6.)

The construction of this kind of jack, which is very ingenious, is as follows. An iron bar fixed in the back of

the chimney, and projecting from it about a foot, serves to support a perpendicular spindle, the extremity of which turns in a cavity formed in the bar; while the other extremity is fitted into a collar in another bar, placed at some distance above the former. This spindle is surrounded with a helix of tin plate, which makes a couple of revolutions, or turns round the spindle, and which is about a foot in breadth. But instead of this helix, it will be sufficient to cut several pieces of tin plate, or sheet iron, and to fix them to the spindle in such a manner that their planes shall form with it an angle of about 60 degrees; they must be disposed in several stories, above each other; so that the upper ones may stand over the vacuity left by the lower ones. The spindle, towards its summit, bears a horizontal wheel, the teeth of which turn a pinion having a horizontal axis, and the latter, at its extremity, is furnished with a pulley, around which is rolled the endless chain that turns the spit. Such is the construction of this machine, the action of which may be explained in the following manner. When a fire is kindled in the chimney, the air which by its rarefaction immediately tends to ascend, meeting with the helicoid surface, or kind of inclined vanes, causes the spindle, to which they are affixed, to turn round, and consequently communicates the same motion to the spit. The brisker the fire becomes, the quicker the machine moves, because the air ascends with greater rapidity.

When the machine is not used, it may be taken down, by raising the vertical spindle a little, and removing the point from its cavity; which will allow the summit to be disengaged from the collar in which it is made to turn. When wanted for use, it may be put up with the same ease.

Remarks.—I. The following mechanical amusement is founded on the same principle. Cut out from a card as large a circle as possible; then cut in this circle a spiral,

making three or four revolutions, and ending at a small circle, reserved around the centre, and of about a line or two in diameter; extend this spiral by raising the centre above the first revolution, as if it were cut into a conical surface or parabolid; then provide a small spit made of iron, terminating in a point, and resting on a supporter. Apply the centre or summit of the helix to this point; and if the whole be placed on the top of a warm stove, the machine will soon put itself in motion, and turn without the assistance of any apparent agent. The agent however in this case is the air, which is rarefied by the contact of a warm body, and which ascending forms a current.

II. There is no doubt that a similar invention might be applied to works of great utility: it might be employed, for example, in the construction of wheels to be always immersed in water, their axis being placed parallel to the current: to give the water more activity this helicoid wheel might be inclosed in a hollow cylinder, where the water, when it had once entered, being impelled by the current above it, would in our opinion act with a great force.

If the cylinder were placed in an erect position, so as to receive a fall of water through the aperture at the top, the water would turn the wheel and its axis, and might thus drive the wheel of a mill, or of any other machine. Such is the principle of motion employed in the wheels of Basacle, a famous mill at Toulouse.

III. The smoke jacks here in England are made somewhat different from that above described; being mostly after the manner of that exhibited in fig. 55 plate 13: where AB is a circle containing the smoke vanes, of thin sheet iron, all fixed in the centre, but set obliquely at a proper angle of inclination. The other end of the spindle has a pinion c, which turns the toothed wheel D, on the spindle of which is fixed the vertical wheel E, over which

passes the chain EF which turns the spit below. There are other forms of this useful machine also made; but all or most of them having the same kind of vanes in the circle AB, instead of the spiral form in the original.

PROBLEM XXXII.

What is it that supports in an upright position, a top or tetotum, while it is revolving?

It is the centrifugal force of the parts of the top or tetotum, put in motion. For a body cannot move circularly without making an effort to fly off from the centre; so that if it be affixed to a string, made fast to that centre, it will stretch it, and in a greater degree according as the circular motion is more rapid.

The top then being in motion, all its parts tend to recede from the axis, and with greater force the more rapidly it revolves; hence it follows that these parts are like so many powers acting in a direction perpendicular to the axis. But as they are all equal, and as they pass all round with rapidity by the rotation, the result must be that the top is in equilibrio on its point of support, or the extremity of the axis on which it turns.

PROBLEM XXXIII.

How comes it that a stick, loaded with a weight at the upper extremity, can be kept in equilibrio, on the point of the finger, much easier than when the weight is near the lower extremity, or that a sword, for example, can be balanced on the finger much better, when the hilt is uppermost?

The reason of this phenomenon, so well known to all those who perform feats of balancing, is as follows. When the weight is at a considerable distance from the point of support, its centre of gravity, in deviating either on the one side or the other from a perpendicular direction, describes a larger circle than when the weight is very

near to the centre of rotation, or the point of support. But in a large circle an arc of a determinate magnitude, such as an inch, describes a curve which deviates much less from a horizontal direction than if the radius of the circle were less. The centre of gravity of the weight then may, in the first case, deviate from the perpendicular the quantity of an inch, for example, without having a tendency or force to deviate more, than it would in the second case; for its tendency to deviate altogether from the perpendicular is greater, according as the tangent to that point of the arc where it happens to be, approaches more to a vertical direction. The greater therefore the circle described by the centre of gravity of the weight, the less is its tendency to fall, and consequently the greater the facility with which it can be kept in equilibrio.

PROBLEM XXXIV.

What is the most advantageous position of the feet for standing with firmness, in an erect posture?

It is customary among well bred people to turn their toes outwards; that is to say, to place their feet in such a manner, that the line passing through the middle of the sole, is more or less oblique to the direction towards which the person is turned. Being induced by this circumstance to enquire whether this custom, to which an idea of gracefulness is attached, be founded on any physical or mechanical reason, we shall here examine it according to the principles of mechanics.

Every body whatever rests with more stability on its base, according as its centre of gravity, on account of its position and the extent of that base, is less exposed to be carried beyond it by the effect of any external shock. The problem then, in consequence of this very simple principle, is reduced to the following: To determine whether the base, within which the line drawn perpendicular to the

horizon from the centre of gravity of the human body ought to fall, is susceptible of increase and diminution, according to the position of the feet; and what is the position of the feet which gives to that base the greatest extent. But this becomes a problem of pure geometry, which might be thus expressed: *Two lines* AD *and* BC (fig. 31 pl. 6) *of equal length, and moveable around the points* A *and* B, *as centres, being given; to determine their position when the trapezium or quadrilateral* ABCD *is the greatest possible.* This problem may be solved with the greatest facility, by methods well known to geometricians; and from the solution the following construction is deduced.

On the line A *d* (fig. 32 pl. 6), equal to AD, or BC, construct the isosceles triangle AH *d*, rightangled at H; and make AK equal to AH. Having then assumed AI equal to one half of AG, or one fourth of AB, draw the line KI, and make IE equal to IK: on GE if an indefinite perpendicular, intersecting in D, the circle described from the point A as a centre, with the radius AD, be then raised, the point D, or the angle DAE, will determine the position of AD, and consequently of BC. If the line AB, and consequently AG or AI, be nothing, or vanish, AE will be found equal to AH; and the angle DAE will be half a right one. Thus, when the heels absolutely touch each other, the angle which the longitudinal lines of the soles of the feet ought to form, is half a right one, or nearly so, on account of the small distance which is then between the two points of rotation, in the middle of the heels.

If the distance AB is equal to AD, the angle DAE ought to be 60 degrees; if AB is equal to twice AD, the angle DAE ought to be nearly 70 degrees; and in the last place, if AB be equal to 3 times the line AD, it will be found that DAE ought to be nearly 74° 30′.

It is hence seen, that in proportion as the feet are at a greater distance from each other, their direction, in order to stand or walk with more stability, ought to approach

nearer to parallelism. But, in general, mechanical principles accord with what is taught by custom and gracefulness, as it is called; that is to say, to turn the toes outwards.

PROBLEM XXXV.

Of the game of Billiards.

It is needless to explain here the nature of billiards. It is well known that this game is played on a table covered with green cloth, properly stretched, and surrounded by a stuffed border, the elasticity of which forces back the ivory balls that impinge against it. The winning strokes at this game, are those which, by driving your ball against that of your adversary, force the latter into one of the holes at the corners, and in the middle of the two longer sides, which are called pockets.

The whole art of this game then consists in being able to know in what manner you must strike your adversary's ball with your own, so as to make it fall into one of the pockets, without driving your own into it also. This problem, and some others belonging to the game of billiards, may be solved by the following principles.

1st. The angle of the incidence of the ball against one of the edges of the table, is equal to the angle of reflection.

2d. When a ball impinges against another, if a straight line be drawn between their centres, which will consequently pass through the point of contact, that line will be the direction of the line described after the stroke.

These things being premised, we shall now give a few of the problems which arise out of this game.

I. *The position of the pocket and that of the two balls* M *and* N *being given* (fig. 33 pl. 7); *to strike your adversary's ball* M *in such a manner, that it shall fall into the pocket.*

Through the centre of the given pocket and that of the ball, draw, or conceive to be drawn, a straight line; the

point where it intersects the surface of the ball, on the side opposite to or farthest from the pocket, will be that where it ought to be touched, in order to make it move in the required direction. If we then suppose the above line continued from one of the radii of the ball, the point O, where it terminates, will be that through which the impinging ball ought to pass. It may be readily conceived, that it is in this that the whole dexterity of the game consists: nothing being necessary, but to strike the ball in a proper manner. It is easy to see what ought to be done, but it is not so easy to perform it.

In the last place, it is evident from what has been said, that provided the angle NOB exceeds a right angle ever so little, it is possible to drive the ball M into the pocket.

II. *To strike the ball by reflection.*

The ball M (fig. 34 pl. 7) being concealed, or almost concealed, behind the iron, in regard to the ball N, so that it would be impossible to touch it directly, without running the risk of striking the iron and failing in the attempt; it is necessary, in that case, to try to touch it by reflection. For this purpose, conceive the line MO, drawn perpendicular from M to the edge DC, to be continued to *m*; so that O *m* shall be equal to OM. If you aim at the point *m*, the ball N, after touching the edge DC, will strike the ball M.

If it were required to strike the ball M (fig. 35 pl. 7) by two reflections, the geometrical solution, in this case, is as follows. Conceive the line MO, drawn perpendicular from the point M, to the edge BC, to be continued till O *m* become equal to OM. Conceive also the line *m* P, drawn perpendicular from the point *m* to the edge continued, to be continued to *q*, until P *q* be equal to P *m*; if the ball N be directed to the point *q*, after impinging against the edges DC and CB, it will strike the ball M.

To those, in the least acquainted with geometry, the demonstration of this problem will be easy.

III. *If a ball strikes against another in any direction whatever, what is the direction of the impinging ball after the shock?*

It is of importance, at the game of billiards, to be able to know what will be the direction of your own ball after it strikes that of your adversary obliquely; for every one knows that it is not sufficient to have touched the latter, or to have driven it into the pocket; you must also prevent your own from falling into it.

Let M and N (fig. 36 pl. 7) be the two balls, the latter of which is to strike the former, touching it in the point O. Through this point O, let there be drawn the tangent OP; and through the centre *n*, of the ball N, when it arrives at the point of contact, draw or conceive to be drawn *n p*, parallel to OP: the direction of the impinging ball, after the shock, will be *n p*. A bad player would here be infallibly lost; and indeed this is often the case in this position of the balls. Expert players, when they find that they have to do with novices, often give them this deceitful chance, which makes them lose, by driving their ball into one of the corner pockets. In this case you must not take the ball of your adversary by halfs, according to the technical term of the game, to drive it to one of the corners at the other end of the table; for in doing so, you will not fail to lose yourself in the other corner.

REMARK.—In reasoning on this game, we set out from common principles; but we must confess that we have some doubts on this subject, the reason of which we shall here explain.

If the balls had only one progressive movement forwards, without rotation around their centres, the above principles would be evidently and sufficiently demonstrated. But every one knows that, independently of this

progresive motion of the centre, a billiard ball rolls on the table in a plane which is perpendicular to it. When a ball then touches the edge, and is repelled with a force nearly equal to that with which it impinged, it would appear that this motion ought to be compounded of the rotary motion it had at the moment of the shock, and that which it has in a direction parallel to the edge. But since the first of these motions compounded with the latter, gives the angle of reflection equal to the angle of incidence, what then becomes of the second, which ought to alter the first result? In our opinion this is a dynamical problem, which has never yet been solved, though it deserves to be so.

However, this rotary motion, in certain circumstances, gives a result which seems contrary to the laws of the impinging of elastic bodies; for according to this law, when an elastic body impinges directly and centrally against another which is equal to it, the first ought to stop, in consequence of having communicated, as is supposed, all its velocity to the second. But at the game of billiards, this does not take place; for here the impinging ball continues to move, instead of stopping short. This effect is partly a consequence of the motion of the impinging ball around its centre; a motion which subsists in a great measure after the shock, and it is this motion partly which makes the ball still move forwards. Another cause of the striking ball's moving forward, is the want of perfect elasticity in them both, on which account that ball still retains some portion of its direct forward motion, the other ball, which is struck, receiving the rest of the motion.

PROBLEM XXXVI.

To construct a Water Clock.

This name is given to a clock shaped like a drum or barrel, as ABCD, (fig. 37 pl. 8), made of metal well soldered, and put in motion by a certain quantity of water contained

in the inside of it. The hours are indicated on two vertical pillars, between which it is suspended by small strings or cords, rolled round an axis, every where of the same thickness. The internal mechanism is exceedingly ingenious, and deserves a better explanation than what has been given of it in the preceding editions of the Mathematical Recreations, where Ozanam does not tell us how the machine goes and is supported, as we may say, in the air, without falling, as it seems it ought to do.

Let the circle 1 2 3 4 (fig. 38) represent a section of the drum or cylinder, by a plane perpendicular to its axis. We shall here suppose the diameter of it to be six inches; and let A, B, C, D, E, F, G represent seven cells, the partitions of which are formed of the same metal, and are well soldered to the two circular ends, and to the circular band which forms the circumference. These partitions ought not to proceed from the centre to the circumference, but to be placed in a somewhat transverse direction, so as to be tangents to an interior circle, of about an inch and a half in diameter: the small square H is a section of the axis, which in that part ought to be square, and to fit very exactly into holes, of the same form, made in the centre of each end of the cylinder. Each partition also ought to have in it a small round hole, as near as possible to the circumference of the cylinder, all pierced with the same piercer, that there may be no difference among them.

Let us now suppose that a certain quantity of water, about 8 or 9 ounces, has been put into the cylinder, and that it has already distributed itself as shown by the horizontal shading lines fig. 38. If the line IK represent the two strings, GH and EF, (fig. 37), rolled round the axis of the cylinder, it may be easily seen that the centre of gravity, which, if the machine were empty, would be in the centre of the figure, being thrown out of the line of suspension, and towards the side where the machine has a tendency to fall, it would indeed fall; but the effect of

the water behind the partition D, is to throw back the centre of gravity, so that if it were on this side the vertical line KI continued, the cylinder would revolve from D to E, in order to be in that vertical; and in this position the machine would remain in equilibrio, if the water could not proceed from the one cavity to the other; for the cylinder cannot revolve in the direction AGF, without making the centre of gravity ascend towards D: in the like manner it cannot revolve in the direction ..., without the centre rising on the opposite side. The machine must then remain in equilibrio, until something is changed.

But, if the water flows gradually through the hole in the partition D, which is between the cells D and E, it is evident that the centre of gravity will advance a little beyond KI continued, and the machine will imperceptibly revolve in the direction AGF; and since by descending in this manner, the centre of gravity is thrown towards the vertical line KI produced, the equilibrium will at the same time be restored, and this motion will continue until the whole of the cord be unrolled from the axis. This movement indeed will not be altogether uniform, for it is evident that when the water is almost entirely behind the partition D, the cylinder will revolve faster than when it has nearly flowed off; and the periods of these inequalities during a whole revolution of the cylinder will be equal in number to the cells; a circumstance which seems not to have been observed by those who have written on clocks of this kind.

To have an exact division of time by these means, it will therefore be necessary to make a mark on the circumference of the cylinder. If the machine be then wound up as high as possible, and disposed in such a manner that the mark shall be at the top of the cylinder, you will have a good clock, with which you must mark, during a whole revolution, the points of the hours elapsed. But care must be taken that the number of hours shall be an integer

number, as 2, 4, 6, &c; and for that purpose the movement of the machine must be retarded or accelerated till the proper precision has been obtained; otherwise it might err some minutes, and perhaps a quarter of an hour. How this movement may be accelerated or retarded, we shall show hereafter.

In the last place, in winding up the clock, care must be taken that when the axis is placed opposite to the first division, the mark made in the cylinder shall be in the same position; otherwise there may be an error, as already said, of some minutes. We shall now add some useful observations in regard to this object.

I. It is absolutely necessary that the water employed be distilled water; otherwise it will soon become corrupted, so as to stop up the holes through which it ought to flow; and the machine will consequently stand still.

II. The substance most proper for constructing the cylinder of these machines, is gold or silver; or, what is cheaper, copper well tinned on the inside, or even tin itself.

III. This machine is apt to go a little faster in summer than in winter, and therefore ought to be regulated from time to time, and retarded or accelerated. For this purpose, it will be necessary to add to it a small weight as a counterpoise, tending to make it revolve outward. This weight ought to have the form of a bucket (fig. 39 pl. 8); and to be of some light substance, so that it can be charged more or less by means of small drops of lead. To accelerate the machine, two or three drops of lead may be added; and when it is necessary to retard it, they may be removed; which will be much more convenient than adding or taking away water.

IV. The place where the axis passes through the cylinder must be well cemented; otherwise the water would gradually evaporate, by which means the machine would be continually retarded, and at length would stop.

V. Notwithstanding all these precautions, it may be readily seen, that a machine of this kind is rather an object of curiosity, than calculated to measure time with accuracy. It may be proper for the cell of a convent, or a cabinet of mechanical curiosities; but it will certainly never be used by the astronomer.

VI. The inventor of this kind of clock is not known. Ozanam, who wrote in 1693, says that the first seen at Paris about that period had been brought from Burgundy; and he adds that father Timothy, a Barnabite, who excelled in mechanics, had given to this machine all the perfection of which it was susceptible. This monk had constructed one about 5 feet in height, which required winding up only once a month. Besides the hours, which were marked on a regular dial plate, at the top of the frame, it indicated the day of the month, the festivals throughout the year, the sun's place, and his rising and setting, as well as the length of the day and night. This was performed by means of a small figure of the sun, which gradually descended, and which, when it reached the bottom of the frame, was raised to the top at the end of every month.

Father Martinelli has treated, at great length, on these clocks, in an Italian work, entitled Horologi Elementari, in which he delivers methods of making clocks by means of the four elements, water, earth, air, and fire. This work was printed at Venice, in 1663, and is very rare. The author shows in it how striking machinery may be adapted to a water clock; with other curiosities, which are sometimes added to common clocks.

PROBLEM XXXVII.

MECHANICAL PARADOX.

How equal weights placed at any distance from the point of support of a balance, shall be in equilibrio.

Provide a frame in the form of a parallelogram, such as

DEFG (fig. 40 pl. 8), constructed of four pieces of wood, joined together in such a manner as to move freely at the angles, so that the frame can change its rectangular form into that represented by the letters *e f g d*. The long sides ought to be about twice the length of the others. This frame is inserted in a cleft formed in the perpendicular stand BC, so as to be moveable on the two points I and H, where it is fixed in the stand by two small axes: in the last place, two pieces of wood, MN and KL, pass through the shorter sides, in which they are well fixed, and the whole apparatus rests on the stand AB.

Now if the weight P, be suspended from the point M, which is almost at the extremity of the arm MN, the most distant from the centre or centres of motion; and if the weight Q, equal to the former, be suspended from any point R, of the other arm KL, nearer the centre, and even within the frame, these two weights will always be in equilibrio; though unequally distant from the point of support or of motion in this kind of balance; and they will remain so, whatever situation may be given to the machine, as *e f d g*.

The reason of this effect, which at first seems to contradict the principles of statics, is however very simple. For two equal bodies will be in equilibrio, whatever movement may be made by the machine from which they are suspended, if the spaces passed over by these two bodies or weights are equal and similar. But it may be readily seen that this must necessarily be the case here, since the two weights, whatever be their position, are obliged to describe equal and parallel lines.

It may be readily seen also, that, in such a machine, whatever be the position of the weights along the arms MN and KL, the case will always be the same, as if they were suspended from the middle of the short sides ED and FG. But in the latter case, the weights would be in equilibrio, therefore in former also.

PROBLEM XXXVIII.

What velocity must be given to a machine, moved by water, in order that it may produce the greatest effect?

That this is not a matter of indifference, will readily appear from the following observation. If the wheel moved with the same velocity as the fluid, it would experience no pressure; consequently the weight it would be capable of raising would be nothing, or infinitely small. On the other hand, if it were immoveable, it would experience the whole pressure of the current; but in this case there would be an equilibrium, and as no weight would be raised, there would consequently be no effect. There is therefore a certain mean velocity, between that of the current and no velocity at all, which will produce the greatest effect—an effect proportional, in a given time, to the product of the weight multiplied by the height to which it is raised.

We shall not here give the analytical reasoning which conducts to the solution of the problem: We shall only observe, that in a machine of the above nature, the velocity of the wheel ought to be equal to a third part of that of the current. Consequently the resistance or the weight must be increased, until the velocity be in this ratio. The machine will then produce the greatest effect possible.

REMARK.—The above proportional part, viz, one third, is an old error which has been properly corrected by a late author, who has shown, both theoretically and practically, that the water wheel works with the greatest effect, when its velocity is equal to half the velocity of the stream which turns it. See the Transactions of the American Philosophical Society, vol. 3. p. 144; or Dr. Hutton's Dictionary, vol. 2, under the word MILL.

PROBLEM XXXIX.

What is the greatest number of float-boards, that ought to be applied to a wheel moved by a current of water, in order to make it produce the greatest effect?

It was long believed, that the float-boards of such a wheel ought to be so proportioned, that when one of them was in a vertical position, or at the middle of its immersion, the next one should be just entering the water. A great many reasons were assigned for this mode of construction, which however are contradicted by calculation, as well as by experience.

It is now demonstrated, that the more float-boards such a wheel has, the greater and more uniform will be its effect. This result is proved by the researches of the abbé de Valernod, of the academy of Lyons, and those of M. du Petit-Vandin, to be found in the first volume of the Mémoires des Sçavans Etrangers.

The abbé Bossut, who examined, by the help of experiments, the greater part of the hydraulic theories, has demonstrated also the same thing. According to the experiments which he made, a wheel furnished with 48 float-boards, produced a much greater effect, than one furnished with 24; and the latter a greater effect than one with 12; their immersion in the water being equal. M. du Petit-Vandin therefore observes, that in Flanders, where running water is so exceedingly scarce, as to render it necessary to turn it to the greatest possible advantage, the wheels of water-mills are furnished with 32 float-boards, at least, and even with 48, when the wheel is from 16 to 19 feet in diameter.

PROBLEM XL.

If there be two cylinders, containing exactly the same quantity of matter, the one solid and the other hollow, and both of the same length; which of them will sustain, without breaking, the greatest weight suspended from one of its extremities, the other being fixed?

Some, and perhaps several of our readers, may be inclined to think that, the base of rupture being the same, every thing else ought to be equal. On the first view, one might be induced to consider the solid cylinder as ca-

pable of presenting greater resistance to being broken: this however would be a mistake.

Galileo, who first examined mathematically the resistance of solids to being broken by a weight, has shown that the hollow cylinder will present the most resistance; and that this resistance will be greater in the transverse direction, according as the hollow part is greater. He even shows, from a theory which approaches very near the truth, that the resistance of the hollow cylinder will be to that of the solid one, as the whole radius of the hollow is to that of the solid. Thus the resistance of a hollow cylinder, having as much vacuity as solid, will be to the resistance of a solid one, as $\sqrt{2}$ to 1, or as 1·141 to 1·000; for the radius of the former will be $\sqrt{2}$, while that of the latter is unity. The resistance of a hollow cylinder, having twice as much vacuity as solid, will be to a solid one, as $\sqrt{3}$ to 1, or as 1·73 to 1·00; for their radii will be in the ratio of $\sqrt{3}$ to 1. The resistance of a hollow cylinder, the solidity of which forms only a 20th part of the whole volume, will be to that of a solid cylinder of the same mass, as $\sqrt{21}$ to 1, or as 4·31 to 1·00; and so on.

Remark.—It may be readily observed, and Galileo does not fail to take notice of it, that this mechanism is that which nature, or its Supreme Author, has employed on various occasions to combine strength with lightness. Thus the bones of the greater part of animals are hollow: by being solid, with the same quantity of matter, they would have lost much of their strength; or to give them the same power of resistance, it would have been necessary to render them more massy; which would have lessened the facility of motion.

The stems of many plants are hollow also, for the very same reason. In the last place, the feathers of birds, in the formation of which it was necessary that great strength should be united with great lightness, are also hollow: and the cavity even occupies the greater part of their whole diameter; so that the sides are exceedingly thin.

PROBLEM XLI.

To construct a lantern, which shall give light at the bottom of the water.

This lantern must be made of leather, which will resist the waves better than any other substance; and must be furnished with two tubes, having a communication with the air above. One of these tubes is destined to admit fresh air for maintaining the combustion of the candle or taper; and the other to serve as a chimney, by affording a passage to the smoke: both must rise to a sufficient height above the surface of the water, so as not to be covered by the waves when the sea is tempestuous. It may be readily conceived, that the tube which serves to admit fresh air, ought to communicate with the lantern at the bottom; and that the one which serves as a chimney, must be connected with it at the top. Any number of holes at pleasure, into which glasses are fitted, may be made in the leather of which the lantern is constructed; and by these means the light will be diffused on all sides. In the last place, the lantern must be suspended from a piece of cork, that it may rise and fall with the waves.

A lantern of this kind, says Ozanam, might be employed for catching fish by means of light; but this method of fishing has, in some countries, been wisely forbidden under severe penalties.

PROBLEM XLII.

To construct a lamp, which shall preserve its oil in every situation, however moved or inclined.

To construct a lamp of this kind, the body of it, or the vase that contains the oil and the wick, must have the form of a spherical segment, with two pivots at the edge, diametrically opposite to each other, and made to turn in two holes at the extremities of the diameter of a brass or iron circle. This circle must, in like manner, be furnished with two pivots exactly opposite to each other, and at the distance of 90° from the holes in which the former are in-

serted. These second pivots must be made to turn in two holes diametrically opposite in a second circle; and this second circle must likewise be furnished with two pivots, inserted in some concave body, proper to serve as a covering to the whole lamp.

It may be readily seen that, by this method of suspension, whatever motion be given to the lamp, unless too abruptly, it will always maintain itself in a horizontal position.

This method of suspension is that employed for the mariners compass, so useful to navigators; and which must always be preserved in a horizontal situation. We have read in some author, that Charles the 5th caused a carriage to be suspended in this manner, to guard against the danger of being overturned.

PROBLEM XLIII.

Method of constructing an anemoscope and an anemometer.

These two machines, which in general are confounded, are not however the same. The anemoscope serves for pointing out the direction of the wind, and therefore, properly speaking, is a weather-cock; but in common this term is used to denote a more complex machine, which indicates the direction of the wind by means of a kind of dial-plate, placed either on the outside of a house, or in an apartment. In regard to the anemometer, it is a machine which serves to indicate, not only the direction, but the duration and force of the wind.

The mechanism of the anemoscope is very simple. (Fig. 41 pl. 9). In consists, in the first place, of a weather-cock, raised above the building, and supported by an axis, one end of which, passing through the roof, is made to turn in a socket fitted to receive it, and with such facility as to obey the least impulse of the wind. On this axis is fixed a crown-wheel, the teeth of which being turned downwards, fit into those of a vertical wheel, exactly of the same size, placed on a horizontal axis, which at its in-

tensity is furnished with an index. It is hence evident, that when the vane makes one turn, the index will make one exactly also. If this index then be placed in such a manner as to be vertical, when the wind is north, and if care be taken to observe in what direction it turns when it changes to the west, it will be easy to divide the dial-plate into 32 points.

An anemometer, if it be required only to measure the intensity or force of the wind, may be constructed with equal ease. We would propose the following. Let AB (fig. 42 pl. 9) be an iron bar, fixed in a horizontal direction to the vertical axis of a vane. The extremities of this bar, which are bent at right angles, serve to support a horizontal axis, around which turns a moveable frame ABCD, of a foot square. To the middle of the lower side of the frame is fastened a very fine but strong silk thread, which passes over a pulley F, fitted into a cleft in the vertical axis of the vane, whence it descends along the axis to an apartment below the roof. The distance GF must be equal to GE. To the end of the silk thread is suspended a small weight, just sufficient to keep it stretched. When the frame which, by the turning of the vane, will be always presented to the wind, is raised up, as will be the case, more or less, according to the force of the wind, the small weight will be raised up also, and will thus indicate, by means of a scale adapted to the axis of the vane, the strength of the wind. It may readily be perceived that the force of the wind will be equal to zero, or nothing, when the small weight is at its lowest point; and that its maximum, or greatest degree, will be when it is at its highest, which will indicate that the wind keeps the frame in a horizontal position, or very nearly so.

The force of the wind, according to the different inclination of the frame, may be determined with still greater precision; for this force will always be equal to the absolute weight of the frame, (which is known) multiplied by

the sine of the angle which it makes with the vertical line, and divided by the square of the same angle. Nothing then will be necessary, but to ascertain, by the motion of the small weight affixed to the thread EFP, the inclination of the frame. But this is easy; for it may be readily seen that the quantity which it rises above the lowest point, will always be equal to the chord of the angle formed by the frame with the vertical plane, or to double the sine of the half of that angle. The extent therefore of this angle may be marked along the scale, and also the force of the wind, calculated according to the foregoing rule.

In the Memoirs of the Academy of Sciences, for the year 1734, may be found the description of an anemometer, invented by M. d'Ons-en-Bray, to indicate at the same time the direction of the wind, its duration in that direction, and its strength. This anemometer merits that we should here give some idea of it.

It consists of three parts, viz, a common clock, and two other machines, one of which serves to mark the direction of the wind and its duration; the other to indicate its force.

The first of these machines consists, like the common anemoscope, of a vertical axis bearing a vane, which by means of some wheels indicates, on a dial-plate, from what quarter the wind blows; the lower part of this axis passes through a cylinder, in which are implanted 32 pins, in a spiral line; and these pins, by the manner in which they present themselves, press against a piece of paper, properly prepared and stretched, between two vertical columns or axes, on one of which it is rolled up, while it is unrolled from the other. This rolling up and unrolling are performed by the simultaneous motion of two axes, which are made to move by the clock above mentioned. It may now be readily conceived that, according to the position of the vane, one of the pins will present itself to the prepared paper, and by pressing gently against it will

leave a mark, the length of which will indicate the duration of the wind. If two neighbouring pins make a mark, at the same time, this will indicate that the wind followed a middle direction.

The part of the anemometer which indicates the force of the wind, consists of a mill, after the Polish manner, which revolves faster, according as the wind is stronger. Its vertical axis is furnished with a wheel that drives a small machine, which, after a certain number of turns, forces a pin against a frame of paper, having a motion similar to that of the anemometer above described. The number of these strokes, each of which is marked by a hole, on a determinate length of this moveable paper, denotes the force of the wind, or rather the velocity of the circulation of the mill, which is nearly in the same proportion. But, for a complete explanation of the mechanism, we must refer to the Memoirs of the Academy of Sciences, above quoted; as want of room will not allow us to give a more minute description of it in this place.

REMARK.—Many other forms of anemometers have been invented, in various countries. Of several of these the descriptions may be seen, with their figures and the calculation of their effects, in Dr. Hutton's Dictionary, under the several articles ANEMOMETER, RESISTANCE, WIND, and WIND-GAGE.

PROBLEM XLIV.

Construction of a Steel-yard, by means of which the weight of a body may be ascertained, without weights.

We shall here describe two instruments of this kind; the one portable and adapted for ascertaining moderate weights, such as from 1 to 25 or 30 pounds; the other fixed, and employed for weights much more considerable, and even of several thousand pounds. One of the latter kind was used in the custom house at Paris; and could be employed, with great convenience, for weights between 100 and 3000 pounds.

The first of these steel yards is represented fig. 43 pl. 9. It consists of a metal tube AB, about 6 inches in length, and 8 lines in diameter, a section of which is here given, to show in the inside of it a spiral steel spring. The upper end A, is pierced with a square hole, to afford a passage to a metal rod, which is also square; and which passes through the spring, so that it is impossible to draw it upwards without compressing the spring against the upper end within the tube. To the lower part of the tube is affixed a hook, from which the body to be weighed is suspended.

It is here evident, that if bodies of different weights be applied to the hook, while the steel-yard is suspended by its ring, they will draw down the tube more or less, by forcing the upper end of it against the spring. The rod therefore must be divided, by suspending successively from the hook different weights, such as one pound, two pounds, &c, to the greatest which it can weigh; and if the part of the rod drawn out of the tube each time be marked by a line, accompanied with a figure denoting the weight, the instrument will be complete. When you intend to use it, nothing is necessary but to put your finger into the ring, to raise up the article you intend to weigh, suspended from the hook, and to observe, on the divided face of the rod, the division exactly opposite to the edge of the hole; the figure belonging to this division will indicate the number of pounds which the proposed body weighs.

The second steel-yard consists of two bars, placed back to back, or of a single one ABCDE bent in the form seen fig. 44 pl. 9. The part AB is suspended by a ring from a strong beam, and the part DE terminates in a hook at E, from which the articles to be weighed are suspended. To the part ED is fixed a rack, fitted into a pinion, connected with a wheel, the teeth of which are fitted into another pinion, having on its axis an index; and this index makes just one revolution, when the weight of 3000 pounds is suspended from the hook E. For it may be readily seen,

that when any weight is suspended from E, the spring DCB must be more or less stretched; this will give motion to the rack DF, and the latter will turn the pinion into which it is fitted; and consequently will give motion to the wheel and second pinion, having on its axis the index. It is also evident, that in constructing the machine, such a force may be given to the spring, or its wheels may be combined in such a manner, that a determinate weight, as 3000 pounds, shall cause the index to perform a complete revolution. The centre of motion of this index is in the centre of a circular plate, marked with the divisions, that serve to indicate the weight. These divisions must be formed by suspending, in succession, weights less than the greatest, in the arithmetical progression, as 29 hundred weight, 28, 27, &c. This will give the principal divisions, which without any considerable error may be then subdivided into equal parts.

When the instrument is thus constructed; then to find the weight of any article that weighs less than 3000 pounds, nothing is necessary but to suspend it to the hook E; and the index will point out, on the circular plate, its weight in quintals, or hundreds, quarters, and pounds.

REMARK.—It may be proper here to observe, that this method of weighing cannot be perfectly exact, unless we suppose that the temperature of the air always remains the same; for during cold weather springs are stiffer, and during hot weather are less so. On this account, we have no doubt that there is a difference between the same article weighed at the custom house at Paris in winter and in summer. In winter it must appear to weigh less than it does in summer.

PROBLEM XLV.

To construct a carriage, which a gouty person may employ for moving from one place to another, without the assistance of men or horses.

A carriage of this kind is represented Plate 2 fig. 45.

The reader may observe, 1st. Two large wheels, which ought to be about 44 inches in diameter; the circumference of these wheels consists of one piece; it is covered with one piece of iron, and ought to be pretty broad, that they may sink less into the ground.

2d. At the distance of about two thirds of each spoke, from the nave of the wheel, is applied a roller, an inch in thickness, and turning on an axis, one end of which is fixed in the spoke, and the other in a flat circular piece of iron, which by means of nuts and screws serves to keep all the rollers in their places.

3d. On each shaft, beyond the place where it is crossed by the axis of the two wheels, is fixed a piece of iron, shaped like a fork, which serves to support the axis of a crank, having at its extremity a wheel with 4 teeth, cut into the form of an epicycloid. These teeth are fitted into the rollers above mentioned, and serve to turn the wheel. The arm of the crank ought to be only 8 or 9 inches in length.

4th. A plan of the same things is represented fig. 46. It exhibits the form of the shafts or frame, consisting of two parallel pieces of wood, a little concave on the upper side, kept together behind by a turned bar of wood, and before by a piece of iron. These two cross pieces serve to support the two springs, on which are placed a small easy chair, furnished with cushions, and a step for the feet. If required, it may be fitted with an umbrella. It ought to stand a little backwards, that the weight of the person may not throw the carriage forward. To the lower part of the foot step is fixed a piece of bent iron, which in case the machine should incline forward, may serve to keep it back by resting against the floor. To maintain it firm behind, a small wheel is connected with the middle of the cross bar of the frame, by a mechanism similar to that of bed-rollers; the vertical axis of which, for the greater strength, passes through a bar of iron fixed to the axis of

the large wheels. In the last place, the extremities of the frame or body are furnished with two handles, by means of which the machine can be pushed forward by a servant in difficult places; and in front there are two hooks, to which traces can be made fast, in order that a horse may be attached to the carriage if necessary. A more minute description of this machine may be found in Mémoires présentés à l'Académie Royale des Sciences, par divers Sçavants, vol. 4.

We are informed by M. Brodier, the inventor of this machine, that having caused one to be constructed, it weighed no more, including the weight of his body, than 378 pounds; and calculating its effect according to the principles of mechanics, he found that on ascending an inclined plane of 8 degrees, it was capable of proceeding at the rate of 400 yards in 23 minutes, which is agreeable to experience. By ascending in this manner, the person could not fail to be fatigued; but on firm ground, and on a horizontal pavement, a person might direct it for a long time; especially if assisted in the difficult places by a child 14 or 15 years of age.

The former editions of the Mathematical Recreations contain very brief descriptions of some machines of the like kind. The first is a small rolling chair of the usual form, with 4 wheels, of which those before are moveable on their axis, and roll only by the impulse of those behind. The latter are strongly fixed to their axis, and this axis has in the middle a pinion fitted into a crown-wheel, which the person who sits in the carriage turns by means of a handle. We much doubt whether this machine was ever attended with success, or rather we consider it to be very defective; since the moving power is applied as near as possible to the centre of motion.

The other carriage, as Ozanam says, was moved by a boy seated behind, who trod alternately with his feet on two moveable treadles. These treadles, in rising and fall-

ing, moved two pieces of wood, fitted into toothed wheels fixed to the axis of the large wheels. But this mechanism is so badly explained by Ozanam, both in the description and the figure, that no one can understand it; for this reason we have thought proper to make a total change in this article, as we have done in many others equally defective both in the form and the matter.

PROBLEM XLVI.

Method of constructing a small figure, which when left to itself descends along a small stair on its hands and its feet.

This small machine, the mechanism of which is very ingenious, was a few years ago brought from India. It is called the tumbler, because its motion has a great resemblance to that of those performers at some of the public places of amusement, who throw themselves backwards resting on their hands; then raise their feet, and complete the circle by resuming their former position; but the figure can perform this movement only descending, and along a sort of steps. The artifice of this small machine, is as follows:

AB (Plate xi fig. 47) is a small piece of light wood, about 2 inches in length, 2 lines in thickness, and 6 in breadth. At its two extremities are two holes c and D, which serve to receive two small axes, around which the legs and arms of the figure are made to play. At each extremity of the piece of wood there is also a small receptacle, of the form seen in the plate, viz, nearly concentric with the holes c and D; having an oblique prolongation towards the middle of the piece of wood, and from the ends of these two prolongations proceed two grooves eg and ef, formed in the thickness of the wood, and nearly a line in diameter.

Quicksilver being put into one of these receptacles till it is nearly full, they are both closed up by means of very

light pieces of pasteboard, applied on the sides. To the axis, passing through one of the holes C, are affixed two supporters, cut into the form of legs, with feet somewhat lengthened, to give them more stability. And to the other axis, passing through D, are affixed two supporters, shaped like arms, with their hands placed in such a manner as to become a base, when the machine is turned backwards. In the last place, to the part GH is applied a sort of head and visage, made of the pith of the elder tree, and dressed after the manner of tumblers. A belly is constructed of the same substance, and the whole figure is clothed in a silk dress, which descends to the middle of the thighs. Having thus given a general account of the construction of this small machine, we shall now proceed to explain its mode of action.

Let us first suppose the machine to be placed upright on its legs, as seen fig. 48, or 49 No. 1. As all the weight is on one side of the axis of rotation C, because the receptacle of the quicksilver on that side is filled, the machine must incline to that side, and would be thrown entirely backwards, did not the arms or supporters, turning around the axis D, present themselves in a vertical direction; but as they are shorter than the legs, the machine assumes the position represented fig. 49 No. 2; and the quicksilver, finding the small groove Gg, inclined to the horizon, flows with impetuosity into the receptacle placed on the side D.

Let us now suppose that at this moment the machine rests on the supports or arms DL, which turn round the axis D: it is evident that, if the empty part of the machine is very light, the quicksilver being entirely beyond the point of rotation D, will, by its considerable preponderance overcome it, and cause the machine to revolve round the axis D, which will raise it, and make it turn on the other side. But as the supporters CK must necessarily be longer than the others DL, that the line CD may have the inclination which is necessary to cause the quicksilver to flow by

the small groove $r f$, from the one receptacle to the other; the base must make a jump double in height to the difference of these supporters; otherwise the line $r f$, instead of assuming a horizontal position, would remain inclined in a direction contrary to that which it ought to have.

The machine having then attained to the situation DE, fig. 49 n°. 3, and the quicksilver having passed into the receptacle on the side c; it is evident that the same mechanism which will raise it up, by making it turn round the point c, will overturn it on the other side, where the two supporters, which revolve round the axis c, present it a base: this will make it resume the position of fig. 49 n°. 2: and so on. Hence this motion will be perpetual, as long as the machine meets with steps like the first.

Remarks.—Some particular conditions are required in order that the supporters of the small figure, that is, its legs and arms, may present themselves in a proper manner, to keep it in the position in which it ought to be.

1st. It is necessary that the great supporters, or legs, when they have arrived at that point at which the figure, after having thrown itself topsy turvy, rests upon them, should meet with some obstacle, to prevent them or the figure from turning any more: this may be done by two small pegs, which meet a prolongation of the thighs.

2d. While the figure is raising itself on its legs, it is necessary that the arms should perform, on their axis, a semi-revolution; that they may present themselves perpendicular to the horizon, and in a firm manner, when the figure throws itself backward. This may be accomplished by furnishing the arms of the figure with two small pulleys, concentric to the axis of the motion of these arms, over which are conveyed two silk threads, that unite under the belly of the figure, and are fixed to a small cross bar, joining the thighs towards the middle: this will greatly contribute to their stability. These threads must be lengthened, or shortened, till this semi-revolution of the

arms is exactly performed; and until the figure, when placed on its four supporters, with its face turned either up or down, does not waver; which it would do if these supporters were not bound together in this manner; and if the large ones, or legs, did not meet with an obstacle to prevent them from inclining any farther.

PROBLEM XLVII.

To arrange three sticks, on a horizontal plane, in such a manner, that while the lower extremities of each rest on that plane, the other three shall mutually support each other in the air.

This depends merely on a little mechanical address, and may be performed in the following manner.

Take the first stick AB (fig. 50 pl. 11), and rest the end A on the table, holding the other raised up, so that the stick shall be inclined at a very acute angle. Place above it the second stick, with the end C resting on the table. And then dispose the third stick EF, in such a manner, that while the end E rests on the table, it shall pass below the stick AB, towards the upper end B, and rest on the stick CD. These three sticks, by this arrangement, will be so connected with each other, that the ends D, B and F will necessarily remain suspended, each supporting the other.

PROBLEM XLVIII.

To construct a cask, into which if three different kinds of liquor be poured, they can be drawn off at pleasure by the same cock, without being mixed.

For this purpose, the cask must be divided into three partitions, or cells, A, B, C, (fig. 51 pl. 11), intended to contain the three different liquors; such as red wine, white wine, and water; which may be introduced each into its proper cell, by the same bung, in the following manner.

In constructing the cask, a funnel D, with three pipes

E, F, G, each conveyed into its cell, must be fitted into the bung; and within this funnel must be placed another H, pierced with three holes, which may be made to correspond, at pleasure, with the apertures of each pipe. If each hole, in the interior funnel H, be made to correspond, in succession, by turning it, with the aperture of the pipe to which it belongs, the liquor poured into the funnel H, will pass into that pipe. In this manner each cell may be filled with the liquor intended for it, without one of them being able to mix with the rest; because when one pipe is open, the other two are shut.

But, to draw off each liquor also, without confusion, at the bottom of the cask, there must be three other pipes K, L, M, each corresponding to a cell; and a kind of cock IN, pierced with three holes, each corresponding to its pipe, that by turning the stopper of the cock I, until one of these holes is brought opposite to its pipe, the liquor of the cell, to which that pipe belongs, may issue alone through it.

PROBLEM XLIX.

To make a soft body, such as the end of a candle, pierce a board.

Load a musket with powder, and instead of a ball put over it the end of a candle; if you then fire it against a board, not very thick, the latter will be pierced by the candle-end, as if by a ball.

The cause of this phenomenon, no doubt, is that the rapid motion with which the candle-end is impelled, does not allow it time to be flattened, and therefore it acts as a hard body. It is the effect of the inertia of the parts of matter, as may be easily proved by experiment. Nothing is easier to be divided than water; yet if the palm of the hand be struck with some velocity against the surface of water, a considerable degree of resistance, and even of pain, is experienced from it, as if a hard body had been

struck. Nay, a musket ball, when fired against water, is repelled by it, and even flattened. If the musket is fired with a certain obliquity, the ball will be reflected; and after this reflection is capable of killing any person who may be in its way. This arises from a certain time being necessary to communicate to any body a sensible motion. When a body then moving with great velocity, meets with another of a size much more considerable, it experiences almost as much resistance as if the latter were fixed.

PROBLEM L.

To break a stick, placed on two drinking glasses, by striking it with another stick, and without breaking the glasses.

We give this problem, and the solution of it, merely because it is found in all the editions of the Mathematical Recreations; but, to speak the truth, those who attempt to perform it, ought to supply themselves with plenty of glasses. However, the solution of it, whether real or false, is as follows.

The stick, intended to be broken, must neither be thick, nor rest with any great hold on the two glasses. Both its extremities must be made to taper to a point; and it ought to be of as uniform a size as possible, in order that its centre of gravity, which in this case will be in the middle, may be more easily known.

The stick, supposed to possess all the above properties, must be placed with its two extremities resting on the edges of the glasses, which ought to be perfectly level, that the stick may remain horizontal, and not inclined to one side more than another. Care also must be taken that the points only shall rest lightly on the edge of each glass. If a speedy and smart blow, but proportioned, as far as can be judged, to the size of the stick and the distance of the glasses, be then given to it in the middle, it will break in two, without either of the glasses being injured.

REMARK.—We are far from warranting the certainty

of this effect: for, in our opinion, those who try the experiment, will break many glasses before they break the stick. There is however a physical reason, which renders the success possible; and this reason is the same as that which causes a musket ball to pierce a weather-cock, or a door moveable on its hinges. The stick indeed, when struck in the middle with a smart and sudden blow, cannot, on account of its mass, immediately acquire that motion necessary to yield to the impetuosity of the blow: it is strongly retained, as it were, by its extremities; and in that case will assuredly be broken. We must however repeat, that we would not advise any one to try the experiment, unless furnished with plenty of glasses.

It might be tried however, in a manner less expensive, by making the extremities of the stick, destined to be broken, to rest on two small bits of wood, fixed perpendicularly on a stool or board. A person, after being exercised in this manner, might perform the experiment with all that appearance of the marvellous, which it acquires when the stick is made to rest on two glasses.

PROBLEM LI.

On the principles by which the possible effect of a machine can be determined.

It is customary for quacks, and those who have not a sufficient knowledge of mechanics, to ascribe to machines prodigious effects, far superior to such as are consistent with the principles of sound philosophy. It may therefore be of utility to explain here those principles by which we ought to be guided, in order to form a rational opinion, respecting any proposed machine.

Whatever may be the construction of a machine, even supposing it to be mathematically perfect, that is immaterial and without friction, its effect, that is to say, the weight put in motion, multiplied by the perpendicular height to which it may be raised, in a determinate time,

cannot exceed the product of the moving power, multiplied by the space it passes over in the same time. Consequently, since every machine is material, and as it is impossible to get entirely rid of friction, which will necessarily destroy a part of the power, it is evident that the first product will always be less than the latter. Let us apply this to an example.

Should a person propose a machine, which by the strength of one man applied to a crank, or the lever of a capstan, shall raise in an hour 3500 gallons of water, to the height of 24 feet; we might tell him, that he was ignorant of the principles of mechanics.

For the strength of a man applied to a crank, or to draw or push any weight, is only equal to about 26 or 28 pounds, with a velocity at most of 11000 feet per hour; and he could labour no more than 7 or 8 hours in succession. Now, as the product of 11000 by 28 is 308000, if this product be divided by 24, the height to which the water is to be raised, the quotient will be 12833 pounds of water, or 206 cubic feet = 1540 gallons raised to that height; which makes about 60 gallons, per minute, to the height of 10 feet. This is all that could be produced by such a power in the most favourable case. But the more complex the machine, the greater is the resistance to be surmounted; so that the product would never be nearly equal to the above effect.

In a machine, where a man should act by his own weight, and in walking, the advantage would not be much greater: for all that a man could do by walking, without any other weight than that of his body, on a plane inclined at an angle of 30 degrees, would be to pass over 6000 feet per hour, especially if he had to walk in this manner for 7 or 8 hours. But here it is the perpendicular height alone, which in this case is 3000 feet, that is to be considered; the product of 3000 by 150 pounds, which is the average weight of a man, is 450000; the greatest effect therefore,

of such a machine, would be 450000 pounds, raised to the height of one foot, or 18750 to the height of 24 feet, or about 90 gallons, per minute, to the height of 10 feet. By taking an arithmetical mean between this determination and the preceding, it will be found that the mean product possible of the strength of a man, employed to put in motion a hydraulic machine, is at most 75 gallons per minute; especially if continued for 7 or 8 hours in the day.

If the power were to act only for a very short time, as 3, 4, or 5 minutes, the product indeed might appear more considerable, and about double. This is one of the artifices employed by mechanicians, to prove the superiority of their machines. They put them in motion for some minutes, by vigorous people, who make a momentary effort, and thus cause the product to appear much greater than it really is.

The above determination agrees pretty well with that given by Desaguliers in his Treatise on Natural Philosophy: for he assured himself, he says, by calculation, that the effect of the simplest and most perfect machines, put in motion by men, never gives, in the ratio of each man, above 72 gallons of water per minute raised to the height of 10 feet.

A circumstance, very necessary to be known in regard to machines which are to be moved by horses, is as follows: a horse is equal to about seven men*, or can make an effort in a horizontal direction of 210 pounds, moving with the velocity of 10000 or 11000 feet per hour, supposing he is

* C. Regnier, in his description of the Dynanometer, an instrument invented by him for the purpose of determining the relative strength of men and horses, published in the Journal de l'Ecole Polytechnique, vol. 2. p. 160, says, that from the result of all his experiments it appears, that the mean term of the maximum of the strength of ordinary men, to raise a weight, is about 265 pounds avoirdupois; which agrees with the experiments of Delahire, but which Desaguliers considered as too small. In regard to horses, he says, that by taking the mean results given by 4 horses, of the middle

to work 8 or 10 hours per day. Desaguliers even gives less, and thinks that the force of a man is to be only quintupled to find that of the horse.

Those who are acquainted with these principles, will run no risk of being deceived by ignorant or pretended mechanicians; and it is no small advantage to be able to avoid becoming the dupe of such men, whose aim is often to pick the pockets of those who are so simple as to listen to them.

PROBLEM LII.

Of the Perpetual Motion.

The perpetual motion has been the quicksand of mechanicians, as the quadrature of the circle, the trisection of an angle, &c, have been that of geometricians: and as those who pretend to have discovered the solution of the latter problems are, in general, persons scarcely acquainted with the principles of geometry, those who search for, or imagine they have found, the perpetual motion, are always men to whom the most certain and invariable truths in mechanics are unknown.

size, subjected to trial one after the other, the strength of ordinary horses may be estimated at 794 pounds averdupois.

In comparing the relative force of men with that of horses, when the former draw a cart or a boat by the help of a rope, after various trials, he found that the maximum of the strength of ordinary men, in dragging a horizontal weight, by the help of a rope, is equal to 110 pounds averdupois, and that of the strongest does not exceed 132 pounds averdupois. These different trials agree pretty well with the general received opinion, that a horse is 7 times as strong as a man This principle, however, cannot be admitted in all cases; for it is known by experiment that a horse would sink under a burden, 7 times as heavy as that which a man can support when standing upright. It may readily be conceived that what has been here said respecting men and horses, is not applicable to daily and incessant labour; but we may deduce from it this very just consequence, that both can act for a whole day, when employing a 5th of their absolute forces. According to the above results therefore, the power which an ordinary man can exert for a continuance in dragging or pulling, is equal to no more than about 22 pounds, and that of the strongest to about 26 pounds.

It may be demonstrated indeed, to all those capable of reasoning in a sound manner on those sciences, that a perpetual motion is impossible: for, to be possible, it is necessary that the effect should become alternately the cause, and the cause the effect. It would be necessary, for example, that a weight, raised to a certain height by another weight, should in its turn raise the second weight to the height from which it descended. But, according to the laws of motion, all that a descending weight could do, in the most perfect machine which the mind can conceive, is to raise another in the same time to a height reciprocally proportional to its mass. But it is impossible to construct a machine in which there shall be neither friction nor the resistance of some medium to be overcome; consequently, at each alternation of ascent and descent, some quantity of motion, however small, will always be lost: each time therefore, the weight to be raised will ascend to a less height; and the motion will gradually slacken, and at length cease entirely.

A moving principle has been sought for, but without success, in the magnet, in the gravity of the atmosphere, and in the elasticity of bodies. If a magnet be disposed in such a manner as to facilitate the ascension of a weight, it will afterwards oppose its descent: Springs, after being unbent, require to be bent by a new force equal to that which they exercised; and the gravity of the atmosphere, after forcing one side of the machine to the lowest point, must be itself raised again, like any other weight, in order to continue its action.

We shall however give an account of various attempts to obtain a perpetual motion, because they may serve to show how much some persons have suffered themselves to be deceived on this subject.

Fig. 52 pl. 12, represents a large wheel, the circumference of which is furnished, at equal distances, with levers,

each bearing at its extremity a weight, and moveable on a hinge, so that in one direction they can rest upon the circumference, while on the opposite side, being carried away by the weight at the extremity, they are obliged to arrange themselves in the direction of the radius continued. This being supposed, it is evident that when the wheel turns in the direction *a b c*, the weights A, B and C will recede from the centre; consequently, as they act with more force, they will carry the wheel towards that side; and as a new lever will be thrown out, in proportion as the wheel revolves, it thence follows, say they, that the wheel will continue to move in the same direction. But, notwithstanding the specious appearance of this reasoning, experience has proved that the machine will not go: and it may indeed be demonstrated that there is a certain position, in which the centre of gravity of all these weights is in the vertical plane passing through the point of suspension, and that therefore it must stop.

The case is the same with the following machine, which it would appear ought to move also incessantly. In a cylindric drum, in perfect equilibrium on its axis, are formed channels as seen in fig. 53, which contain balls of lead, or a certain quantity of quicksilver. In consequence of this disposition, the balls or quicksilver must, on the one side, ascend by approaching the centre; and on the other must roll towards the circumference. The machine then ought to turn incessantly towards that side.

A third machine of this kind is represented fig. 54. It consists of a kind of wheel formed of six or eight arms, proceeding from a centre, where the axis of motion is placed. Each of these arms is furnished with a receptacle in the form of a pair of bellows; but those on the opposite arms stand in contrary directions, as seen in the figure. The moveable top of each receptacle has affixed to it a weight, which shuts it in one situation and opens it in the

other. In the last place, the bellows of the opposite arms have a communication by means of a canal, and one of them is filled with quicksilver.

These things being supposed, it is visible, that the bellows on the one side must open, and those on the other must shut; consequently the mercury will pass from the latter into the former, while the contrary will be the case on the opposite side.

It might be difficult to point out the deficiency of this reasoning; but those acquainted with the true principles of mechanics will not hesitate to bet a hundred to one that the machine, when constructed, will not answer the intended purpose.

The description of a pretended perpetual motion, in which bellows, to be alternately filled with and emptied of quicksilver, were employed, may be seen in the Journal des Sçavans for 1685. It was refuted by Bernoulli, and some others, and it gave rise to a long dispute. The best method which the inventor could have employed to defend his invention, would have been to construct it, and show it in motion; but this was never done.

We shall here add another curious anecdote on this subject. One Orfyreus announced, at Leipsic, in the year 1717, a perpetual motion, consisting of a wheel, which would continually revolve. This machine was constructed for the landgrave of Hesse-Cassel, who caused it to be shut up in a place of safety, and the door to be sealed with his own seal. At the end of 40 days, the door was opened, and the machine was found in motion. This however affords no proof in favour of a perpetual motion; for as clocks can be made to go a year without being wound up, Orfyreus's wheel might easily go 40 days, and even more.

The result of this pretended discovery is not known: we are informed, by one of the journals, that an Englishman offered 80000 crowns for this machine; but Orfyreus refused to sell it at that price; in this he certainly acted

wrong, as there is reason to think that he obtained by his invention, neither money, nor even the honour of having discovered the perpetual motion.

The Academy of Painting, at Paris, possessed a clock, which had no need of being wound up, and which might be considered as a perpetual motion, though it was not so. But this requires some explanation. The ingenious author of this clock employed the variations in the state of the atmosphere, for winding up his moving weight: various artifices might be devised for this purpose; but this is no more a perpetual motion, than if the flux and reflux of the sea were employed to keep the machine continually going; for this principle of motion is exterior to the machine, and forms no part of it.

But enough has been said on this chimera of mechanics. We sincerely hope that none of our readers will ever lose themselves in the ridiculous and unfortunate labyrinth of such a research.

To conclude, it is false that any reward has been promised by the European powers to the person who shall discover the perpetual motion; and the case is the same in regard to the quadrature of the circle. It is this idea, no doubt, that excites so many to attempt the solution of these problems; and it is proper they should be undeceived.

PROBLEM LIII.

To determine the height of the arched ceiling of a church, by the vibrations of the lamps suspended from it.

For this invention we are indebted, it is said, to Gallileo, who first ascertained the ratio of the duration of the oscillations made by pendulums of different lengths*. But

* Indeed, it seems it was by that author accidentally observing the uniformity in the intervals of the swing of the suspended lamps, that he first took the hint of employing the oscillations of pendulous bodies, or pendulums, for the purpose of measuring time. And hence the invention of pendulum clocks.

in order that this method may have a certain degree of exactness, the weight of the lamp ought to be several times greater than that of the cord by which it is supported.

This being supposed, put the lamp in motion by removing it a very little from its perpendicular direction, or carefully observe that communicated to it by the air, whch is very common; and with a stop-watch find how many seconds one vibration continues, or, if a stop-watch is not at hand, count the number of vibrations performed in a certain number of minutes: the greater the number of minutes, the more exact will the duration of each vibration be determined; for nothing will then be necessary, but to divide those minutes by the number of vibrations, and the quotient will be the duration of each in minutes or seconds.

We shall here suppose that it has been found, by either of these methods, that the time of each vibration is $5\frac{1}{2}$ seconds; square $5\frac{1}{2}$, which is $30\frac{1}{4}$, and multiply by it $39\frac{1}{8}$ inches, the length of a pendulum that swings seconds in the latitude of London, the product will be 98 ft. 7 inc. 6 lin., which will be nearly the height from the point of suspension to the bottom or rather centre of the lamp.

If the distance from the bottom of the lamp to the pavement be then measured, which may be done by means of a stick, and added to the former result, the sum will give the height of the arch above the pavement.

This solution is founded on a property of pendulums, demonstrated in mechanics; which is, that the squares of the times of the vibrations are as the lengths; so that a pendulum 4 times the length of another, performs vibrations which last twice as long.

But on account of the irregular form of the lamp, and the weight of the rope, which sustains it, we must confess that this method is rather curious than exact. We shall however present the reader with another problem of the same kind.

PROBLEM LIV.

To measure the depth of a well, by the time elapsed between the commencement of the fall of a heavy body, and that when the sound of its fall is conveyed to the ear.

Have in readiness a small pendulum that swings half seconds, that is, $9\frac{3}{4}\frac{1}{2}$ inches in length, between the centre of the ball, and the point of suspension. You must also employ a weight of some substance as heavy as possible, such for example as lead; as a common stone or pebble experiences a considerable retardation in falling, and therefore would not answer the purpose so well.

Let go the weight and the ball of the pendulum at the same moment of time, and count the number of the vibrations the latter makes, till the moment when you hear the sound. We shall here suppose that there were 10 vibrations, which make 5 seconds.

As a heavy body near the earth's surface falls about $16\frac{1}{12}$ feet in one second of time, or for this purpose 16 feet will be exact enough; and as sound moves at the rate of 1142 feet per second; multiply together 1142, 16 and 5, which will give 91360, and to four times this product, or 365440, add the square of 1142, which is 1304164, and the sum will be 1669604; then if from the square root of the last number = 1292 the number 1142 be subtracted, the remainder 150 divided by 32 will give 4·69 for the number of seconds which elapsed during the fall of the body; if this remainder be subtracted from 5, the number of seconds during which the body was falling and the sound returning, we shall have 0·31 for the time which the sound alone employed before it reached the ear; and this number multiplied by 1142, will give for product 354 feet = the depth of the well.

This rule, which we must allow to be rather complex, is founded on the property of falling bodies, which are accelerated in the ratio of the times, so that the spaces passed

over increase as the squares of the times*. But as the resistance of the air, which in considerable heights, such as those of several hundred feet, does not fail to retard the fall in a sensible manner, has been neglected, the case of this problem is nearly the same as with the preceding; that is to say, the solution is rather curious than useful.

* For the sake of our algebraical readers we shall here show how to find the formula from which the above rule is deduced: Let $a=5$, $b=16\frac{1}{12}$, $c=1142$, and let x be the time which the body employs in falling, consequently $a-x$ will be the time of the sound returning. Then as $1^2 : b :: x^2 . bx^2 =$ depth of the well; and $1 : c :: a-x : ca-cx =$ depth of the well also; therefore $bx^2=ca-cx$, and by transposition and division, $x^2+\frac{c}{b}x=\frac{ca}{b}$. Completing the square, $x^2+\frac{c}{b}x+\frac{c^2}{4b^2}=\frac{ca}{b}+\frac{c^2}{4b^2}=\frac{4bca+c^2}{4b^2}$. Hence, $x+\frac{c}{2b}=\sqrt{\frac{4bca+c^2}{4b^2}}$ and $x=\sqrt{\frac{4bca+c^2}{4b^2}}-\frac{c}{2b}=\frac{\sqrt{(c^2+4abc)}-c}{2b}=$ nearly $\frac{ac}{ab+c}$ the time of descent. Consequently $a-\frac{ac}{ab+c}=\frac{a^2b}{ab+c}$ is nearly the time of the sound's ascent.

Hence, from the expression $\frac{ac}{ab+c}$ a much simpler rule is obtained for the time of the descent, which is as follows: Multiply 1142 by 5, which gives for product 5710, then multiply also 16 by 5 which gives 80, to which add 1142, this gives 1222, by which sum divide the first product 5710, and the quotient 4·68 will be the time of descent, nearly the same as before. This taken from 5 leaves 0·32 for the time of the ascent; which multiplied by 1142, gives 365 for the depth, differing but little from the former more exact number.

Historical account of some extraordinary and celebrated mechanical works.

An essential part might seem wanting to this work if we neglected to give some account of the various machines, most celebrated both among the ancients and moderns. We shall therefore take a cursory view of the rarest and most singular inventions, produced by mechanical genius, in different ages.

§ I. *Of the machines or automatons of Archytas, Archimedes, Hero and Ctesibius.*

Some machines of this kind are mentioned in ancient history, in terms of the utmost admiration. Such were the tripod automatons of Vulcan; and the dove of Archytas, which, as we are told, could fly like a real animal. We have no doubt however, that the wonderful properties of these machines, if they ever really existed, have been greatly exaggerated by credulity; and by the accounts of them being handed down through such a long series of ages. We are told also of the moving sphere of Archimedes, in which, as appears, that celebrated philosopher had represented all the celestial motions, as they were then known; and this, no doubt, was a master-piece of mechanism for that remote period. Every one is acquainted with the famous verses of Claudian on this machine.

Several wonderful machines were constructed also by Hero and Ctesibius of Alexandria. An account of some of those invented by Hero may be seen in a book called Spiritalia. Some of them are very ingenious, and do honour to the talents of that mechanician.

§ II. *Of the machines ascribed to Albert the Great, and to Regiomontanus.*

That ignorance, in the darkness of which all Europe was

involved, from the 6th or 7th century to the 15th, did not entirely extinguish mechanical genius. We are told that the embassadors sent by the king of Persia to Charlemagne brought, as a present to the latter, a machine, which, according to the description given of it, would have done honour to our modern mechanicians; for it appears to have been a striking clock, which had figures that performed various movements. It is indeed true that, while Europe was immersed in ignorance, the arts and sciences diffused a gleam of light among the nations of the East. In regard to those of the West, if we can believe what is related of Albert the Great, who lived in the 13th century, that mathematician constructed an automaton in the human form, which when any one knocked at the door of its cell, came to open it, and sent forth some sounds, as if addressing the person who entered. At a period later by some centuries, Regiomontanus, or John Muller of Konigsberg, a celebrated astronomer, constructed an automaton in the figure of a fly, which walked around a table. But these accounts are probably very much disfigured by ignorance and credulity. The following however are instances of mechanical skill, in which there is much more of reality.

§ III. *Of various celebrated Clocks.*

In the 14th century, James Dondi constructed for the city of Padua a clock, which was long considered as the wonder of that period. Besides indicating the hours, it represented the motion of the sun, moon, and planets, as well as pointed out the different festivals of the year. On this account, Dondi got the surname of Horologio, which became that of his posterity. A little time after, William Zelandin constructed, for the same city, one still more complex; which was repaired in the 16th century by Janellus Turrianus, the mechanician of Charles the 5th.

But the most celebrated works of this kind are the clocks of the cathedrals of Strasburgh and Lyons. That of Stras-

burgh was the work of Conrad Dasypodius, a mathematician of that city, who lived towards the end of the 16th century, and who finished it about the year 1573. It is considered as the first in Europe. At any rate there is none but that of Lyons which can dispute pre-eminence with it, or be compared to it in regard to the variety of its effects.

The face of the basement of the clock of Strasburgh exhibits three dial-plates; one of which is round, and consists of several concentric circles; the two interior ones of which perform their revolutions in a year, and serve to mark the days of the year, the festivals, and other circumstances of the calendar. The two lateral dial-plates are square, and serve to indicate the eclipses, both of the sun and the moon.

Above the middle dial-plate, and in the attic space of the basement, the days of the week are represented by different divinities, supposed to preside over the planets from which their common appellations are derived. The divinity of the current day appears in a car rolling over the clouds, and at midnight retires to give place to the succeeding one.

Before the basement is seen a globe, borne on the wings of a pelican, around which the sun and moon revolved; and which in that manner represented the motion of these planets; but this part of the machine, as well as several others, has been deranged for a long time.

The ornamented turret, above this basement, exhibits chiefly a large dial, in the form of an astrolabe; which shows the annual motion of the sun and moon through the ecliptic, the hours of the day &c. The phases of the moon are seen also marked out on a particular dial-plate above.

This work is remarkable also for a considerable assemblage of bells and figures, which perform different motions. Above the dial-plate last mentioned, for example, the four ages of man are represented by symbolical figures: one passes every quarter of an hour, and marks the quarter by striking on small bells; these figures are followed by

death, who is expelled by Jesus Christ risen from the grave; who however permits it to sound the hour, in order to warn man that time is on the wing. Two small angels perform movements also; one striking a bell with a sceptre, while the other turns an hour-glass, at the expiration of an hour.

In the last place, this work was decorated with various animals, which emitted sounds, similar to their natural voices; but none of them now remain except the cock, which crows immediately before the hour strikes, first stretching out its neck and clapping its wings. The voice of this figure however is become so hoarse as to be much less harmonious than the voice of that at Lyons, though the latter is attended, in a considerable degree, with the same defect. It is to be regretted that a great part of this machine is entirely deranged. It would be worthy of the illustrious metropolitan chapter of Strasburgh to cause it to be repaired: we have heard indeed that it has been attempted, but that no artist could be found capable of performing it.

The clock of the cathedral of Lyons is of less size than that of Strasburgh; but is not inferior to it in the variety of its movements; and it has the advantage also of being in a good condition. It is the work of Lippius de Basle, and was exceedingly well repaired in the last century by an ingenious clock-maker of Lyons, named Nourisson. Like that of Strasburgh, it exhibits, on different dial-plates, the annual and diurnal progress of the sun and moon, the days of the year, their length, and the whole calendar, civil as well as ecclesiastical. The days of the week are indicated by symbols more analogous to the place where the clock is erected; the hours are announced by the crowing of the cock, three times repeated, after it has clapped its wings, and made various other movements. When the cock has done crowing, angels appear, who, by striking various bells, perform the air of a hymn; the annunciation of the

Virgin is represented also by moving figures, and by the descent of a dove from the clouds; and after this mechanical exhibition, the hour strikes. On one of the sides of the clock is seen an oval dial-plate, where the hours and minutes are indicated by means of an index, which lengthens or contracts itself, according to the length of the semi-diameter of the ellipsis over which it moves.

A very curious clock, the work of Martinot, a celebrated clock-maker of the 17th century, was to be seen in the royal apartments at Versailles. Before it struck the hour, two cocks on the corners of a small edifice crowed alternately, clapping their wings; soon after two lateral doors of the edifice opened, at which appeared two figures bearing cymbals, beaten on by a kind of guards with clubs. When these figures had retired, the centre door was thrown open, and a pedestal, supporting an equestrian statue of Louis 14th, issued from it, while a group of clouds separating gave a passage to a figure of Fame, which came and hovered over the statue. An air was then performed by bells; after which the two figures re-entered; the two guards raised up their clubs, which they had lowered as if out of respect for the presence of the king, and the hour was then struck. Though all these things are easy for ingenious clock-makers of the present day, when we come to treat of Astronomy, we shall give an account of some machines of this kind, purely astronomical, which do honour to the inventive genius of those by whom they were constructed.

§ IV. *Automaton machines of Father Truchet, M. Camus, and M. de Vaucanson.*

Towards the end of the 17th century, Father Truchet, of the royal Academy of Sciences, constructed for the amusement of Louis 14th, moving pictures, which were considered as very remarkable master pieces of mechanics. One of these pictures, which that monarch called his little

opera, represented an opera of five acts, and changed the decorations at the commencement of each. The actors performed their parts in pantomime. The representation could be stopped at pleasure; this effect was produced by letting go a catch, and by means of another the scene could be made to re-commence at the place where it had been interrupted. This moving picture was 16 inches and a half in breadth, 13 inches 4 lines in height, and 1 inch 3 lines in thickness, for the play of the machinery. An account of this piece of mechanism may be found in the eulogy on Father Truchet, published in the Memoirs of the Academy of Sciences, for the year 1729.

Another very ingenious machine, and in our opinion much more difficult to be conceived, is that described by M. Camus, a gentleman of Lorrain, who says he constructed it for the amusement of Louis 14th, when a child. It consisted of a small coach drawn by two horses, in which was the figure of a lady, with a footman and page behind. If we can give credit to what is stated in the work of M. Camus, this coach being placed at the extremity of a table of a determinate size, the coachman smacked his whip, and the horses immediately set out, moving their legs in the same manner as real horses do. When the carriage reached the edge of the table, it turned at a right angle, and proceeded along that edge. When it arrived opposite to the place where the king was seated, it stopped, and the page getting down opened the door, upon which the lady alighted, having in her hand a petition which she presented with a curtsey. After waiting some time, she again curtsied, and re-entered the carriage; the page then resumed his place, the coachman whipped his horses, which began to move, and the footman, running after the carriage, jumped up behind it.

It is much to be regretted that M. Camus, instead of confining himself to a general account of the mechanism, which he employed to produce these effects, did not enter

into a more minute description; for, if they are true, it must have required a very singular artifice to produce them, and the same means might be applied to machines of greater utility.

About 30 or 35 years ago, three very curious machines were exhibited by M. de Vaucanson, viz, an automaton flute-player, a player on the flageolet and tambourine, and an artificial duck. The first played several airs on the flute, with a precision greater perhaps than was ever attained to by the best living player, and even executed the tonguing, which serves to distinguish the notes. According to M. de Vaucanson, this part of the machinery cost him the greatest trouble. In short, the tones were really produced in the flute by the proper motion of the fingers.

The player on the flageolet and tambourine performed also some airs on the first of these instruments, and at the same time kept continually beating on the latter.

But the motion of the artificial duck, in our opinion, was still more astonishing; for it extended its neck, raised up its wings, and dressed its feathers with its bill; it picked up barley from a trough, and swallowed it; drank from another, and, after various other movements, voided some matter resembling excrements. The first time I saw these machines I immediately discovered some of the artifices employed in regard to the two former, but I confess that the latter baffled my penetration.

We have also of late been amused, by M. Droz and M. Maillardet, &c, with the surprising performances of the chess-players, the small but sweet singing-bird, the writing figure, the musical lady, the conjurer, the tumbler, &c, &c.

§ V. *Of the Machine at Marly.*

It will doubtless be allowed, that the machines above mentioned are, in general, more curious than useful; but there are other two, the celebrity and utility of which re-

quire that we should here give them a place. These are the machine of Marly, and that known under the name of the steam engine. We shall begin with the former, of the construction and effects of which the following brief description will give some idea.

The machine of Marly consists of 14 wheels, each about 36 feet in diameter, moved by a stream of water, confined by an estacade, and received into so many separate channels. Each wheel has at the extremities of its axis, two cranks, and this forms 28 powers, distributed in the following manner.

It must however be first observed, that the water is raised, to the place to which it is to be conveyed, by three different stages; first from the river to a reservoir, at the elevation of 160 English feet above the level of the Seine; then to a second reservoir 346 feet higher; and from the latter to the summit of a tower, somewhat more than 533 feet above the river.

Of the 28 cranks, above mentioned, 8 are employed to give motion to 64 pumps; which is done by means of working beams, having 4 pistons at each extremity of their arms: this makes 8 to each working beam, which are drawn up and pushed down alternately. These 64 pumps force up the water to the first reservoir; and this reservoir furnishes water to the first well, on which is established the second set of pumps.

Eleven more cranks are employed to force the water from the first well to the second reservoir. This is done by means of long arms adapted to these cranks, which move large frames, to one of the arms of which are attached strong iron chains, that extend from the bottom of the mountain to the first well. These chains, called chevalets, are formed of parallel bars of iron, the extremities of which are bound together by iron bolts, and are supported at certain intervals by transversal pieces of wood, moveable on an axis, that passes through the middle of

each; so that when the upper bar of iron, for example, is drawn down by the lower end, all these pieces of wood incline in one direction, and the lower part moves backwards and pushes in a direction contrary to the upper one. These bars or chains serve to put in motion the working beams, or squares, and the latter move the pistons of 80 sucking and forcing pumps, which raise the water from the first well to the second reservoir.

In the last place, 9 other cranks, by a similar mechanism, put in motion those chains, called the grands chevalets, which move the pumps of the second well, and raise the water from it to the summit of the tower. These pumps are in number 72.

Such, in a few words, is the mechanism of the machine of Marly. Its mean product, as said, is from 30000 to 40000 gallons of water, per hour. We make use of the term mean product, because at certain times it raises 60000 gallons, but only under very favourable circumstances. During inundations, when the Seine is frozen, when the water is very low, or when any repairs are making, the machinery stops either entirely, or in part. We have read that in the year 1685 it raised 70000 gallons per hour; but this we can scarcely believe; if by that quantity is understood its mean product; as it would be above 1000 gallons per minute.

However this may be, the following calculation is founded on details collected on purpose. The annual expence of the machine, including the salaries of those who superintend it, and the wages of the workmen employed, together with repairs, necessary articles, &c, may amount to about 3300*l* sterling, or 9*l*. per day; which makes about 1 farthing per 90 gallons. But if we take into this account, the interest of the 333000*l*. which, it is said, were expended in the construction of it, 90 gallons will cost 3 halfpence, which is at the rate of a farthing for 15 gallons. This is very far from the price which the King of Denmark thought

he might set on this water; for that prince, when he paid a visit to Marly, in the year 1769, being astonished, no doubt, at the immensity of the machine, the multitude of its movements, and the number of the workmen it employed, he observed that the water perhaps cost as much as wine. By the above calculation the reader may see how far his majesty was mistaken.

It is an important question to know, whether the machine at Marly could be simplified. On this subject we shall give a few observations, which from some experiments made, and a minute examination of the different parts of the machine, appear to be founded on probability.

People in general are surprized that the inventor of this machine should cause the water, in some measure, to make two rests, before it is conveyed to the summit of the tower. It has been humorously said, that he no doubt thought the water would be too much fatigued to ascend to the perpendicular height of more than 533 feet, all at one breath. It is more probable that he thought his moving force would not be sufficient to raise the water to that height; but this is not agreeable to theory; for it is found by calculation, that the force of one crank is more than sufficient to raise a cylinder of water of that altitude, and above 8 inches in diameter. Able mechanicians however are of opinion, that though this be not impossible, to carry it into execution would be attended with great inconveniences, which it would be too tedious to explain.

But it appears certain at present, that the water might be raised in one jet to the second well. This results from two experiments, one made in 1738, and the other in 1775. In the first, M. Camus of the Royal Academy of Sciences, endeavoured to make the water rise in one jet to the tower: his attempt was not attended with success, but he made it rise to the foot of the tower, which is considerably higher than the second reservoir; hence it follows, that if he had confined himself to making the water rise in one jet to the

second reservoir, he would have succeeded. It is said that, during this experiment, the machine was prodigiously strained; that it was even found necessary to secure some parts of it with chains; that it required 24 hours to force it to that height, which is about 480 feet, and that it was not possible to make it go farther. The object of the second trial, made in 1775, was to raise the water only to the second well. It indeed ascended thither at different times, and in abundance; but the pipes were exceedingly strained at the bottom, so that several of them burst; and it was necessary to suspend and recommence the experiment several times. It is however evident that this arose from the age of the tubes and their want of strength, as they had not the proper thickness; a fault which might have been easily remedied. Here then we have one step towards the improvement of the machine; and it results from this trial, that the chains which proceed from the river to the first well, might be suppressed, and even the first well itself.

It still remains to be determined, whether the water could be made to ascend, in one jet, to the summit of the tower. This would be a very curious experiment; but no doubt difficult and expensive, because it would be necessary to make considerable changes in different parts of the machine; and even in the case of its succeeding, the water raised might perhaps be in such small quantity, that it would be better to retain the present mechanism.

It is probable that various improvements might be made in different parts of the machine. In several positions, the moving forces act only obliquely, which occasions a great loss of power, and must tend to render the machine less effectual. The form of the pistons, valves, and aspiration tubes, might perhaps admit also of some change. But as this is not the place for entering into these details, we shall proceed to the Steam Engine, of which we promised to give a short description.

§ VI. *Of the Steam Engine.*

The Steam Engine is that perhaps in which the genius of mechanism has been manifested in the highest degree; for no idea could be more happy than that of employing alternately, as moving powers, the expansive force of the steam of water, and the weight of the atmosphere. Such indeed is the principle of this ingenious machine, which is at present employed with so much success in pumping water from mines, and for a variety of other purposes in the arts and manufactures.

The first part of this machine is a large boiler, to the cover of which is adapted a hollow cylinder, 2, 3, or 4 feet in diameter. A communication is formed between the boiler and the cylinder by an aperture, capable of being opened or shut. Into this cylinder is fitted a piston, the rod of which is made fast to the extremity of one of the arms of a working beam, having at the extremity of its other arm, the weight to be raised, which is generally the piston of a sucking pump, adapted to raise water from a great depth. The whole must be combined in such a manner, that when the air or steam has free access into the cylinder, which communicates with the boiler, the weight alone of the apparatus affixed to the opposite arm shall be capable of raising that piston.

Let us now suppose the boiler filled with water to a certain height, and that it is brought to a state of complete ebullition, by a large fire kindled below the boiler. As a part of this water will continually rise in steam, when the communication between the boiler and the cylinder is opened, this vapour, which is elastic, will introduce itself into it, and raise the piston; as its force is equivalent to that of air. Let us suppose also that the piston, when it attains to a certain height, by means of some mechanism, which may be easily conceived, moves a certain part of the machine, which intercepts the communication between

the boiler and the cylinder; and in the last place, that by the same cause a jet of cold water is thrown beneath the bottom of the piston in the cylinder, so as to fall down through the vapour in the form of rain. At that moment the steam will be condensed into water; a vacuum will be formed in the cylinder, and consequently the piston will be then charged with the weight of the atmosphere above it, or a weight equivalent to a column of water of the same base and 32 feet in height. If the piston, for example, be 52 inches in diameter, as is the case in the steam-engines of Montrelais, near Ingrande, this weight will be equal to 29450 pounds: the piston will consequently be obliged to descend with a force equal to nearly 30000 pounds, and the other arm of the working beam, if it be of the same length, will act with an equal force to overcome the resistance opposed to it. When the piston has made this first stroke, the communication between the boiler and the cylinder is restored; the steam of the boiling water again enters it, and the equilibrium between the air of the atmosphere and the inside of the cylinder being re-established, the weight of the apparatus affixed to the other end of the working beam descends, and raises the piston; the same play as before is renewed; the piston again falls, and the machine continues to produce its effect.

It may be readily conceived, that we must here confine ourselves to this short sketch; for a long description and a variety of figures would be necessary to give a correct idea of the many different parts requisite to produce this effect; such as that which opens and shuts the communication between the boiler and the cylinder; that which injects cold water into the cylinder; those which serve to evacuate the air and water formed in the inside of the cylinder; the regulator necessary to prevent the steam, when it becomes too strong, from bursting the machine, &c. For farther details therefore we must refer the reader to those authors who have purposely treated of this machine;

such as Belidor in his *Architecture Hydraulique*, vol. II; Desaguliers, in his *Cours de Physique Experimentale*, vol. II; M. Prony, in his *Nouvelle Architecture Hydraulique*; and several others.

The machine here described is very different from that mentioned by Muschenbroek, in his *Cours de Physique Experimentale*. In the latter, the steam acts by its compression on a cylinder of water, which it causes to ascend. This requires steam highly elastic, and very much heated; but in this case there is great danger of the machine bursting. In the new machine, that above described, it is sufficient if the steam has the elasticity of the air: this it will acquire if the water boils only briskly; and therefore the danger of the machine bursting is not nearly so great: it is not even said that this accident ever happened to any of the large Steam-Engines, which have been long established.

The largest Steam Engine with which I am acquainted, is that of Montrelais, near Ingrande, which is employed in freeing the coal mines from water. The cylinder is 52½ inches in diameter*. It raises per hour, to the height of 652 feet, by eight different stages, 1145 cubic feet of water, or 10800 gallons; and as it is estimated, after deducting the time lost by putting it in motion, during accidental repairs, which are necessary from time to time, &c, that it works 22 hours in the 24, its daily effect is to raise, to the above height, and evacuate, 237600 gallons of water. In the same time it consumes about 266 cubic feet of coals. The other expences attending it must also be considerable.

In the same place is another machine which, in some respects, appears to be constructed on a better principle. Though the cylinder is only 34 inches in diameter, it raises, in 22 hours, to the same height, and at one jet, 22000 cubic feet; or about 165000 gallons, which is above

* In some Steam Engines in England the cylinder is 63, and even 72 inches in diameter, and their power is equal to that of 250 horses.

two-thirds of the quantity raised by the former, while the moving power, which is in the ratio of the squares of the diameters of the pistons, is only about $\frac{2}{5}$ of that of the other.

An attempt was made, some years ago, to employ the steam engine to move carriages, and an experiment on this subject was tried at the arsenal of Paris. The carriage indeed moved, but in our opinion this idea must be considered rather as ingenious, than susceptible of being put in practice. It would not be very agreeable to travellers to hear, behind them, the noise of a machine capable, if it should burst, of blowing them to atoms; and we much doubt whether this invention would meet with encouragement. A boat also which, it is said, could be made to move against the current by means of a Steam Engine, was seen for a long time in the middle of the Seine, opposite to Passy. Nothing less was hoped from this invention, than to be able to convey a boat, laden with merchandise, in two or three days, from Rouen to Paris; but scarcely was the machine in motion when the wheels, the float-boards of which were to serve as oars, were broke in pieces by the effect of the too violent and sudden impression they received. Such was the result of this attempt, the failure of which had been predicted by the greater part of those mechanicians, who had seen the preparations*.

REMARK.—As Montucla has given but a short and imperfect account of that truly noble English invention, we have subjoined the following brief history of it. The Steam Engine was invented by the marquis of Worcester, in the year 1655. And an account of it was printed in a little book, intitled, A Century of the Names and Scant-

* The learned and ingenious Earl Stanhope, however, has lately succeeded, in constructing a useful sailing vessel, which is impelled by the force of steam.

lings of such Inventions as at present I can call to mind, &c, in the year 1663.

In the 68th article of that work, the marquis describes the invention in the following words—"An admirable "and most forcible way to drive up water by fire. Not "by drawing or sucking it upwards, for that must be, as "the Philosopher calleth it, Intra Sphæram Activitatis, "which is but at such a distance; but this way hath no "bounds, if the vessel be strong enough; for I have taken "a piece of a whole cannon, whereof the end was burst, "and filled it three quarters full of water, stopping and "securing up the broken end, as also the touch hole, and "making a constant fire under it, within 24 hours it burst "and made a great crack; so that having a way to make "my vessels, so that they are strengthened by the force "within them, and the one to fill after the other. I have "seen the water run like a constant fountain stream 40 "feet high; one vessel of water rarefied by fire driveth "up 40 of cold water. And a man that tends the work "is but to turn two cocks, that one vessel of water being "consumed, another begins to force and refill with cold "water, and so successively, the fire being tended and "kept constant, which the self same person may likewise "abundantly perform in the interim between the necessity "of turning the said cocks."

But though the above description is a distinct and intelligible one, of the manner of applying steam for raising of water, and though it appears that Sir Samuel Morland, in the year 1682, wrote a treatise on the Steam Engine, yet no person, that I have heard of, attempted to erect a machine on these principles until the year 1699; when Captain Savary produced, the 14th of June in that year, a model which was worked before the Royal Society, at their weekly meeting at Gresham College. He afterwards published an account of this machine in the year 1702, in a work intitled The Miner's Friend.

In Savary's machine, the steam is used for making a vacuum in a vessel placed near to the water to be raised, and communicating with it by a pipe, which has a cock or valve adapted to it. This valve or cock being opened when there is a vacuum in the vessel, the atmosphere presses the water into the vessel; and when this is filled, the valve or cock is shut; and steam being let into it, this presses on the surface of the water, and forces it upwards through a pipe adapted to the vessel for this purpose.

The disadvantages attending this method of construction were so great, that Captain Savary never succeeded further, than in making some engines for the supply of gentlemen's seats; but he did not succeed for mines, or the supplying of towns with water. This discouragement stopped the progress and improvement of the Steam Engine, till Mr Newcomen, an Ironmonger, and John Ceudley, a Glazier at Dartmouth, about the year 1712, invented what is called the Lever or Newcomen engine. In this machine, the steam is made to act in a cylinder distinct from the pumps, and is used merely for the purpose of making and unmaking a vacuum, in this manner, namely, there is a piston in the cylinder, fitted so nicely to it, that it can slide easily up and down without the admission of any air, or other fluid, to pass between its edge and the cylinder. The steam is admitted below the piston, which, being of a strength equal to the atmosphere, brings it into a state of equilibrium, when the weight of the pump rods and volumes of water, at the other end of the lever or balance, raises it up, when the piston has got to the top of the cylinder, a jet of cold water is thrown among the steam, which condenses it, and forms a partial vacuum. The atmosphere then acting on the upper side of the piston, forces it down, and raises the column of water at the other end of the beam.

No improvement on this principle took place for above half a century, except in the construction of a variety of

contrivances for the purpose of opening and shutting the different cocks and valves, necessary to admit the steam into the cylinder, the water to condense it, to carry off the condensed steam, to make the piston more air tight, and in general to improve the various working parts of the engine.

Machines of this kind have been constructed in a variety of places; particularly in Great Britain, for the purpose of raising water from mines or for supplying towns, and for raising water to turn wheels. One of the largest of this kind is that which was constructed by the late ingenious Mr. Smeaton, for raising water to turn the wheels of the Blast Furnaces at Carron—the cylinder of this engine is 72 inches diameter, and I believe it is reckoned the most perfect engine that has been constructed on Newcomen's principle.—But though Mr. Smeaton spent much time in the improvements of these engines, and succeeded to a very considerable extent, yet the manner of employing the steam in a cylinder where cold water is to be admitted, for the purpose of condensing it at each stroke, and the piston and cylinder being exposed to the atmosphere, render it so imperfect, that above one half of the power of the steam is lost by this construction. And therefore, even with Mr. Smeaton's ingenious improvements, the Steam Engine at that time was but a very imperfect machine, and by no means applicable to such a variety of purposes as it is now in its improved state.

The ingenious Mr. James Watt of Glasgow, perceiving the great loss of steam which was sustained in its use, in Newcomen's engine, about 1768, made a variety of experiments on this subject, and in 1770 obtained a patent for a new mode of applying it; in which the cylinder was made close both at bottom and top, and the rod which connected the piston with the lever, was made to work through a collar of hemp and tallow, so as to be perfectly air tight. The atmosphere being thus excluded from the

cylinder, both the vacuum is made by the steam, and the piston is moved by it. Also the steam is not condensed by throwing cold water into the cylinder, but it is taken out by an air pump, and condensed in a separate vessel; and, in order to keep the cylinder as hot as possible, it is surrounded with steam, and covered with non-conducting substances. By this construction, the engine has been made to perform at least double the effect, with the same quantity of fuel, as the best engines on Newcomen's construction. Mr. Watt obtained an extension of his patent right in the year 1775, by an act of parliament, for 25 years, and was joined by the ingenious Mr. Bolton of Soho, near Birmingham; since which, the same principle has still been followed; but the working parts have undergone various modifications, by the joint abilities of these able mechanicians. The principle which was applied to the working of the piston, only one way, that is, by pushing it downwards, as the atmosphere did in Newcomen's engine, has also been applied to the forcing it up; by which means, engines, where cylinders are of a given diameter, are now made to perform double the effect. This has not only saved great expence in the original construction of the engines, but has enabled them to be applied in cases where immense power has been wanted, and which could not have been performed at all by them on Newcomen's construction. By the same mode of applying the steam, it can now not only be used of the strength of the atmosphere, but as much stronger as necessity or convenience may require; which is a still farther consolidation of the power. The celerity also with which the condensation of the steam, and the discharging of the condensed steam and water, are performed, enables them to work quicker, and so to be applied to all kinds of mill work, which are used in the numerous manufactories of this country. Corn is ground by them, cotton spun, silk twisted, the immense machinery used in the new manu-

factories are worked, and including every kind of mill work to which water can be applied. They are also used in the various branches of the civil engineer. Thus the water is taken from the foundations of Locks, Bridges, Docks, &c. The piles are driven for the foundations, as the mortar manufactured for the building of the walls; earth taken from their canals; and docks and works have been of late performed by their means, which could not have been executed without them.

They are also made so portable for some purposes, that they are even constructed on boats and carriages, to be moved from one place to another; while in others they are made on a large and magnificent scale. Messrs. Bolton and Watt have made them from the power of one, to that of 250 horses; and by their late contrivances in the execution of their different parts, they are so manageable, that even a lad may attend and direct their operations; and so regular in their motions, that water itself cannot be more so.

The quantity of fuel which they consume is comparatively small, to the effect they produce.—One bushel of the best Newcastle coal applied to the working of an engine for pumping, will raise about 30 million of pounds one foot high.——But in these engines, when the steam acts on the piston, both in its ascent and descent, the same quantity of fuel will not produce quite so great an effect, as there is not so much time for performing the condensation, on which account the vacuum is not so complete.

For a more full account of Steam Engines, see Dr. Hutton's Dictionary.

OF BALLOONS, TELEGRAPHS, &c.

THE latter part of the last century, among many ingenious mechanical inventions, has produced the two remarkable ones relating to air balloons, and to telegraphs,

with other means of distant, quick or secret intelligence; concerning which a brief account may here be added: and first of Aerostation and Air Balloons.

The fundamental principles of aerostation have been long and generally known, as well as speculations on the theory of it; but the successful application of them to practice seems to be altogether a modern discovery. These principles chiefly respect the pressure and elasticity of the air, with its specific gravity, and that of the other bodies to be floated in it. Now any body that is specifically, or bulk for bulk, lighter than the atmosphere, is buoyed up by it, and ascends to such height where the air, by always diminishing in its density upward, becomes of the same specific gravity as the rising body; here this body will float, and move along with the wind or current of air, like clouds at that height. This body then is an aerostatic machine, whatever its form or nature may be; such as an air-balloon, the whole mass of which, including its covering and contents, with the weights annexed to it, is of less weight than the same bulk of air in which it rises.

We know of no solid bodies however that are light enough thus to ascend and float in the atmosphere; and therefore recourse must be had to some fluid or aeriform substance. Among these, that which is called inflammable air is the most proper for that purpose; it is very elastic, and is 6, 8, or 10 times lighter than common air. So that, if a sufficient quantity of that kind of air be inclosed in any thin bag or covering, the weight of the two together will be less than the weight of the same bulk of common air: consequently this compound mass will rise in the atmosphere, till it attain the height at which the atmosphere is of the same specific gravity as itself; where it will remain or float with the current of air, as long as the inflammable gas does not too much escape through the pores of its covering. And this is an inflammable-air balloon.

Another way is, to make use of common air rendered lighter, by heating it, instead of the inflammable air. Heat rarefies and expands common air, and consequently lessens its specific gravity. So that, if the air, inclosed in any kind of a bag or covering, be heated, and thus dilated, to such a degree, that the excess of the weight of an equal volume of common air, above the weight of the heated air, be greater than the weight of the covering and its appendages, the whole compound mass will ascend in the atmosphere, till it arrive at a height where the atmosphere has the same specific gravity with it; where it will remain till, by the cooling and condensation of the included air, the balloon shall gradually contract, and descend again, unless the heat be renewed or kept up. And this is a heated-air or a fire balloon, which is also called a Montgolfier, after the name of its inventor.

Various schemes for rising up in the air, and passing through it, have been devised and attempted, both by the ancients and the moderns, on different principles, and with various success. Of these attempts, some have been on mechanical principles, or by the powers of mechanism; and such, it is conceived, were the instances related of the flying pigeon made by Archytas, also the flying eagle, and the fly by Regiomontanus, with many others, both among the ancients and moderns.

Other projects have been vainly formed, by attaching wings to some part of the human body, to be moved either by the hands or the feet, by mean of mechanical powers; so that striking the air with them, after the manner of the wings in a bird, the person might raise himself in the air, and transport himself through it, in imitation of that animal. But these attempts belong rather to that species or principle of motion called artificial flying, than to the subject of aerostation, which is properly the sailing or floating in the air by means of a machine rendered specifically lighter than that element, in imitation of aqueous

navigatión, or the sailing on the water in a ship, or vessel, which is specifically lighter than this element.

The first rational account to be found on record, for this kind of sailing, is perhaps that of our countryman Roger Bacon, who died in the year 1292. He not only affirms that the art is feasible, but assures us that he himself knew how to make a machine, in which a man sitting might be able to convey himself through the air like a bird: and he farther affirms that there was another person who had tried it with success. The secret it seems consisted in a couple of large thin shells, or hollow globes, of copper, exhausted of air; so that the whole being thus rendered lighter than air, they would support a chair, in which a person might sit.

Bishop Wilkins too, who died in 1672, in several of his works, makes mention of similar ideas being entertained by divers persons. "It is a pretty notion to this purpose, says he, (in his Discovery of a New World), mentioned by Albertus de Saxonia, and out of him by Francis Mendoza, that the air is in some part of it navigable. And that upon this static principle, any brass or iron vessel, suppose a kettle, whose substance is much heavier than that of the water; yet being filled with the lighter air, it will swim upon it, and not sink." And again, in his Dedalies, he says, "Scaliger conceives the framing of such volant automata to be very easy. Those ancient motions were thought to be contrived by the force of some included air. As if there had been some lamp or other fire within it; which might produce such a forcible rarefaction, as should give a motion to the whole frame." Hence it would seem that bishop Wilkins had some confused notion of such a thing as a heated-air balloon.

Again, father Francisco Lana, in his Prodroma, printed in 1670, proposes the same method with that of Roger Bacon, as his own thought. He considers that a hollow vessel, exhausted of air, would weigh less than when filled

with that fluid. He also reasoned that, as the capacity of spherical vessels increases much faster than their surface, the former increasing as the cube of the diameter, but the latter only as the square of the same, it is therefore possible to make a spherical vessel of any given matter and thickness, and of such a size as, when emptied of air, it will be lighter than an equal bulk of that air, and consequently that it will ascend in the atmosphere. After stating these principles, father Lana computes that a round vessel of plate-brass, 14 feet in diameter, weighing 3 ounces the square foot, will only weigh 1848 ounces; whereas a quantity of air of the same bulk will weigh 2156 ounces, allowing only one ounce to the cubic foot; so that the globe will not only ascend in the air, but will also carry up a weight of 308 ounces: and by increasing the bulk of the globe, without increasing the thickness of the metal, he adds, a vessel might be made to carry up a much greater weight.

Such then were the speculations of ingenious men, and the gradual approaches towards this art. But one thing more was yet wanting: though in some degree acquainted with the weight of any quantity of air, considered as a detached substance, it seems they were not aware of its great elasticity, and the universal pressure of the atmosphere; a pressure by which a globe, of the dimensions above described, and exhausted of its air, would immediately be crushed inwards, for want of the equivalent internal counter pressure, to be sought for in some element, much lighter than common air, and yet nearly of equal pressure or elasticity with it; a property and circumstance attending inflammable gas, and also common air when considerably heated.

It is evident then that the schemes of ingenious men hitherto must have gone no farther than mere speculation; otherwise they could never have recorded fancies which, on the first attempt to be put in practice, must have ma-

nifested their own insufficiency, by an immediate failure of success. For, instead of exhausting the vessel of air, it must be filled either with common air heated, or with some other equally elastic but lighter air. So that on the whole it appears, that the art of traversing the atmosphere, is an invention of our own time; and the whole history of it is comprehended within a very short period.

The rarefaction and expansion of air by heat is a property of it that has been long known, not only to philosophers, but even to the vulgar. By this means it is, that the smoke is continually carried up our chimneys: and the effect of heat upon air, is made very sensible by bringing a bladder, only partially filled with air, near a fire; when the air presently expands with the heat, and distends the bladder so as almost to burst it. Indeed, so well are the common people acquainted with this effect, that it is the constant practice of those who kick about blown bladders, for foot balls, to bring them from time to time to the fire, to restore the spring of the air, and the distension of the ball, lost by the continual cooling and waste of that fluid.

But the great levity, or rather small weight, of inflammable gas, is a very modern discovery, namely within the last 40 or 50 years; a discovery chiefly owing to our own countrymen, Mr. Cavendish and Dr. Black, the latter of whom frequently mentioned also the feasibility of inclosing it in a very thin bag, so as that it might ascend into the atmosphere; an idea which was first put in practice, on a very small scale, by Mr. Cavallo, another ingenious philosopher.

It was however two brothers, of the name of Montgolfier, near the city of Lyons in France, who, in the year 1782, first exhibited to the world what may properly be called air-balloons, of large dimensions, being silken bags of many feet in diameter. These were on the principle of common air heated, by passing through a fire, made near

the orifice or bottom of the balloon. This heated air and the smoke thus ascended straight up into the bag, and gradually distended it, till it became quite full, and so much lighter than the atmosphere that the balloon rapidly ascended, and carried up other weights with it, to very great heights. After attaining its utmost height however, partly by the cooling of the included air, and partly by its escape through the pores of the covering, the balloon gradually descends very slowly, and comes at length to the ground, after being sometimes carried to great distances by the wind, or currents of air in the atmosphere.

Other balloons were also soon made by the philosophers in France, and after them in other countries; namely, by filling the balloon case with inflammable gas, a more troublesome and expensive process, but of much better effect; because, having only to guard against the waste of the fluid through the pores, but not its cooling, these balloons continue much longer in the air, sometimes for the space of many hours, enabling the passengers to pass over large tracts of country. On one of these occasions, Mr. Blanchard, a noted operator, with a favourable wind, passed over from Dover to Calais, accompanied by another gentleman.

Many other persons exhibited balloons, of large dimensions, particularly in France and other parts of the continent, with various success. The people of that country have also successfully applied balloons to the examination of the state of the higher regions of the atmosphere; and also in their armies, to discover the dispositions and operations of an enemy's position and camp. In England they have been less attended to, perhaps owing at first to an unfortunate prejudice, and an idea thrown out, that they could not be turned to any useful purpose in life.— A representation of several different balloons is exhibited in plate 14.

TELEGRAPHS.

A Telegraph is a machine lately brought into use by the French nation, namely in the year 1793; being contrived to communicate words or signals, from one person to another, at a great distance, and in a very short time.

The object proposed is, to obtain an intelligible figurative language, to be distinguished at a distance, to avoid the obvious delay in the dispatch of orders or information by messengers.

On first reflection, we find the practical modes of such distant communication must be confined to sound and vision, but chiefly the latter. Each of these is in a great degree affected by the state of the atmosphere: as, independent of the wind's direction, the air is sometimes so far deprived of its elasticity, or whatever other quality the conveyance of sound depends on, that the heaviest ordnance is scarcely heard farther than the shot flies; and, on the other hand, in thick hazy weather, the largest objects become quite obscured at a short distance. No instrument therefore, designed for the purpose, can be perfect. We can only endeavour to overcome these defects as much as may be.

Some kind of distant signals must have been employed from the earliest antiquity. It seems the Romans had a method in their walled cities, either by a hollow formed in the masonry, or by tubes affixed to it, so to confine and augment sound, as to convey information to any part they wished; and in lofty houses it is now sometimes the custom to have a pipe, by way of speaking trumpet, to give orders from the upper apartments to the lower: by this mode of confining sound, its effect may be carried to a very great distance; but beyond a certain extent the sound, losing articulation, would only convey alarm, and not give directions.

Every city among the ancients had its watch towers; and the castra stativa of the Romans had always some

spot, elevated either by art or nature, from which signals were given to the troops, cantoned or foraging in the neighbourhood. But they had probably not arrived at greater refinement than that, on seeing a certain signal, they were immediately to repair to their appointed stations.

A beacon, or bonfire made of the first inflammable materials that offered, as the most obvious, is perhaps the most ancient mode of general alarm, and by being previously concerted, the number or point where the fires appeared might have its particular intelligence affixed. The same observations may be referred to the throwing up of rockets, whose number or the point from whence thrown may have its affixed signification.

Flags or ensigns, with their various devices, are of earliest invention, especially at sea; where, from the first idea, which was probably that of a vane to show the direction of the wind, they have been long adopted as the distinguishing mark of nations, and are now so neatly combined by the ingenuity of a great naval commander, that by his system every requisite order and question is received and answered by the most distant ships of a fleet.

To the adopting this, or a similar mode, in land service, the following are objections: that in the latter case, the variety of matter necessary to be conveyed is so exceedingly great, that the combinations would become too complicated. And if the person for whom the information is intended should be in the direction of the wind, the flag would then present a straight line only, and at a little distance be invisible. The Romans were so well aware of this inconvenience of flags, that many of their standards were solid; and the name manipulus denotes the rudest of their modes, which was a truss of hay fixed on a pole.

The principle of water always keeping its own level has been suggested, as a possible mode of conveying intelligence, by an ingenious gentleman, and put in practice on

a small scale with a very pleasing effect. As for example, suppose a leaden pipe to reach between two distant places, and to have a perpendicular tube connected to each extremity. Then, if the pipe be constantly filled with water to a certain height, it will always rise to its level on the opposite end; and if but one inch of water be added at one extremity, it will almost instantly produce a similar elevation in the tube at the other end: so that by corresponding letters being adapted to the vertical tubes, at different heights, intelligence may be quickly conveyed. But this method is liable to such objections, that it is not likely it can ever be adopted to facilitate the object of very distant communication.

Full as many, if not greater objections, will perhaps operate against every mode of electricity being used as the vehicle of information.—And the requisite magnitude of painted or illuminated letters, offers an insurmountable obstacle; besides in them one object would be lost, that of the language being figurative.

Another idea is perfectly numerical, which is, to raise and depress a flag or curtain a certain number of times for each letter, according to a previously concerted system: as, suppose one elevation to mean A, two to mean B, and so on through the alphabet. But in this case, the least inaccuracy in giving or noting the number, changes the letters; and besides, the last letters of the alphabet would be a tedious operation.

Another method that has been proposed, is an ingenious combination of the magnetical experiment of Comus, and the telescopic micrometer. But as this is only an imperfect idea of Mr. Garnet's very ingenious machine, described below, no farther notice need be taken of it here.

Mr. Garnet's contrivance is merely a bar or plank, turning on a centre like the arm of a windmill; which being moved into any position, an observer or correspondent at a distance turns the tube of a telescope round its axis, into

the same position, by bringing a fixed wire within it to coincide with, or become parallel to, the bar, which is a thing extremely easy to do. The centre of motion of the bar has a small circle fixed on it, with letters and figures around the circumference, and a moveable index turning together with the bar, pointing to any letter or mark the operator may wish to set the bar to, or to communicate to the observer. The eye end of the telescope has a like index and circle fixed on the outside of it with the corresponding letters or other marks. The consequence is obvious; the telescope being turned round its axis, till its wire cover, or become parallel to the bar, the index of the former necessarily points out the same letter or mark on its circle as that of the latter, and the communication of sentiment is immediate and perfect. The use of this machine is so easy, that we have seen it put into the hands of two common labouring men, who had never seen it before, when they have immediately held a quick and distant conversation together.

Fig. 1 pl. 15 represents the principal parts of this telescope: ABDE is the telegraph or bar, having on the centre of gravity C, about which it turns, a fixed pin, going through a hole or socket in the firm upright post G, and on the opposite side is fixed an index EI. Concentric to C on the same post, is fixed a brass circle, of 6 or 8 inches diameter, divided into 48 equal parts, 24 of which represent the letters of the alphabet, and in the other 24, between the letters, are numbers. So that the index, by means of the arm AB, may be set or moved to any letter or number. The length of the arm or bar should be 2½ or 3 feet for every mile of distance. Two revolving lamps of different colours, suspended occasionally at A and B, the ends of the arm, would serve equally at night.

Let *ss* (fig. 2 pl. 15) represent a transverse section of the outward tube of a telescope, and *xx* the like section of the sliding or adjusting tube, on which is fixed an index II.

On the part of the outward tube next to the observer, is fixed a circle of letters and numbers, similarly divided and situated as the former circle in fig. 1; so that the index II, by means of the sliding or adjusting tube, may be turned to any letter or number. Now there being a hair, or fine silver wire, *fg*, fixed in the focus of the eye-glass; when the arm AB of the telegraph is viewed at a distance through the telescope, the hair may be turned, by means of the sliding tube, to the same position as the arm AB; then the index II (fig. 2) will point to the same letter or number on its own circle, as the index I (fig. 1) points to on the telegraphic circle.

If, instead of using the letters and numbers to form words at length, they be used as signals, three motions of the arm will give a hundred thousand different signals.

But a telegraph, combined with a telescope, it seems was originally the invention of M. Amontons, an ingenious French philosopher, about the middle of the 17th century; when he pointed out a method to acquaint people at a great distance, and in a very little time, with whatever we please. This method was as follows: Let persons be placed in several stations, at such distances from each other, that, by the help of a telescope, a man in one station may see a signal made by the next before him; this person immediately repeats the signal to the third man; and this again to a fourth, and so on through all the stations, to the last.

This, with considerable improvements, it seems has lately been brought into use by the French, and called a Telegraph. It is said they have availed themselves of this contrivance to good purpose, in the late war; which has induced the English also to employ a like instrument, in a different form.

The new invented telegraphic language of signals, says a French author, is an artful contrivance to transmit thoughts, in a peculiar way, from one distance to another, by means of machines, which are placed at different dis-

tances, of from 12 to 15 miles each, so that the expression reaches a very distant place in the space of a few minutes. The only thing which can interrupt their effects is, if the weather be so bad and turbid, that the objects and signals cannot be distinguished. By this invention, remoteness and distance almost disappear; and all the communications of correspondence are effected with the rapidity of the twinkling of an eye. The greatest advantage which can be derived from this correspondence, is that, if we choose, its object shall be known to certain individuals only, or to one individual alone, or to the extremities of any distance.

Fig. 3 pl. 15 represents the form of the French Telegraph. AA is a beam or mast of wood, placed upright on a rising ground, and is 15 or 16 feet high. BB is a beam or balance, moving on the centre AA. This balance beam may be placed vertically, or horizontally, or any how inclined, by means of strong cords, which are fixed to the wheel D, on the edge of which is a double groove to receive the two cords. This balance is 11 or 12 feet long, and 9 inches broad, having at the end two bars CC, which likewise turn on the angles by means of four other cords passing through the axis of the main balance. The pieces C are each about three feet long, and may be turned and placed either to the right or left, straight or square with the balance beam. By means of these three, the combination of movements is said to be very extensive, remarkably simple, and easy to perform. Below is a small wooden hut, in which a person is employed to attend the movements of the machine. In the mountain nearest to this, another person is to repeat these movements, and a third to write them down. The signs are sometimes made in words, and sometimes in letters; when in words, a small flag is hoisted; and, as the alphabet may be changed at pleasure, it is only the corresponding person who knows the meaning of the signs. The alphabet, as well as the

numbers to 10, are exhibited in the middle of fig. 3, annexed to the different forms and positions into which the bars of the machine may be put.

Many improvements and additional contrivances have been since made in England. The following one is by the Rev. J. Gamble. The principle of it is simply that of a Venetian window-blind, or rather what are called the lever boards of a brewhouse, which when horizontal, present so small a surface to the distant observer, as to be lost to his view, but are capable of being in an instant changed into a screen of a magnitude adapted to the required distance of vision. AEBDFC (fig. 4 pl. 15) is a firm upright frame, supporting 9 lever boards, working on centres in BE and DF, and opening in three divisions by iron rods. And *a b c d*, *e f g h* are two lesser frames, fixed to the great one, having also three lever boards in each, and moving by iron rods, in the same manner as the others. If all these rods be brought so near the ground, as to be in the management of the operator, he will then have 5 keys to play on. Now as each of the handles *i k l m n* commands three lever boards, by raising any one of them, and fixing it in its place by a catch or hook, it will give a different appearance in the machine; and by the proper variation of these 5 movements, there will be more than 25 of what may be called mutations, in each of which the machine exhibits a different appearance, and to which any letter or figure may be annexed at pleasure.

Should it be required to give intelligence in more than one direction, the whole machine may be easily made to turn to different points, on a strong centre, after the manner of a single post windmill.—To use this machine by night, another frame must be connected with the back part of the telegraph, for raising 5 lamps, of different colours, behind the openings of the lever boards, these lamps by night answering for the openings by day.

Fig. 5 pl. 15, represents a front view of the latest form

of the telegraph, now employed by the English government, by which a signal is conveyed between London and Deal, being 72 miles, by repetition, in 3 minutes. The corresponding boards forming a scale for the alphabet, and for numbering, is annexed in the engraving.

We shall limit ourselves to what has been here said respecting those machines, which have acquired the greatest celebrity; but we shall point out a few books, which those who are fond of machines, and who wish to instruct themselves by example, may consult for that purpose. The first of these, which we shall mention, is the Theatrum Mechanicum of Leupold, in several volumes folio, the last of which appeared in 1725. This is a curious work, but the author's theory is not always well founded; for he seems not to be entirely convinced of the impossibility of the perpetual motion. The next is the Théâtre des Machines of James Besson, in Italian and French. And to these we shall add, Bockler's work, in Latin; that of Ramelli in Italian and French, which is rare, and in great request. The Cabinet des Machines of de Servieres, 4to, Paris 1738, is one of the most curious works of this kind, on account of the great number of machines described in it, and which were invented by the author. Some of them are very ingenious, and the principles on which they are constructed deserved to have been better explained; but, in general, they are more curious than useful.

The description of the method in which the Chevalier Carlo Fontana raised the famous obelisk, now before St. Peter's at Rome, is likewise a work worthy of a place in the library of every person fond of mechanics. M. Loriot, who has a collection of machines, the invention of which displays great ingenuity, has promised to publish some day a description of them. This, in our opinion, would be a curious and useful work; for the most of his machines bear the stamp of genius. We have seen one invented by him for driving piles, which acts by a motion always in

the same direction, without being obliged to stop or to retrograde, in order to raise up again the weight. Nothing, in our opinion, can be more ingenious than the method in which, after the fall of the weight or rammer, the hook, that serves to raise it again, lays hold of it, and by which the cable lengthens itself in order to reach lower and lower in proportion as the pile sinks deeper. If this mode of construction be compared with those hitherto employed, no one can refuse to give it the preference.

There is also the Collection, in 6 vols. 4to, of Machines and Inventions approved by the Royal Academy of Sciences, containing the engravings and descriptions of a great multitude of machines. In English too we have Desaguliers's Course of Experimental Philosophy, in 2 vols. 4to. also Emerson's Mechanics, both containing the figures and descriptions of many curious and useful machines. Besides some others, of less note.

A TABLE

Of the Specific Gravities of different bodies, that of rain or distilled water being supposed 1000.

METALS.

Gold.

	Specific Gravity.
Pure gold of 24 carats, melted but not hammered	19258
The same hammered	19362
Gold, of the Parisian standard, 22 carats fine, not hammered *	17486
The same hammered	17589
Gold of the standard of French coin, 21 $\frac{2}{3}\frac{2}{2}$ carats fine, not hammered	17402
The same coined	17647
Gold of the French trinket standard, 20 carats fine, not hammered	15709
The same hammered	15775

Silver.

Pure or virgin silver, 12 deniers fine, not hammered	10474
The same hammered	10511
Silver of the Paris standard, 11 deniers 10 grains fine, not hammered †	10175
The same hammered	10377
Silver, standard of the French coin 10 deniers 21 grains fine, not hammered . . .	10048
The same coined	10408

Platina.

Crude platina, in grains	15602
Purified platina, not hammered . . .	19500

* This is the same as sterling gold. † This is 10 grs. finer than sterling.

The same hammered	20337
The same drawn into wire . . .	21042
The same rolled	22069

Copper and Brass.

Copper not hammered . . .	7788
The same wire-drawn . . .	8879
Brass, not hammered	8396
The same wire-drawn	8544
Common cast brass	7824

Iron and Steel.

Cast iron	7207
Bar iron, either hardened or not . .	7788
Steel, neither tempered nor hardened . .	7833
Steel hardened under the hammer, but not tempered	7840
Steel tempered and hardened . . .	7818
Steel tempered and not hardened . .	7816

Other Metals.

Pure tin from Cornwall, melted and not hardened	7291
The same hardened	7299
Malacca tin, not hardened . . .	7296
The same hardened	7307
Molten lead	11352
Molten zinc	7191
Molten bismuth	9823
Molten cobalt	7812
Molten arsenic	5763
Molten nikel	7807
Molten antimony	6702
Crude antimony	4064
Glass of antimony	4946
Molybdena	4739
Tungsten	6067

Mercury	13568
Uranium	6440

PRECIOUS STONES.

White oriental diamond	3521
Rose coloured ditto	3531
Oriental ruby	4283
Spinell ditto	3760
Ballas ditto	3646
Brasilian ditto	3531
Oriental topaze	4011
Saxon ditto	3564
Oriental sapphire	3994
Brasilian ditto	3131
Girasol	4000
Jargon of Ceylon	4416
Hyacinth	3687
Vermilion	4230
Bohemian garnet	4189
Syrian ditto	4000
Volcanic ditto with 24 sides	2468
Peruvian emerald	2776
Chrysolite of the Jewellers	2782
Brasilian ditto	2692
Beryl or oriental aqua-marine	3549
Occidental ditto	2723

SILICEOUS STONES.

Pure rock crystal of Madagascar	2653
Ditto of Europe	2655
Crystallized quartz	2655
Oriental agate	2590
Agate onyx	2638
Transparent calcedony	2664
Carnelian	2614
Sardonyx	2603

Prasium	2581
Onyx pebble	2664
White Jade	2950
Green ditto	2966
Red Jasper	2661
Brown ditto	2691
Yellow ditto	2710
Violet ditto	2711
Grey ditto	2764
Black prismatic hexaedral schorl	3385
Black amorphous schorl, called antique basaltes	2923
Paving stone	2416
Grind-stone	2143
Cutler's stone	2111
Mill-stone	2484
White flint	2594
Blackish ditto	2582

VARIOUS STONES, &c.

Opake green Italian serpentine	2430
Coarse Briançon chalk	2727
Spanish chalk	2790
Muscovy talc	2792
Common schist or slate	2672
New slate	2854
White razor hone	2876
Black and White ditto	3131
Icelandic crystal	2715
Pyramidal calcareous spar	2730
Oriental or white antique alabaster	2714
Green Campanian marble	2742
Red ditto	2724
White Casara marble	2717
White Parian marble	2838
Ponderous spar	4430
White fluor	3156

Red fluor	3191
Green ditto	3182
Blue ditto	3169
Violet ditto	3176
Red porphyry	2765
Red Egyptian granite	2654
Pumice stone	915
Obsidian stone	2348
Basaltes from the Giants causway	2864
Touch stone	2415
Bottle glass	2733
Green glass	2642
White glass	2892
Leith crystal	3189
Flint glass	3329
Seves porcelain	2146
China ditto	2385
Native sulphur	2033
Melted ditto	1991
Phosphorus	1714
Hard peat	1329
Ambergris	926
Yellow transparent amber	1078

LIQUORS.

Distilled water	1000
Rain water	1000
Sea water *	1026
Burgundy wine	992
Malmsey Madeira	1038
Cyder	1018
Red beer	1034
White ditto	1023

* Sea water differs in weight, according to the climate. It is heavier in the torrid zone, and at a distance from the coasts, than in the northern seas, and near land.

Highly rectified alcohol . . .	829
Common spirit of wine . . .	837
Sulphuric ether	739
Nitric ditto	909
Muriatic ditto	730
Acetic ditto	866
Highly concentrated sulphuric acid . .	2125
Common sulphuric acid . . .	1841
Highly concentrated nitric acid . .	1580
Common nitric acid . . .	1272
Muriatic acid	1194
Fluoric ditto	1500
Red acetous ditto . . .	1025
White acetous ditto	1014
Distilled ditto ditto	1010
Acetic ditto	1063
Formic ditto	994
Solution of caustic ammonia, or volatile alkali fluor	897
Essential oil of turpentine . . .	870
Liquid turpentine	991
Volatile oil of lavender . . .	894
Volatile oil of cloves . . .	1036
Volatile oil of cinnamon . . .	1044
Oil of olives	'915
Oil of sweet almonds . . .	917
Linseed oil	940
Whale oil	923
Woman's milk	1020
Cow's milk . . .	1032
Mare's milk	1035
Ass milk	1036
Goat's milk	1034
Ewe milk	1041

RESINS AND GUMS.

Common yellow resin . . .	1073

Mastic	1074
Storax	1110
Opake copal	1140
Madagascar ditto	1060
Chinese ditto	1063
Elemi	1018
Labdanum	1186
Dragon's blood	1205
Gum lac	1139
Gum elastic	934
Camphor	989
Gum ammoniac	1207
Gamboge	1222
Myrrh	1360
Galbanum	1212
Assafœtida	1328
Gum arabic	1452
Tragacanth	1316
Terra Japonica	1398
Socotrine aloes	1380
Opium	1337
Indigo	769
Yellow wax	965
White ditto	969
Spermaceti	943
Beef fat	923
Veal fat	934
Mutton fat	924
Tallow	942
Hog's fat	937
Lard	948
Butter	942

WOODS.

Heart of Oak 60 years old	1170
Cork	240

Elm plank	671
Ash ditto	845
Beech	852
Alder	800
Walnut	671
Willow	585
Male fir	550
Female ditto	498
Poplar	383
White Spanish ditto	529
Apple tree	793
Pear tree	661
Quince tree	705
Medlar	944
Plum tree	785
Cherry tree	715
Filbert tree	600
French box	912
Dutch ditto	1328
Dutch yew	788
Spanish ditto	807
Spanish cypress	644
American cedar	561
Spanish Mulberry tree	897
Pomegranate tree	1354
Lignum vitæ	1333
Orange tree	705

Note, We may here observe, that the numbers in the above table, express nearly the absolute weight of an English cubic foot, of each substance, in averdupois ounces.

TABLE OF WEIGHTS,

Both ancient and modern, as compared with the English Troy pound, which contains 12 *ounces, or* 5760 *grains.*

As we gave, at the end of that part which relates to Geometry, a comparative table of the principal longitu-

dinal measures, we think it our duty to give here a similar table of the ancient Hebrew, Greek, and Roman weights; and also of the modern weights, of different countries, particularly in Europe, as compared with the English Troy pound.

ANCIENT WEIGHTS.

Hebrew Weights.

	Grs. Troy.	lib.	oz.	dwt.	grs.
The obolus called gerah	10·66	0	0	0	10·66
Half shekel or beka	103·37	0	0	4	7·37
Shekel	206·74	0	0	8	14·74
Mina or maneh	12453·67	2	1	18	21·67
Talent or cicar	622683·6	108	1	5	3·6

Attic Greek Weights.*

	Grs. Troy.	lib.	oz.	dwt.	grs.
Chalcus	·82	0	0	0	0·82
Obolus	8·20	0	0	0	8·20
Drachma	51·89	0	0	2	3·89
Didrachma	103·78	0	0	4	7·78
Tetradrachma	207·56	0	0	8	15·56
Lesser mina of 75 drachms	3891·77	0	8	2	3·77
Greater mina of 100 drachms	5189·03	0	10	16	5·03
Lesser talent of 60 lesser minæ	233506·20	40	6	9	10·20
Greater talent of 60 greater minæ	311341·8	54	0	12	13·8

Roman Weights.

	Grs. Troy.	lib.	oz.	dwt.	grs.
The denarius	51·89	0	0	2	3·89
Ounce, equal to 12 denarii	415·12	0	0	17	7·12
As or pound, equal to 12 ounces	4981·44	0	10	7	13·44

* It may be proper here to observe, that these weights were at the same time money.

	Grs. Troy.	lib.	oz.	dwt.	grs.
Another pound of 10 ounces	4151·2	0	8	12	23·2
The lesser talent	233506·20	40	6	9	10·20
The greater talent	311341·8	54	0	12	13·8

The above tables are taken from a work by M. Christiani, entitled, Delle Misure d'ògni genere, antiche è moderne, &c; printed in quarto, at Venice, in the year 1760. As this is an obscure subject, and as some difference prevails among the learned in regard to the value of the ancient weights, the translator has added the following tables from Arbuthnot, in order to render this article more complete.

Jewish weights reduced to English Troy weight.

	lib.	oz.	dwt.	gr.
The shekel	0	0	9	$2\frac{4}{7}$
Maneh	2	3	6	$10\frac{2}{7}$
Talent	113	10	1	$10\frac{2}{7}$

The most ancient Grecian weights, reduced to English Troy weight.

	lib.	oz.	dwt.	gr.
Drachma	0	0	6	$2\frac{22}{49}$
Mina	1	1	0	$4\frac{44}{49}$
Talent	65	0	12	$5\frac{43}{49}$

Less ancient Grecian and Roman weights reduced to English Troy weight.

	lib.	oz	dwt.	gr.
Lentes	0	0	0	$0\frac{85}{116}$
Siliquæ	0	0	0	$3\frac{1}{28}$
Obolus	0	0	0	$9\frac{3}{28}$
Scriptulum	0	0	0	$18\frac{3}{14}$
Drachma	0	0	2	$6\frac{9}{14}$
Sextula	0	0	3	$0\frac{6}{7}$
Sicilicus	0	0	4	$13\frac{2}{7}$
Duella	0	0	6	$1\frac{5}{7}$
Uncia	0	0	18	$5\frac{1}{7}$
Libra	0	10	18	$13\frac{5}{7}$

The Roman ounce is the English avoirdupois ounce, which they divided into 7 denarii, as well as 8 drachms: and since they reckoned their denarius equal to the Attic drachm, this will make the Attic weights $\frac{1}{7}$ heavier than the correspondent Roman weights.

We shall here observe, that the Greeks divided their obolus into chalci and lepta: thus, Diodorus and Suidas divide the obolus into 6 chalci, and every chalcus into 7 lepta: others divided the obolus into 8 chalci, and every chalcus into 8 lepta, or minuta.

The greater Attic weights, reduced to English Troy weight.

	lib.	oz.	dwt.	grs.
Libra or pound	0	10	18	$13\frac{5}{7}$
Common Attic mina . .	0	11	7	$16\frac{}{7}$
Another mina used in medicine	1	2	11	$10\frac{2}{7}$
The common Attic talent .	56	11	0	$17\frac{1}{7}$

It is here to be remarked, that there was another Attic talent, said by some to consist of 80, and by others of 100 minæ. Every mina contains 100 drachmæ, and every talent 60 minæ; but the talents differ in weight, according to the different standard of the drachmæ and minæ of which they are composed. The value of different minæ and talents, in English Troy weight, is exhibited in the following tables:

Table of different minæ.

	lib.	oz.	dwt.	grs.
Egyptian mina	1	5	6	$22\frac{26}{49}$
Antiochic	1	5	6	$22\frac{26}{49}$
Ptolemaic of Cleopatra .	1	6	14	$16\frac{32}{49}$
Alexandrian of Dioscorides .	1	8	16	$7\frac{41}{49}$

Table of different Talents.

	lib.	oz.	dwt.	grs.
Egyptian	86	8	16	8
Antiochic	86	8	16	8

	lib.	oz.	dwt.	gr.
Ptolemaic of Cleopatra . .	93	11	11	0
Alexandrian	104	0	19	14
Of the Islands . . .	130	1	4	12
Antiochian	390	3	13	11

Modern weights of the principal countries in the world, and particularly in Europe.

	Grs. Troy.	lib	oz.	dwt.	gr.
Aleppo, the pound, called *rotolo* . .	30984·86	5	4	11	0·86
Alexandria in Egypt .	6158·74	1	0	16	14·74
Alicant	6908·58	1	2	7	20·58
Amsterdam . . .	7460·71	1	3	10	20·71
Antwerp, and the Netherlands . .	7048·15	1	2	15	4·15
Avignon . . .	6216·99	1	0	19	0·99
Basle	7713·31	1	4	1	9·31
Bayonne . . .	7460·71	1	3	10	20·71
Bergamo { . . .	4663·97	0	9	14	7·97
Bergamo { . . .	11659·52	2	0	5	19·52
Berghen . . .	7833·17	1	4	6	9·17
Berne	6721·53	1	2	0	1·53
Bilboa	7460·71	1	3	10	20·71
Bois-le-Duc . . .	7105·48	1	2	16	1·48
Bourdeaux, *see* Bayonne.					
Bourg	7073·57	1	2	14	17·57
Brescia : . . .	4496·61	0	9	7	8·61
Cadiz	7038·21	1	2	13	6·21
China *(the kin)* .	9222·93	1	7	4	6·93
Cologne	7220·34	1	3	0	20·34
Constantinople . .	7578·03	1	3	15	18·03
Copenhagen . . .	6940·58	1	2	9	4·58
Damascus . . .	25612·88	4	5	7	4·88
Dantzic . . .	6573·86	1	1	13	21·86
Dublin	7774·11	1	3	19	18·11
Florence	5286·65	0	11	0	6·65

	Grs. Troy.	lib.	oz.	dwt.	gr.
Genoa	4426·05	0	9	4	10·05
	6637·85	1	1	16	3·85
Geneva	8407·45	1	5	10	7·45
Hamburgh	7314·68	1	3	4	18·68
Konigsberg	5968·41	1	0	8	16·41
Leghorn	5145·54	0	10	14	9·54
Leyden	7038·21	1	2	13	6·21
Liege	7089·07	1	2	15	9·07
Lille	6544·33	1	1	12	16·33
Lisbon	7005·39	1	2	11	21·39
Lucca	5272·71	0	10	19	16·71
Lyons { Silk weight	6946·32	1	2	9	10·32
Lyons { Town weight	6431·93	1	1	7	23·93
Madrid	6544·33	1	1	12	16·33
Malo St. *see* Bayonne.					
Marseilles	6041·42	1	0	11	17·42
Mechlin, *see* Antwerp.					
Mêlun	4440·82	0	9	5	0·82
Messina	4844·46	0	10	1	20·46
Montpellier	6217·81	1	0	19	1·81
Namur	7174·39	1	2	18	22·39
Nancy	7038·21	1	2	13	6·21
Nantes, *see* Bayonne.					
Naples	4951·93	0	10	6	7·93
Nuremberg	7870·91	1	4	7	22·91
Paris	7560·80	1	3	15	0·8
Pisa, *see* Florence.					
Revel	6573·86	1	1	13	21·86
Riga	6148·89	1	0	16	4·89
Rome	5257·12	0	10	19	1·12
Rouen	7771·64	1	4	3	19·64
Saragossa	4707·45	0	9	16	3·45
Seville	7038·21	1	2	13	6·21
Smyrna	6544·33	1	1	12	16·33
Stettin	6782·24	1	2	2	14·24

	Grs. Troy.		lib.	oz.	dwt.	gr.
Stockholm . . .	9211·45	.	1	7	3	19·45
Strasburg . . .	7276·94	.	1	3	3	4·94
Toulouse, and upper Languedoc . . .	6322·82	.	1	1	3	10·82
Turin and Piedmont, in general . . .	4939·62	.	0	10	5	19·62
Tunis and Tripoli, in Barbary . . .	7139·94	.	1	2	17	11·94
Venice { lesser pound .	4215·21	.	0	8	15	15·21
Venice { greater do. .	6826·54	.	1	2	4	10·54
Verona . . .	5374·44	.	0	11	3	22·44
Vicenza { lesser pound .	4676·28	.	0	9	14	20·28
Vicenza { greater do. .	6879·05	.	1	2	6	15·05

To reduce any of the weights in the preceding table to English averdupois pounds, nothing will be necessary but to divide the grains Troy, in the first column, by 7000.

FRENCH WEIGHTS.

The Paris pound, poids de mark of Charlemagne, contains 9216 Paris grains: it is divided into 16 ounces, each ounce into 8 gros, and each gros into 72 grains*. It is equal to 7561 English Troy grains.

The English Troy pound, of 12 ounces, contains 5760 English grains, and is equal to 7021 Paris grains.

The English averdupois pound, of 16 ounces, contains 7000 English Troy grains; and is equal to 8538 Paris grains.

NEW FRENCH WEIGHTS.

	Eng. Troy grains.
Milligramme	·01544
Centigramme	·15445
Decigramme	1·54457
Gramme	15·44579

* Sometimes the gros is divided into 3 deniers, and each denier into 24 grains.

	Eng. Troy grains.
Decagramme	154·45[illegible]93
Hectogramme	1544·57938
Chiliogramme	15445·79386
Myriagramme . . .	154457·93860

A decagramme is 6 dwts. 10·45 grs. Troy, or 2 drs. 1 scr. 14·45 grs. apoth. weight, or 5·648 drams, averdupois.

A hectogramme is 3 oz. 8·48 drs. averdup.

A chiliogramme is 2 lbs. 3 oz. 4·87 drs. aver.

A myriagramme is 22 lbs. 1 oz. 0·73 drs. aver.

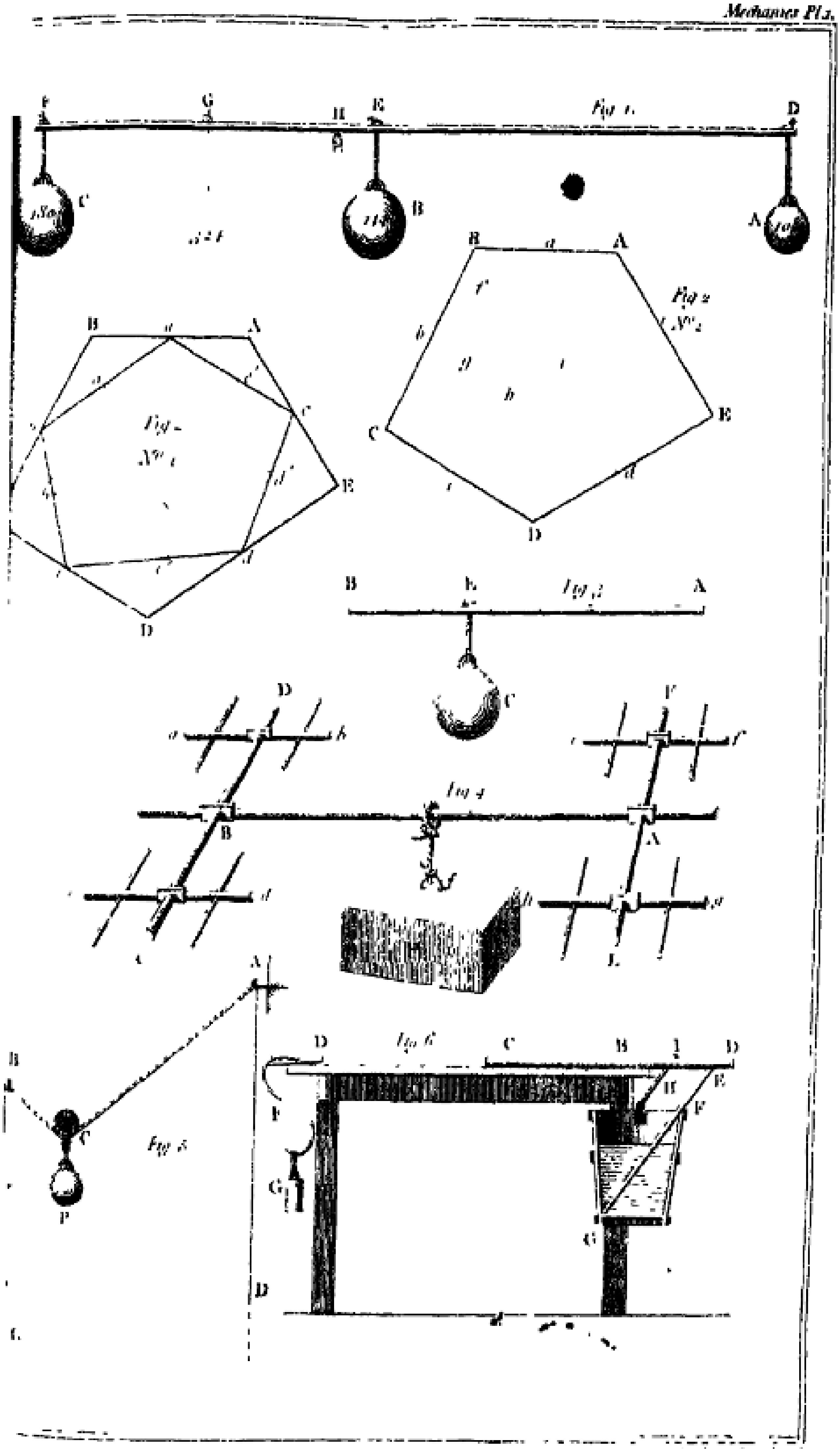


Fig. 1.
Fig. 2
N°. 1
N°. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6

Fig 7.

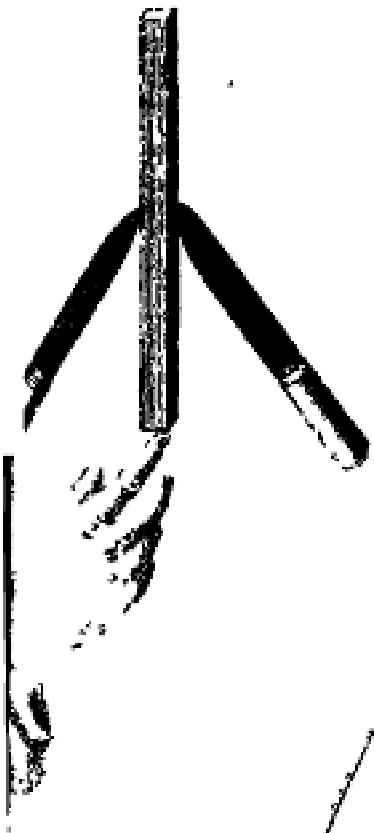
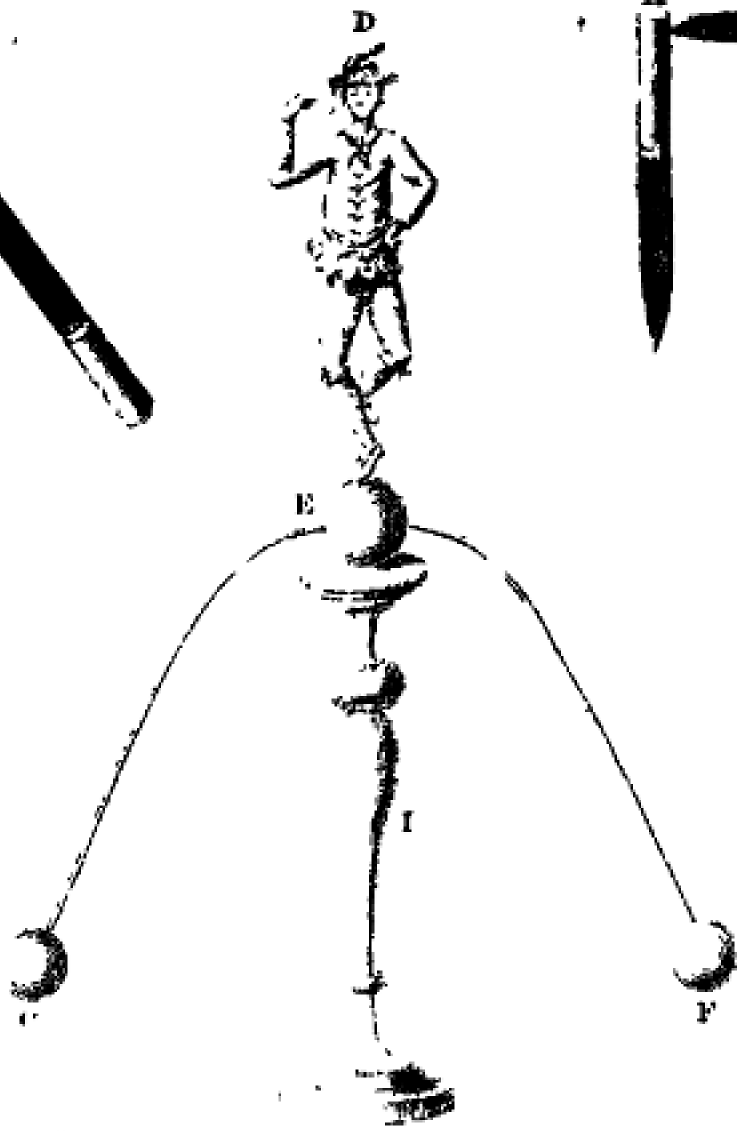


Fig 8



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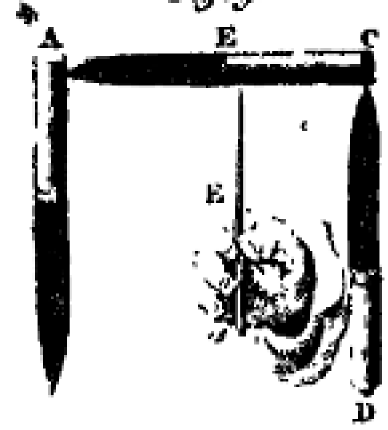
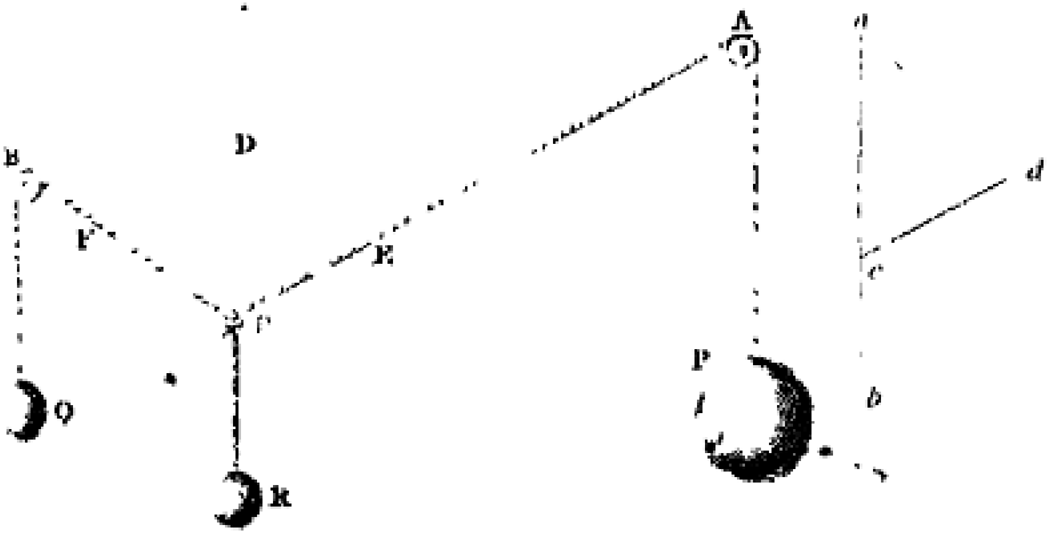


Fig 10.

Fig 11



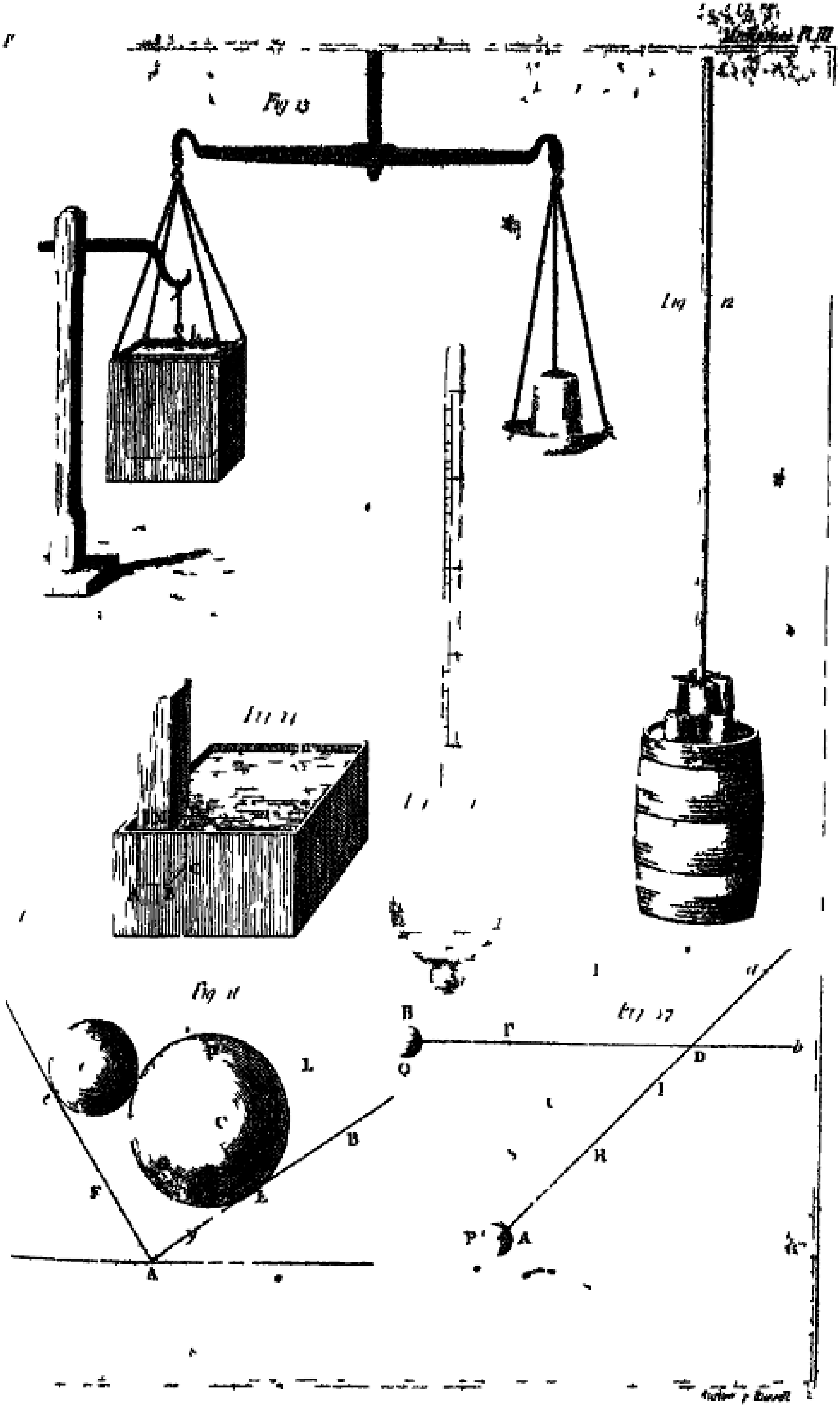


Fig 13
Fig 12
Fig 17
B
F
Q
D
b
I
H
A
P
C
L
E

Fig. 18.

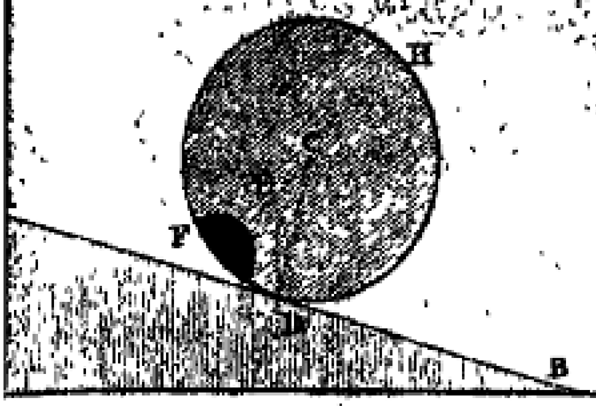
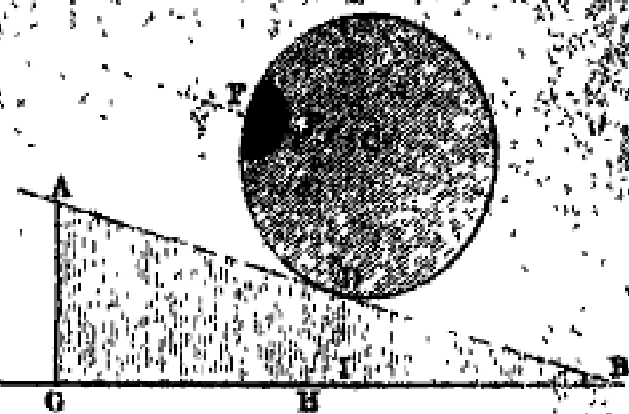


Fig. 19.



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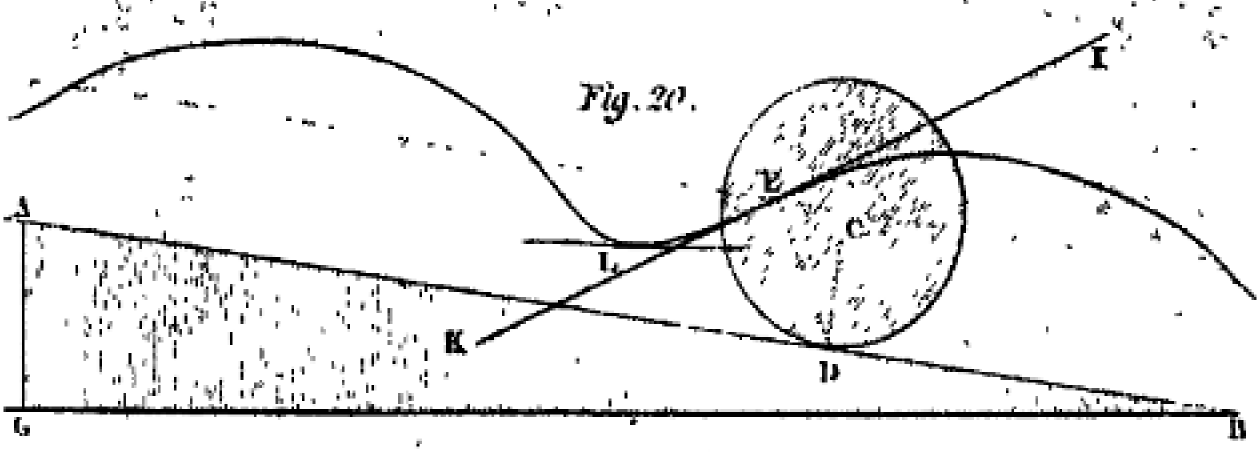

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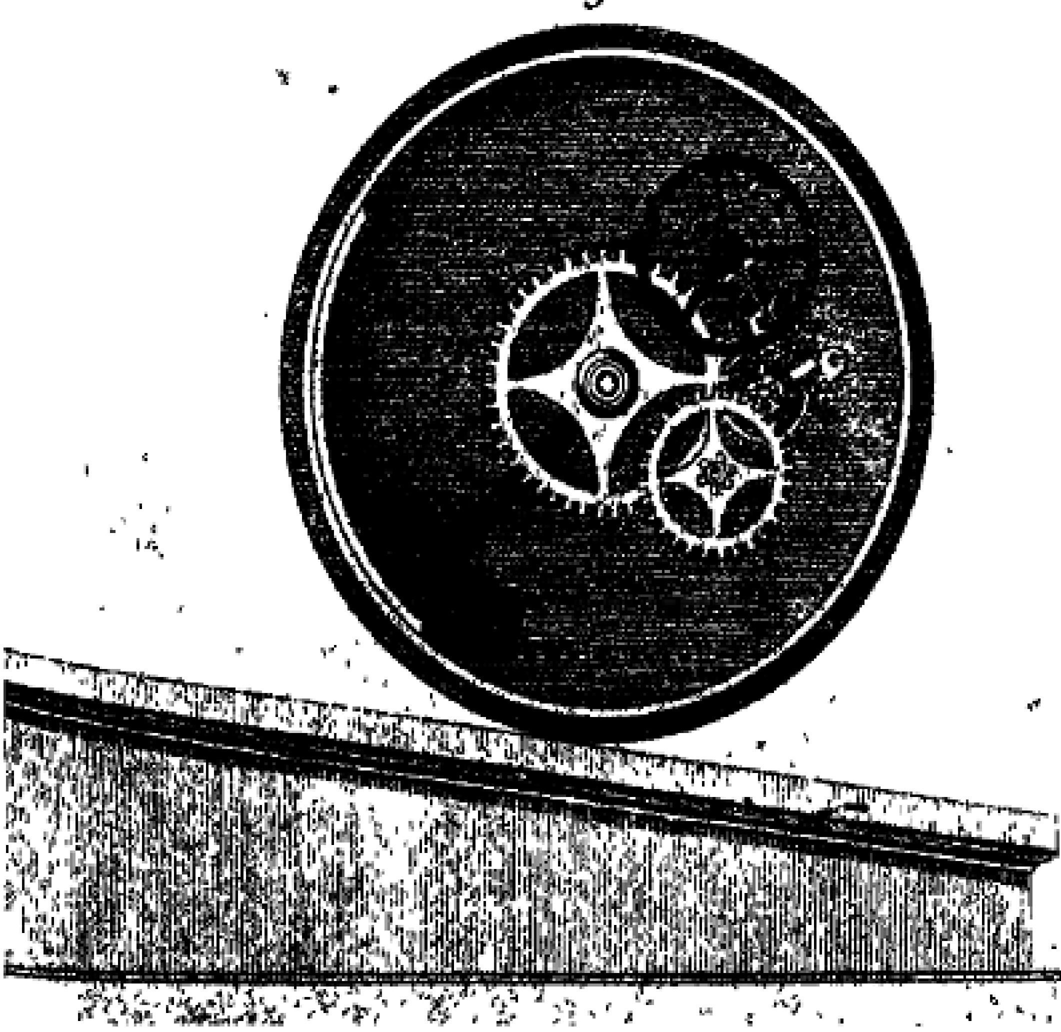

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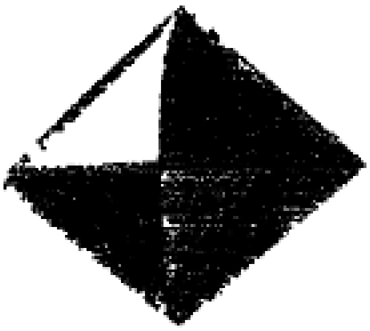

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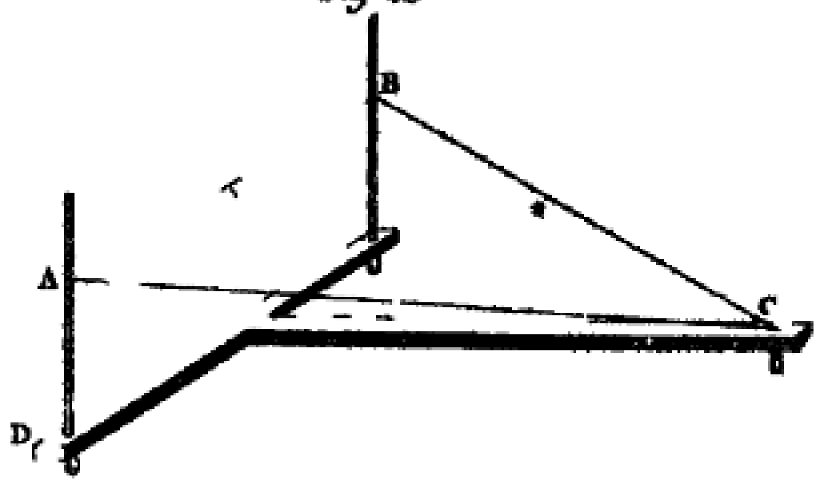

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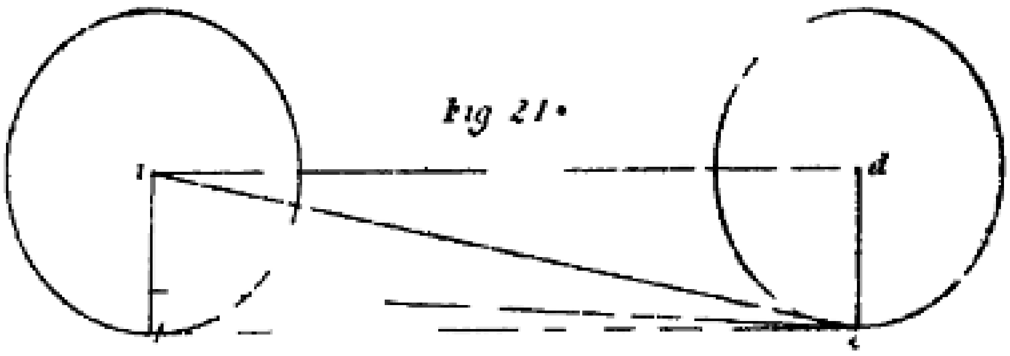

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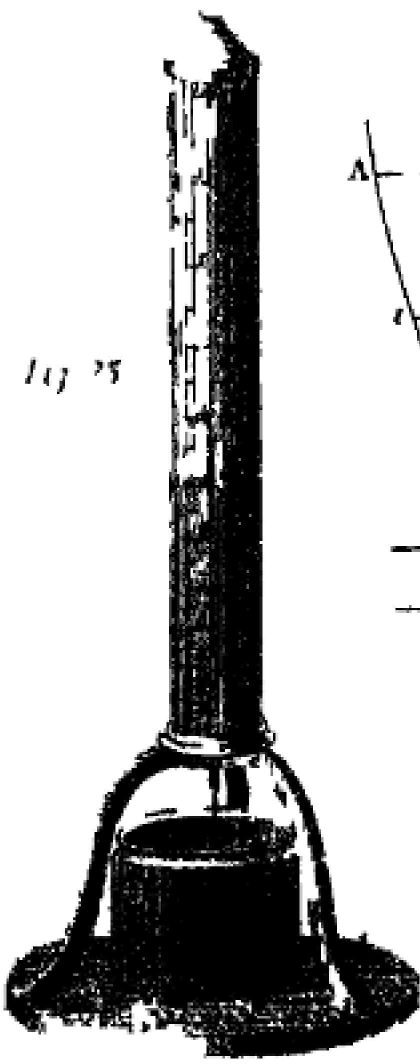
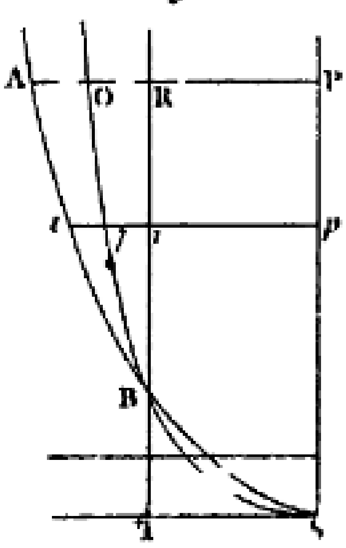


Fig 26



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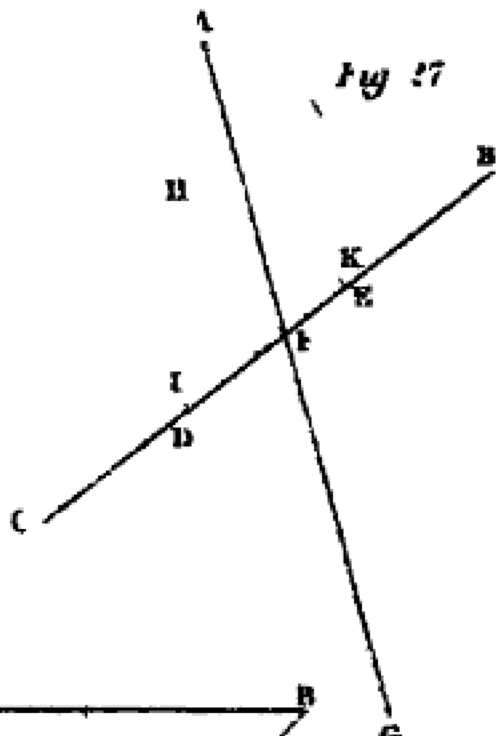

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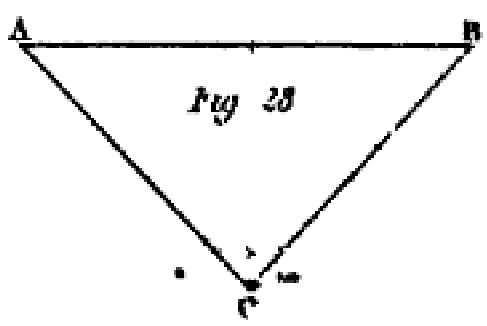

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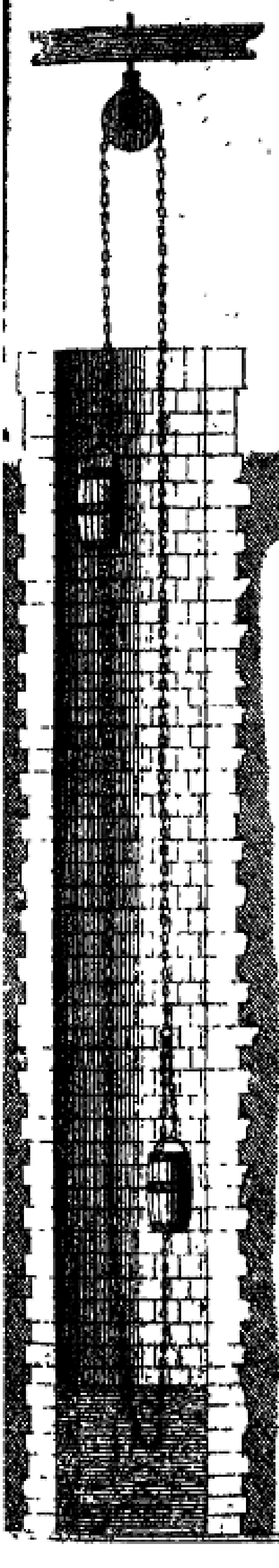
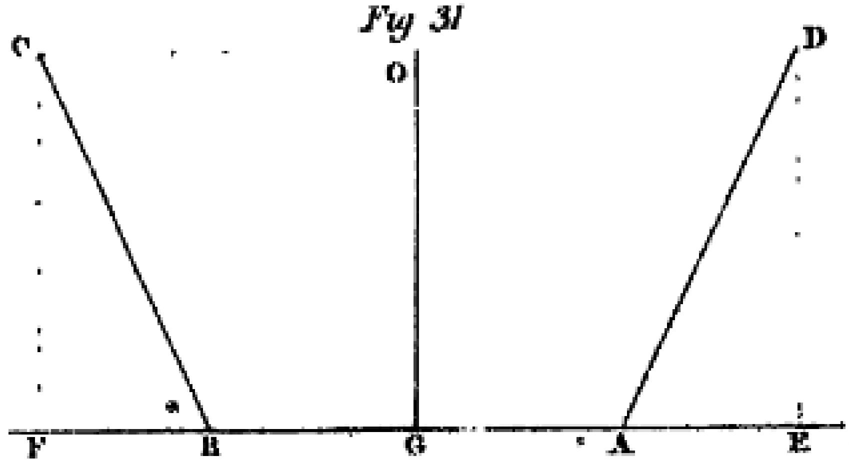
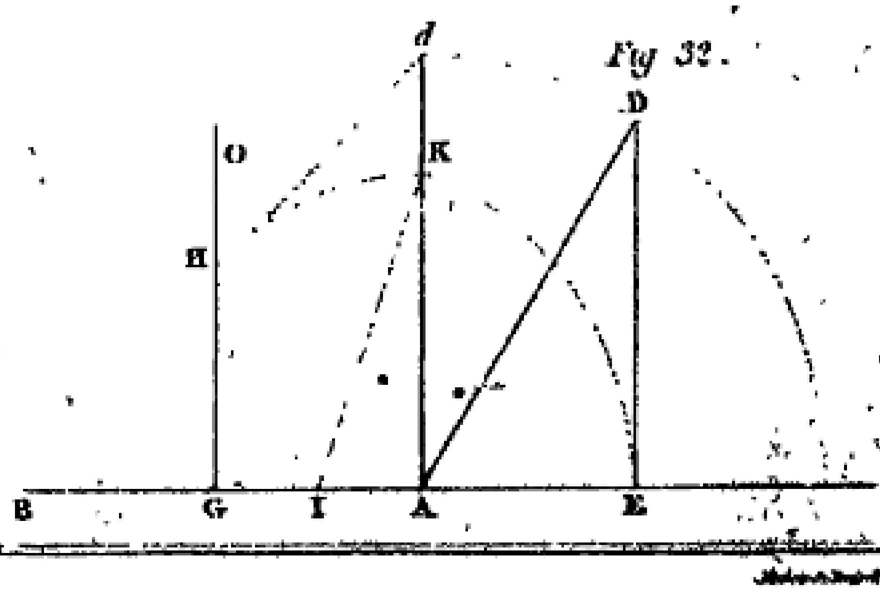


Fig 31





MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART FOURTH.

Containing many curious Problems in Optics.

THE properties of light, and the phenomena of vision, form the object of that part of the mixed mathematics, called *optics*; which is commonly divided into four branches, viz, direct optics, or vision, catoptrics, dioptrics, and perspective.

Light indeed may reach the eye three ways: either directly; or after having been reflected, or after having been refracted. Considered under the first point of view, it gives rise to the first branch of optics, called direct optics, or vision; in which is explained every thing that relates to the direct propagation of light, or by a straight line from the object to the eye, with the manner in which objects are perceived, &c.

Catoptrics treat of the effects of reflected light, and the phenomena produced by the reflection of light from surfaces of different forms; plane, concave, convex, &c.

When light, by passing through transparent bodies, is

turned aside from its direct course, which is called refraction, it becomes the object of dioptrics. It is this branch of optics that explains the effects of refracting telescopes, and of microscopes.

Perspective ought to form a part of direct optics, as it is merely a solution of the different cases of the following problem: On a given surface to trace out the image of an object in such a manner, that it shall make on the eye, when placed in a proper station, the same impression as the object itself—a problem purely geometrical, and in which nothing is required but to determine, on a plane given in position, the points where it is intersected by straight lines drawn to the eye from every point of the object. Consequently, the only thing here borrowed from optics, is the principle of the rectitude of the rays of light, as long as they pass through the same medium: the rest is pure geometry.

Without confining ourselves to any other order than that of method, we shall now take a view of the most curious problems and phenomena in this interesting part of the mathematics.

On the nature of light.

Before we enter into any details respecting optics, we cannot help saying a few words on the nature and properties of light in general.

Philosophers are still divided, and in all probability will be so for a long time to come, in regard to the nature of light. Some are of opinion, that it is produced by an extremely fine and elastic fluid, in consequence of an undulatory motion communicated to it by the vibrations of luminous bodies, and which is propagated circularly to immense distances, and with an inconceivable rapidity. Light, according to this hypothesis, is entirely analogous to sound, which, as is well known, consists in a similar undulation of the air, the vehicle of it. Several very

specious reasons give to this opinion a considerable degree of probability, notwithstanding some physical difficulties which it is not easy to obviate.

According to Newton, light is produced from luminous bodies by the emission of particles highly rarefied, and projected with prodigious velocity. The physical difficulties which militate against the former opinion, seem to serve as proofs of the present one; for the nature and propagation of light can be conceived only in these two ways.

But, whatever may be the nature of light, it is proved that it moves with astonishing velocity, since it is well known that it employs only 7 or 8 minutes in passing from the sun to the earth; and as the distance of the sun from the earth, according to the best observations, is 24000 semi-diameters of the latter, or about 95 millions of miles, light moves at the rate of about two hundred thousand miles per second: at which rate it goes from the earth to the moon, and returns from the moon to the earth, in less than 3 seconds.

The principal properties of light, or those which form the foundation of optics, are the following:

1st. *Light moves in a straight line, as long as it passes through the same transparent medium.*

This property is a necessary consequence of the nature of light; for whatever it may be, it is a body in motion. But a body moves in a straight line if nothing obstructs or tends to turn it aside from its course; and as every thing in the same medium is equal in all directions, the light which passes through it must move in a straight lined course.

This principle of optics, as well as the following, may be proved by experiment.

2d. *Light, when it meets with a polished plane, is reflected, making the angle of reflection equal to the angle of*

incidence; and the reflection always takes place in a plane perpendicular to the reflecting surface, at the point of reflection.

That is, if AB (plate 1 fig. 1) be a ray of light, falling on a plane surface DE; and if B be the point of reflection, to find the direction of the reflected ray BC, we must conceive to be drawn through the line AB, a plane perpendicular to the surface DE, and intersecting it in the point B: if the angle CBE in this plane be then made equal to ABD, the line CB will be the reflected ray.

If the reflecting surface be a curve, as *d* B *e*, a plane touching that surface, must be conceived passing through B, the point of reflection: the reflection will take place the same as if it were produced by the point B; for it is evident that the curved surface and the plane, a tangent to it in the point B, coincide in that infinitely small part, which may be considered as a plane common to the curved surface and to the tangent plane: the ray of light therefore must be reflected from the curved surface, in the same manner as from the point B of the plane which touches it.

3d. *Light, in passing obliquely from one medium into another of a different density, is turned aside from its rectilineal direction, so as to incline towards the perpendicular when it passes from a rare medium into one that is denser, as from the air into glass or water; and vice versa.*

This proposition may be proved by two experiments, which are a kind of optical illusions.

EXPERIMENT 1.

Expose to the sun, or to any other light, a vessel ABCD (fig. 2 pl. 1), the sides of which are opake, and examine at what point of the bottom the shadow terminates. We shall here suppose that it is at E. Then fill it to the brim with water or oil, and it will be found that the shadow, instead of terminating at the point E, will reach no farther than to F.

This difference can arise only from the inflection of the ray of light SA, which touches the edge of the vessel. When the vessel is empty, this ray, proceeding in the straight line SAE, makes the shadow terminate at the point E; but when the vessel is filled with a fluid denser than air, it falls back to AF. This inflection of a ray of light, in passing obliquely from one medium into another, is called Refraction.

EXPERIMENT II.

Place at the bottom of a vessel, the sides of which are opake, at C for example, (fig. 3 pl. 1) a piece of money or any other object, and move backwards from the vessel till the object disappears; if water be then poured into the vessel, the object will immediately become visible, as well as that part of the bottom which was concealed from your sight. The reason of this is as follows:

When the vessel is empty, the eye at O can see the point C only by the direct ray CAO, which is intercepted by the edge A of the vessel; but when the vessel is full of water, the ray CD, instead of continuing its course directly to E, is refracted into DO, by diverging further from the perpendicular DP. This ray conveys to the eye the appearance of the point C, which is seen at *c*, in the straight line OD continued: the bottom therefore, in this case, appears to be raised. For the same reason, a straight stick or rod, when immersed in water, appears to be bent at the point where it meets with the surface, unless it be immersed in a perpendicular direction.

Philosophers have carefully examined the law according to which this inflection takes place, and have found that when a ray, as EF (fig. 4 pl. 1), passes from air into glass, it is refracted into FI, in such a manner, that the sine of the angle CFE and that of DFI, are in a constant ratio. Thus, if the ray EF be refracted into FI, and the ray *eF* into F*i*, the sine of the angle CFE will have the same ratio to the

sine of DFI, as the sine of the angle GFE has to that of DFI. This ratio, when the ray passes from air into common glass, is always as 3 to 2; that is to say, the sine of the angle which the refracted ray forms with the perpendicular to the refracting substance, is always two thirds of the sine of the angle formed by the incident ray with the same perpendicular.

It is to be observed, that when the latter angle, that is the distance of the incident ray from the perpendicular, which is called the angle of inclination, is very small, the angle of refraction may be considered as two thirds of it, because small angles have nearly the same ratio as their sines. We here suppose that the ray passes from air into glass; for it is well known, and may be easily proved by the table of sines, that when two angles are very small, that is, if they do not exceed 5 or 6 degrees, they are sensibly in the same ratio as their sines. Thus, in the case above, the angle of refraction IFD, will be two thirds of the angle of inclination GFE; and consequently the angle formed by the refracted ray and the incident, continued in a straight line, will be one third of it.

When the passage takes place from air into water, the ratio of the sine of the angle of inclination, and that of the angle of refraction, is that of 4 to 3; that is, the sine of the angle DFI is constantly ¾ of the sine of GFE, the angle of inclination, of the ray incident in air. Consequently, when these angles are very small, they may be considered as being in the same ratio; and the angle of refraction will be ¾ of the angle of inclination.

This proportion is the basis of all the calculations of dioptrics; and on that account ought to be well imprinted in the memory. For the discovery of it we are indebted to the celebrated Descartes; though it appears certain, by the testimony of Huygens, that a law of refraction equally constant, and which in fact is the same, was discovered before by Willebrod Snell, a Dutch mathematician. But

Vossius is wrong when he asserts, as he does in his book De Natura Lucis, that the expression of Snellius was more convenient. This learned man did not know what he said, when he attempted to speak of natural philosophy.

PROBLEM I.

To exhibit, in a darkened room, external objects, in their natural colours and proportions.

Shut the door and windows of the apartment, in such a manner, that no light can enter it, but through a small hole very neatly cut in one of the window shutters, opposite to some well frequented place or landscape; then hold a white cloth or piece of white paper opposite to the hole, and if the external objects are strongly illuminated, and the room very dark, they will appear as if painted on the cloth or paper, in their natural colours, but inverted.

The experiment, performed in this simple manner, will succeed well enough to surprise those who see it for the first time; but it may be rendered much more striking by means of a lens.

Adapt to the hole of the shutter, which in this case must be some inches in diameter, a tube having at its internal extremity a convex lens, of 4, 5, or 6 feet focus; if a piece of white cloth, or a sheet of paper, be then held at that distance from the glass, and in a direction perpendicular to the axis of the tube, the external objects will be painted on the cloth or paper, with much more distinctness and vivacity of colouring, than in the preceding experiment; and in so accurate a manner, that the features of the person seen may be distinguished. This spectacle is highly amusing, especially when a public place, a promenade filled with people, &c, are exhibited.

This painting indeed is inverted, which destroys a little of the effect; but different methods may be employed to make it appear in its natural position: it is however to be regretted that this cannot be done without injuring the

distinctness, or lessening the field of the picture. Those who may be desirous of seeing the objects erect, must proceed in the following manner:

At about half the focal distance of the lens place a plane mirror, inclined at an angle of 45°, so that it may reflect downwards the rays proceeding from the lens; if you then place horizontally below it a sheet of paper, the image of the external objects will appear painted on the paper, and in their natural situation to those who have their backs turned towards the window. Fig. 5, represents the mechanism of this inversion, of which a clear idea cannot be formed without some knowledge of catoptrics.

The sheet of paper may be extended on a table, and nothing will be necessary but to dispose the glass and mirror at such a height from the paper, that the objects may be distinctly painted on it. By these means a landscape, or edifice, &c, may be exactly delineated with great ease.

PROBLEM II.

To construct a portable Camera Obscura.

Construct a wooden box ABCD (fig. 6 pl. 1), about a foot in height, as much in breadth, and 2 or 3 feet in length, according to the focus of the lenses employed. To one of the sides adapt a tube EF, consisting of two, one thrust within the other, that it may be lengthened or shortened at pleasure; and in the anterior aperture of the first tube fix two lenses, convex on both sides, and about 7 inches in diameter, so as almost to touch each other; place another of about 5 inches focal distance in the interior aperture; and at about the middle of the box, taken lengthwise, dispose in a perpendicular direction a piece of oiled paper GH, stretched on a frame: in the last place, make a round hole I, in the side opposite to the tube, and sufficiently large to receive both eyes.

When you are desirous of viewing any objects, turn the

tube, furnished with the lenses, towards them; and adjust it, either by drawing it out or pushing it in, till the image of the objects is painted distinctly on the oiled paper.

The following is the description of another camera obscura, invented by Gravesande, and of which he gave an account at the end of his Essay on Perspective.

This machine is shaped almost like a hackney chair; the top of it is rounded off towards the back part, and before it swells out into an arch at about the middle of its height. See plate II fig. 7, where this machine is represented with the side opposite to the door taken off, in order that the interior part of it may be exhibited to view.

1st. The board A, in the inside, serves as a table: it turns on two iron hinges fastened to the fore part of the machine, and is supported by two small chains, that it may be raised to facilitate entrance into the machine.

2d. To the back of the machine, on the outside, are affixed four small staples C, C, C, C, in which slide two pieces of wood DE, DE, 3 inches in breadth; and through these pass two other pieces, serving to keep fast a small board F, which by their means can be moved forwards or backwards.

3d. At the top of the machine is a slit PMOQ, 9 or 10 inches in length, and 4 in breadth, to the edges of which are affixed two rules in the form of a dove tail: between these slides a board of the same length, having a round hole, of about 3 inches diameter, in the middle, furnished with a nut, that serves to raise or lower a tube about 4 inches in height, which has a screw corresponding to the nut. This tube is intended for receiving a convex glass.

4th. The moveable board, above described, supports a square box X, about 7 and a half inches in breadth, and 10 in height, the fore part of which can be opened by a small door, and in the back part of the box towards the bottom is a square aperture N, of about 4 inches in breadth, which may be shut at pleasure by a moveable board.

5th. Above this square aperture is a slit parallel to the horizon, and which occupies the whole breadth of the box. It serves for introducing into the box a plane mirror, which slides between two rules so that the angle it makes with the horizon towards the door B is $112\frac{1}{2}°$, or $\frac{5}{4}$ of a right angle.

6th. The same mirror, when necessary, may be placed in a direction perpendicular to the horizon, as seen at H, by means of a small iron plate adapted to one of its sides, and furnished with a screw which enters a slit formed in the top of the machine, and which may be screwed fast by a nut.

7th. Within the box is another small mirror LL, which turns on two pivots, fixed a little above the slit of No. 5, and which, being drawn up or pushed down by the small rod S, may be inclined to the horizon at any angle whatever.

8th. That the machine may be supplied with air, a tube of tin-plate, bent at both ends, as seen fig. 8, may be fitted into one of the sides: this will give access to the air without admitting light. But, if this should not be sufficient, a small pair of bellows, to be moved by the foot, may be placed below the seat, and in this manner the air may be continually renewed.

The different uses of this machine are as follow.

I. *To represent objects in their natural situation.*

When objects are to be represented in this machine, extend a sheet of paper on the table, or rather stretched on a frame, or you may employ a piece of strong card, and fix it in such a manner as to remain immoveable.

In the tube C, (fig. 7) place a convex glass, the focus of which is nearly equal to the height of the machine above the table; open the back part of the box X, and having removed the mirror H, as well as the board F, and the rules DE, incline the moveable mirror LL, till it make with the

horizon an angle of nearly 45°, if you intend to represent objects at a considerable distance, and which form a perpendicular landscape. When this is done, all those objects which transmit rays to the mirror LL, so as to be reflected on the convex glass, will appear painted on the paper frame: the point where the images are most distinct may be found, if the tube which contains the lens be lowered or raised, by screwing it up or down.

By these means any landscape, or view of a city, &c, may be exhibited with the greatest precision.

II. *To represent objects in such a manner, as to make that which is on the right appear on the left, and vice versa.*

The box X being in the situation represented in the figure, open the door B, and having placed the mirror H in the slit, and in the situation already mentioned No. 5, raise the mirror LL till it make with the horizon an angle of $22\frac{1}{2}$ degrees; if the fore part of the machine be then turned towards the objects to be represented, which we here suppose to be at a considerable distance, they will be seen painted on the paper, but transposed from right to left.

It may sometimes be useful to make a drawing where the objects are transposed in this manner; for example, in the case when it is intended to be engraved; for as the impression of the plate will transpose the figures from right to left, they will then appear in their natural position.

III. *To represent in succession all the objects in the neighbourhood, and quite around the machine.*

Place the mirror H in a vertical position, as seen in the figure, and incline the mirror L at an angle of 45 degrees; if the former be then turned round vertically, the lateral objects will be seen painted in succession on the paper, in a very pleasant manner.

It must here be observed, that it will be necessary to cover the mirror H with a kind of box made of pasteboard, open towards the objects, and also towards the aperture M of the box X; for if the mirror H were left entirely exposed, it would reflect on the mirror L a great many lateral rays, which would considerably weaken the effect.

IV. *To represent the image of paintings or prints.*

Affix the painting or print to the side of the board F, which is next to the mirror L, and in such a manner that it may be illuminated by the sun. But as the object in this case will be at a very small distance, the tube must be furnished with a glass, having its focal distance nearly equal to half the height of the machine above the paper: if the distance of the painting from the glass be then equal to that of the glass from the paper, the figures of the painting will be represented on the paper exactly of the same size.

The point at which the figures have the greatest distinctness, may be found, by moving backwards or forwards the board F, till the representation be very distinct.

Some attention is necessary in regard to the aperture of the convex glass.

In the first place, the same aperture may in general be given to the glass as to a telescope of the same length.

Secondly, this aperture must be diminished when the objects are very much illuminated; and vice versa.

Thirdly, as the traits appear more distinct when the aperture is small, than when it is large, if you intend to delineate the objects, it will be necessary to give to the glass as small an aperture as possible; but taking care not to extenuate the light: it will therefore be proper to have different circles of copper or of blackened pasteboard, to be employed for altering the size of the aperture, according to circumstances.

REMARK.—On the top of the Royal Observatory, at Greenwich, is an excellent camera obscura, capable of

containing 5 or 6 persons, all viewing the exhibition together. All the motions of the glasses are easily performed by one of the persons within, by means of attached rods; and the images are thrown on a large and smooth concave table, cast of plaster of Paris, and moveable up and down so as to suit the distances of the objects.

PROBLEM III.

To explain the nature of vision, and its principal phenomena.

Before we explain in what manner objects are perceived, it will be necessary to begin with a description of the wonderful organ destined for that purpose.

The eye is a hollow globe, formed of three membranes, which contain humours of different densities, and which produces in regard to external objects the same effect as the camera obscura. The first or outermost of these membranes, called the Sclerotica, is only a prolongation of that which lines the inside of the eye-lids. The second, called the Choroides, is a prolongation of the membrane which covers the optic nerve, as well as all the other nerves. And the third, which lines the inside of the eye, is an expansion of the optic nerve itself: it is this membrane, entirely nervous, which is the organ of vision; for notwithstanding the experiments in consequence of which this function has been ascribed to the choroides, we cannot look for sensation any where else than in the nerves and nervous parts.

In the front of the eye the sclerotica changes its nature, and assumes a more convex form than the ball of the eye, forming here what is called the Transparent Cornea. The choroides, by being prolonged below the cornea, must necessarily leave a small vacuity, which forms the anterior receptacle of the aqueous humour. This prolongation of the choroides terminates at a circular aperture well known under the name of the Pupil. The coloured part which surrounds this aperture is called the Iris or Uvea;

it is susceptible of dilatation and contraction, so that, when exposed to a strong light, the aperture of the pupil contracts, and in a dark place it dilates.

This aperture of the pupil is similar to that of the camera obscura. Behind it is suspended, by a circular ligament, a transparent body of a certain consistence, and having the form of a lens: it is called the Crystalline Humour, and, in this natural camera obscura, performs the same office as the glass in the artificial one.

By this description it may be seen that, between the cornea and the crystalline humour, there is a sort of chamber, divided into nearly two equal parts, and another between the crystalline humour and the retina. The first is filled with a transparent humour similar to water, on which account it has been called the Aqueous Humour. The second chamber is filled with a humour of the same consistence almost as the white of an egg: it is known by the name of the Vitreous Humour. All these parts may be seen represented plate 3, fig. 9; where *a* is the sclerotica, *b* the cornea, *c* the choroides, *d* the retina, *e* the aperture of the pupil, *ff* the uvea, *h* the crystalline humour, *ii* the aqueous, *kk* the vitreous, and *l* the optic nerve.

As it is evident, from the above description, that the eye is a camera obscura, but more complex than the artificial one before described, it may readily be conceived that the images of the external objects will be painted in an inverted situation, on the retina, at the bottom of it; and these images, by affecting the nervous membrane, excite in the mind the perception of light, colours, and figures. If the image be distinct and lively, the impression received by the mind is the same; but if it be confused and obscure, the perception is confused and obscure also: this is sufficiently proved by experiment. That such images really exist, may be easily shown by employing the eye of any animal, such as that of a sheep or bullock; for if the back part of it be cut off, so as to leave only the retina; and if the

cornea of it be placed before the hole of a camera obscura, the images of the external objects will be seen painted on the retina at the bottom of it.

But it may here be asked, since the images of the objects are inverted, how comes it that they are seen in their proper position? This question can have no difficulty but to those who are ignorant of metaphysics. The ideas indeed which we have of the upright or inverted situation of objects, in regard to ourselves, as well as of their distance, are merely the result of the two senses, seeing and touching combined. The moment we begin to make use of our sight, we experience by means of touching, that the objects which affect the upper part of the retina, are towards our feet in regard to those that affect the lower part, which touching tells us are at a greater distance. Hence is established the invariable connection that subsists between the sensation of an object which affects the upper part of the eye, and the idea of the lowness of that object.

But what is meant by lowness? It is being nearer the lower part of our body. In the representation of any object, the image of the lower part is painted nearer that of our feet than the image of the upper part: in whatever place the image of our feet may be painted on the retina, this image is necessarily connected with the idea of inferiority; consequently, whatever is nearest to it necessarily produces in the mind the same idea. The two sticks of the blind man of Descartes prove nothing here, and Descartes would certainly have expressed himself in the same manner, had he not adapted the doctrine of innate ideas, proscribed by modern metaphysics.

PROBLEM IV.

To construct an artificial eye, for exhibiting and explaining all the phenomena of vision.

This machine may be easily constructed from the following description. ABDE is a hollow ball of wood (fig. 10

pl. 5), 5 or 6 inches in diameter, formed of two hemispheres joined together at LM, and in such a manner, that they can be brought nearer to or separated from each other about half an inch. The segment AB of the anterior hemisphere is a glass of uniform thickness, like that of a watch; below which is a diaphragm, with a round hole, about 6 lines in diameter, in the middle of it; F is a lens, convex on both sides, supported by a diaphragm, and having its focus equal to FC when the two hemispheres are at their mean distance. In the last place, the part DCE is formed of a glass of uniform thickness, and concentric to the sphere, the interior surface of which, instead of being polished, is only rendered smooth, so as to be semi-transparent. Such is the artificial eye, to which scarcely any thing is wanting but the aqueous and vitreous humours; and these might be represented also, by putting into the first cavity common water, and into the other water charged with a strong solution of salt. But for the experiments we have in view, this is entirely useless.

This small machine however may be greatly simplified by reducing it to two tubes of an inch and a half or two inches in diameter, one thrust into the other. The first, or anterior one, ought to be furnished with a lens of about 3 inches focus; but care must be taken to cover the whole of it except the part nearest the axis, which may be done by means of a circular piece of card, having in the middle of it a hole about half an inch in diameter. The extremity of the other tube may be covered with oiled paper, which will perform the part of the retina. The whole must then be arranged in such a manner, that the distance of the glass from the oiled paper may be varied, from about 2 to 4 inches, by pushing in or drawing out one of the tubes. A machine of this kind may easily be procured by any one, and at a very small expence.

Experiment I.

The glass or the oiled paper being exactly in the focus

of the lens, if the machine be turned towards very distant objects, they will be seen painted with great distinctness on the retina. If the machine be lengthened or shortened, till the bottom part be no longer in the focus of the lens, the objects will be seen painted, not in a distinct, but in a confused manner.

Experiment II.

Present a taper, or any other enlightened object, to the machine at a moderate distance, such as 3 or 4 feet, and cause it to be painted in a distinct manner on the retina, by moving the bottom of the machine nearer to or farther from the glass. If you then bring the object nearer, it will cease to be painted distinctly; but the image will become distinct if the machine be lengthened. On the other hand, if the object be removed to a considerable distance, it will cease to be painted distinctly, and the image will not become distinct till the machine is shortened.

Experiment III.

A distinct image however may be obtained in another manner, without touching the machine. In the first case, if a concave glass be presented to the eye, at a distance which must be found by trial, the painting of the object will be seen to become distinct. In the second case, if a convex glass be presented to it, the same effect will be produced.

These experiments serve to explain, in the most sensible manner, all the phenomena of vision, as well as the origin of those defects to which the sight is subject, and the means by which they may be remedied.

Objects are only seen distinctly when they are painted in a distinct manner on the retina; but when the conformation of the eye is such, that objects, at a moderate distance, are painted in a distinct manner, those which are much nearer, or at a much greater distance, cannot be painted

with distinctness. In the first case, the point of distinct vision is beyond the retina; and if it were possible to change the form of the eye, so as to move the retina farther from that point, or the crystalline humour farther from the retina, the objects would be painted in a distinct manner. In the second case, the effect is contrary: the point of distinct vision is on this side of the retina; and, to produce distinct vision, the retina ought to be brought nearer to the crystalline humour, or the latter nearer to the retina. We are taught by experiment that in either case a change is produced, which is not made without some effort. But in what does this change consist? Is it in a prolongation or flattening of the eye? or is it in a displacement of the crystalline humour? This has never yet been properly ascertained.

In regard to sight, there are two defects, of a contrary nature, one of which consists in not seeing distinctly any objects but such as are at a distance; and as this is generally a failing in old persons, those subject to it are called Presbytæ: the other consists in only seeing distinctly very near objects; and those who have this failing are called Myopes.

The cause of the first of these defects, is a certain conformation of the eye, in consequence of which the image of near objects is only painted in a distinct manner beyond the retina. But the image of distant objects is nearer than that of neighbouring objects, or objects at a moderate distance: the image of the former may therefore fall on the retina, and distant objects will then be distinctly seen, while neighbouring objects will be seen only in a confused manner.

But to render the view of neighbouring objects distinct, nothing else is necessary than to employ a convex glass, as has been seen in the third experiment: for a convex glass, by making the rays converge sooner, brings a distinct image of the objects nearer; consequently it will

produce on the retina a distinct picture, which otherwise would have fallen beyond it.

In regard to the myopes, the case will be exactly the reverse. As the defect of their sight is occasioned by a conformation of the eye which unites the rays too soon, and causes the point where the image of objects moderately distant are painted with distinctness, to fall on this side the retina, they will receive relief from concave glasses interposed between the eye and the object; for these glasses, by causing the rays to diverge, remove to a greater distance the distinct image, according to the third experiment: the distinct image of objects which was before painted on this side the retina, will be painted distinctly on that membrane when a concave glass is employed.

Besides, myopes will discern small objects within the reach of their sight much better than the presbytæ, or persons endowed with common sight; for an object placed at a smaller distance from the eye, forms in the bottom of it, a larger image, nearly in the reciprocal ratio of the distance. Thus a myope, who sees distinctly an object placed at the distance of 6 inches, receives in the bottom of the eye an image 3 times as large as that painted in the eye of the person who does not see distinctly but at the distance of 18 inches: consequently all the small parts of this object will be magnified in the same proportion, and will become sensible to the myope, while they will escape the observation of the presbytæ. If a myope were in such a state as only to see distinctly at the distance of half an inch, objects would appear to him 16 times as large as to persons of ordinary sight, whose boundary of distinct vision is about 8 inches: his eye would be an excellent microscope, and he would observe in objects what persons of ordinary sight cannot discover, without the assistance of that instrument.

PROBLEM V.

To cause an object, whether seen near hand or at a great distance, to appear always of the same size.

The apparent magnitude of objects, every thing else being alike, is greater according as the image of the object painted on the retina occupies a greater space. But the space occupied by an image on the retina, is nearly proportioned to the angle formed by the extremities of the object, as may be readily seen by inspecting fig. 11; consequently it is on the size of the angle formed by the extreme rays, proceeding from the object, which cross each other in the eye, that the apparent magnitude of the object depends.

This being premised, let AB be the object, which is to be viewed at different distances, and always under the same angle. On AB, as a chord, describe any arc of a circle, as ACDB: from every point of this arc, as A, C, D, B, the object AB will be seen under the same angle, and consequently of the same size; for every one knows that all the angles having AB for their base, and their summits in the segment ACDB, are equal.

The case will be the same with any other arc, as A*cd*B.

PROBLEM VI.

Two unequal parts of the same straight line being given, whether adjacent or not; to find the point where they will appear equal.

On AB and BC (fig. 12 pl. 3), and on the same side, construct the two similar isosceles triangles AFB and BGC; then from the centre F, with the radius FB, describe a circle, and from the point G, with the radius GB, describe another circle; intersecting the former in D: the point D will be the place required, where the two lines appear equal.

For, the circular arcs AEBD and BeCD are, by construc-

tion, similar; and hence it follows, that the angle ADB is equal to BDC, as the point D is common to both the arcs.

REMARKS.—1st. There are a great many points, such as D, which will answer the problem; and it may be demonstrated, that all these points are in the circumference of a semi-circle, described from the point I as a centre. This centre may be found by drawing, through the summits F and G of the similar triangles AFB and BGC, the line FG, till it meet AC produced, in I.

2d. If the lines AB and BC form an angle, the solution of the problem will be still the same; the two similar arcs described on AB and BC will necessarily intersect each other in some point D, unless they touch in B; and this point D will, in like manner, give the solution of the problem.

3d. The solution of the problem will be still the same, even if the unequal lines proposed, AB and bc (fig. 13 pl. 4), are not contiguous; only care must be taken that the radii FB and Gb, of the two circles, be such, that the circles shall at least touch each other. If $AB = a$, $Bb = c$, $bc = b$, and $AC = d = a + b + c$, that the two circles touch each other, FB must be at least =

$$\tfrac{1}{2}a\sqrt{\frac{ab + ac + bc + c^2}{ab}}, \text{ or } \tfrac{1}{2}a\sqrt{\frac{ad + cd}{ab}},$$

and $Gb = \frac{1}{2}b\sqrt{\frac{ab + ac + bc + c^2}{ab}}$, or $\frac{1}{2}b\sqrt{\frac{ab + cd}{ab}}$. If these lines be less, the two circles will neither touch nor cut each other. If they be greater, the circles will intersect each other in two points, which will each give a solution of the problem. Let a, for example, be $= 3$, $b = 2$, and $c = 1$: in this case Gb will be found $= \sqrt{2} = 1{\cdot}4142$, and $BF = \frac{3}{2}\sqrt{2} = 2{\cdot}1213$, when the circles just touch each other *.

4th. In the last place, if we suppose three unequal and

* A considerable error in the original has been here corrected, both in the algebraical expressions and in the numeral values.

contiguous lines, as AB, BC, CD (fig. 14 pl. 4), and if the point from which they shall all appear under the same angle, be required, find, by the first article, the circumference BEF &c, from every point of which the lines AB and BC appear under the same angle; find also the arc CEG from which BC and CD appear under the same angle; then the point where these two arcs intersect each other, will be the point required. But to make these two circles touch each other, the least of the given lines must be between the other two, or they must follow each other in this order, the greatest, the mean, and the least.

If the lines AB, BC, and CD be not contiguous, or in one straight line, the problem becomes too difficult to be admitted into this work. We shall therefore leave it to the ingenuity of such of our readers as have made a more considerable progress in the mathematics.

PROBLEM VII.

If AB *be the length of a parterre, situated before an edifice, the front of which is* CD, *required the point in that front from which the apparent magnitude of the parterre* AB *will be the greatest* (fig. 15 pl. 4).

Take the height CE a mean proportional between CB and CA: this height will give the point required. For if a circle be described through the points A, B, E, it will touch the line CE, in consequence of a well-known property of tangents and secants. But it may be readily seen that the angle AEB is greater than any other AeB, the summit of which is in the line CD; for the angle AeB is less than AgB, which is equal to AEB.

PROBLEM VIII.

A circle on a horizontal plane being given; it is required to find the position of the eye where its image on the perspective plane will be still a circle.

We here suppose that the reader is acquainted with the

fundamental principle of perspective representation, which consists in supposing a vertical transparent plane between the eye and the object, called the perspective plane. As rays are supposed to proceed from every point of the object to the eye, if these rays leave traces on the vertical plane, it is evident that they will there produce the same effect on the eye as the object itself, since they will paint the same image on the retina. The traces made by these rays are called the Perspective Image.

Let AC (fig. 16 pl. 4) then be the diameter of the circle on the horizontal plane ACP, perpendicular to the perspective plane; QR a section of the perspective plane, and PO a plane perpendicular to the horizon and to the line AP, in which it is required to find the point O, where, if the eye be placed, the representation *ac*, of the circle AC, shall be also a circle.

If PO be made a mean proportional between AP and CP, the point O will be the one required.

For, if PA be to PO, as PO to PC, the triangles PAO and PCO will be similar, and the angles PAO and COP will be equal: the angles PAO and CCQ, or PAO and RCO, will also be equal; hence it follows that in the small triangle *aco*, the angle at *c* will be equal to the angle OAC, and the angle at O being common to the triangles AOC and *aoc*, the other two, ACO and *cao*, will be also equal: AO then will be to CO, as *co* to *ao*; hence the oblique cone ACO will be cut in a sub-contrary manner, or sub-contrary position, by the vertical plane QR, and consequently the new section will be a circle, as is demonstrated in conic sections.

PROBLEM IX.

Why is the image of the sun, which passes into a darkened apartment through a square or triangular hole, always circular?

This problem was formerly proposed by Aristotle, who gave a very bad solution of it; for he said it arose from

the rays of the sun affecting a certain roundness, which they resumed when they had surmounted the restraint imposed on them by the hole being of a different figure. This reason is entirely void of foundation.

To account for this phenomenon, it must be observed that the rays proceeding from any object, whether luminous or not, which pass through a very small hole into a darkened chamber, form there an image exactly similar to the object itself; for these rays, passing through the same point, form beyond it a kind of pyramid similar to the first, and having its summit joined to that of the first, and which, being cut by a plane parallel to that of the object, must give the same figure but inverted.

This being understood, it may be readily conceived that each point of the triangular hole, for example, paints on paper, or on the floor, its solar image round; for every one of these points is the summit of a cone of which the solar disk is the base.

Describe then on paper a figure similar and equal to that of the hole, whether square or triangular, and from every point of its periphery, as a centre, describe equal circles: while these circles are small, you will have at first a triangular figure with rounded angles; but if the magnitude of the circles be increased more and more, till the radius be much greater than any of the dimensions of the figure, it will be observed to become rounder and rounder, and at length to be sensibly converted into a circle.

But this is exactly what takes place in the darkened apartment; for when the paper is held very near to the triangular hole, you have a mixed image of the triangle and the circle; but if it be removed to a considerable distance, as each circular image of the sun becomes then very large, in regard to the diameter of the hole, the image is sensibly round. If the disk of the sun were square, and the hole round, the image at a certain distance would, for the same reason, be a square, or in general of

the same figure as the disk. The image of the moon therefore, when increasing, is always, at a sufficient distance, a similar crescent, as is proved by experience.

PROBLEM X.

To make an object which is too near the eye to be distinctly perceived, to be seen in a distinct manner, without the interposition of any glass.

Make a hole in a card with a needle, and without changing the place of the eye or of the object, look at the latter through the hole; the object will then be seen distinctly, and even considerably magnified.

The reason of this phenomenon may be deduced from the following observations: When an object is not distinctly seen, on account of its nearness to the eye, it is because the rays proceeding from each of its points, and falling on the aperture of the pupil, do not converge to a point, as when the object is at a proper distance: the image of each point is a small circle, and as all the small circles, produced by the different points of the object, encroach on each other, all distinction is destroyed. But, when the object is viewed through a very small hole, each pencil of rays, proceeding from each point of the object, has no other diameter than that of the hole; and consequently the image of that point is considerably confined, in an extent which scarcely surpasses the size it would have, if the object were at the necessary distance; it must therefore be seen distinctly.

PROBLEM XI.

When the eyes are directed in such a manner as to see a very distant object; why do near objects appear double, and vice versa?

The reason of this appearance is as follows. When we look at an object, we are accustomed, from habit, to direct the optical axis of our eyes towards that point which we

principally consider. As the images of objects are, in other respects, entirely similar, it thence results that, being painted around that principal point of the retina at which the optical axis terminates, the lateral parts of an object, those on the right for example, are painted in each eye to the left of that axis; and the parts on the left are painted on the right of it. Hence there has been established between these parts of the eye such a correspondence, that when an object is painted at the same time in the left part of each eye, and at the same distance from the optical axis, we think there is only one, and on the right; but if by a forced movement of the eyes we cause the image of an object to be painted in one eye, on the right of the optical axis, and in the other on the left, we see double. But this is what takes place when, in directing our sight to a distant object, we pay attention to a neighbouring object situated between the optical axes: it may be easily seen that the two images which are formed in the two eyes are placed, one to the right and the other to the left of the optical axis; that is, on the right of it in the right eye, and on the left of it in the left. If the optical axis be directed to a near object, and if attention be at the same time paid to a distant object, in a direct line, the contrary will be the case. By the effect then of the habit, above mentioned, we must by one eye judge the object to be on the right, and by the other to be on the left; the two eyes are thus in contradiction to each other, and the object appears double.

This explanation, founded on the manner in which we acquire ideas by sight, is confirmed by the following fact. Cheselden relates that a man having sustained a hurt in one of his eyes by a blow, so that he could not direct the optical axes of both eyes to the same point, saw all objects double; but this inconvenience was not lasting: the most familiar objects gradually began to appear single, and his sight was at length restored to its natural state.

What takes place here in regard to the sight, takes place also in regard to the touch; for when two parts of the body which do not habitually correspond, in feeling one and the same object, are employed to touch the same body, we imagine it to be double. This is a common experiment. If one of the fingers be placed over the other, and if any small body, such as a pea for example, be put between them, so as to touch the one on the right side and the other on the left, you would almost swear that you felt two peas. The explanation of this illusion depends on the same principles.

PROBLEM XII.

To cause an object, seen distinctly, and without the interposition of any opake or diaphonous body, to appear to the naked eye inverted.

Construct a small machine, such as that represented fig. 17 pl. 4. It consists of two parallel ends, AB and CD, joined together by a third piece AC, half an inch in breadth, and an inch and a half in length. This may be easily done by means of a slip of card. In the middle of the end AB make a round hole E, about a line and a half in diameter; and in the centre of it fix the head of a pin, or the point of a needle, as seen in the figure: opposite to it in the other end make a hole F with a large pin; if you then apply your eye to E, turning the hole F towards the light or the flame of a candle, you will see the head of the pin greatly magnified, and in an inverted position, as represented at G.

The reason of this inversion is, that the head of the pin being exceedingly near the pupil of the eye, the rays which proceed from the point F are greatly divergent, on account of the hole F; and instead of a distinct and inverted image, there is painted at the bottom of the eye a kind of shadow in an upright position. But inverted images on the retina convey to the mind the idea of up-

right objects: consequently, as this kind of image is upright, it must convey to the mind the idea of an inverted object.

PROBLEM XIII.

To cause an object, without the interposition of any body, to disappear from the naked eye, when turned towards it.

For this experiment we are indebted to Mariotte: and though the consequences he deduced from it have not been adopted, it is no less singular, and seems to prove a particular fact in the animal economy.

Fix, at the height of the eye, on a dark ground, a small round piece of white paper, and a little lower, at the distance of 2 feet to the right, fix up another, of about 3 inches in diameter; then place yourself opposite to the first piece of paper, and, having shut the left eye, retire backwards, keeping your eye still fixed on the first object: when you have got to the distance of 9 or 10 feet, the second will entirely disappear from your sight.

This phenomenon is accounted for by observing, that when the eye has got to the above distance, the image of the second paper falls on the place where the optic nerve is inserted into the eye, and that according to every appearance this place of the retina does not possess the property of transmitting the impression of objects; for while the nervous fibres in the rest of the retina are struck directly on the side by the rays proceeding from the objects, they are struck here altogether obliquely, which destroys the shock of the particle of light.

PROBLEM XIV.

To cause an object to disappear to both eyes at once, though it may be seen by each of them separately.

Affix to a dark wall a round piece of paper, an inch or two in diameter, and a little lower, at the distance of two feet on each side, make two marks: then place yourself

directly opposite to the paper, and hold the end of your finger before your face in such a manner, that when the right eye is open, it shall conceal the mark on the left, and when the left eye is open, the mark on the right. If you then look with both eyes to the end of your finger, the paper, which is not at all concealed by it from either of your eyes, will nevertheless disappear.

This experiment is explained in the same manner as the former; for, by the means here employed, the image of the paper is made to fall on the insertion of the optic nerve of each eye, and hence the disappearance of the object from both.

PROBLEM XV.

An optical game, which proves that with one eye a person cannot judge well of the distance of an object.

Present to any one a ring, or place it at some distance, and in such a manner that the plane of it shall be turned towards the person's face: then bid him shut one of his eyes, and try to push through it a crooked stick, of sufficient length to reach it: he will very seldom succeed.

The reason of this difficulty may be easily given: it depends on the habit we have acquired of judging of the distances of objects by means of both our eyes; but when we use only one, we judge of them very imperfectly.

A person with one eye would not experience the same difficulty: being accustomed to make use of only one eye, he acquires the habit of judging of distances with great correctness.

PROBLEM XVI.

A person born blind having recovered the use of his sight; if a globe and a cube which he has learnt to distinguish by the touch are presented to him, will he be able on the first view without the aid of touching, to tell which is the cube, and which is the globe?

This is the famous problem of Mr. Molyneux, which he proposed to Locke, and which has much exercised the ingenuity of metaphysicians.

Both these celebrated men thought, not without reason, and it is the general opinion, that the blind man, on acquiring the use of his sight, would not be able to distinguish the cube from the globe, or at least without the aid of reasoning; and indeed, as Mr. Molyneux said, though this blind man has learned by experience in what manner the cube and the globe affect his sense of touching, he does not yet know how those objects which affect the touch will affect the sight; nor that the salient angle, which presses on his hand unequally when he feels the cube, ought to make the same impression on his eyes that it does on his sense of touching. He has no means therefore of discerning the globe from the cube.

The most he could do, would be to reason in the following manner, after carefully examining the two bodies on all sides: "On whatever side I feel the globe," he would say, "I find it absolutely uniform; all its faces in regard to my touch are the same; one of these bodies, on whatever side I examine it, presents the same figure, and the same face; consequently it must be the globe." But is not this reasoning, which supposes a sort of analogy between the sense of touching and that of seeing, rather too learned for a man born blind? It could only be expected from a Saunderson. But it would be improper here to enter into further details respecting this question, which has been discussed by Molyneux, Locke, and the greater part of the modern metaphysicians.

What was observed in regard to the blind man, restored to sight by the celebrated Cheselden, has since confirmed the justness of the solution given by Locke and Molyneux.

When this man, who had been born blind, recovered his sight, the impressions he experienced, immediately after

the operation, were carefully observed; and the following is a short account of them.

When he began to see, he at first imagined that all objects touched his eyes, as those with which he was acquainted by feeling touched his skin. He knew no figure, and was incapable of distinguishing one body from another. He had an idea that soft and polished bodies, which affected his sense of touching in an agreeable manner, ought to affect his eyes in the same way; and he was much surprised to find that these two things had no sort of connection. In short, some months elapsed before he was able to distinguish any form in a painting; for a long time it appeared to him a surface daubed over with colours; and he was greatly astonished when he at length saw his father in a miniature picture: he could not comprehend how so large a visage could be put into so small a space; it appeared to him as impossible, says the author from whom this account is extracted, as to put a cask of liquor into a pint bottle.

PROBLEM XVII.

To construct a machine by means of which any objects whatever may be delineated in perspective, by any person, though unacquainted with the rules of that science.

The principle of this machine consists in making the point of a pencil, which continually presses against a piece of paper, to describe a line parallel to that described by a point made to pass over the outlines of the objects, the eye being in a fixed position, and looking through an immoveable sight.

The frame T, T, T, T (fig. 18 pl. 5), supported in a perpendicular direction by the two pieces of wood SG, SG, passing through the two lower corners of it, is adapted for receiving a sheet of paper, on which the objects are to be traced out in perspective. The paper is extended on it, and kept in that position by being cemented at the four

corners. EE is a cross bar perpendicular to the two pieces SG, SG, and having at its extremity another piece KD, moveable on an axis at K. The latter serves to support the perpendicular rod DC, bearing a moveable sight BA, to which the eye is applied.

The piece of wood NP is moveable, and its extremity P is furnished with a slender point, terminating in a small button. Near its two extremities are fixed two pulleys, under which pass two small cords MM: these two cords are conveyed over the pulleys L, L, fixed at the corners T, T of the frame, and then around two horizontal ones R, R: by these means they fall on the other side of the frame, where they are fastened to the weight Q, which moves in a groove, so that when the weight Q rises or falls, the moveable piece of wood NP remains always in a situation parallel to itself. This piece of wood ought to be nearly in equilibrium with the weight, that it may be easily moved, when it is necessary to raise or to lower it a little: in the middle of it is fixed the pencil or crayon I.

It may now be readily conceived that, if the eye be applied to the hole A, and if the moveable piece of wood NP be moved with the hand, in such a manner as to make the end P pass over the outlines of a distant object, the point of the pencil I, will necessarily describe a line parallel and equal to that described by the point P; and consequently will trace out on the paper O, O, against which it presses, the image of the object in exact perspective.

This machine was invented by Sir Christopher Wren, a celebrated mathematician, and the architect who built St. Paul's. But if it be required to trace out any object whatever, according to the rules of perspective, the very simple means described in the following problem may be employed.

PROBLEM XVIII.

Another method, by which a person may represent an object

in perspective, without any knowledge of the principles of the art.

This method of representing an object in perspective requires, in the same manner as the preceding, an acquaintance with the rules of the art; and the kind of machine employed is much simpler; but it supposes a considerable degree of expertness in the art of drawing, or at least enough to be able to delineate in one small space what is seen in another.

To put this method in practice, construct a frame of such a size, that when looking at the object from a determinate point, it may be contained within that frame. Then fix the place of the eye before the frame, and, in regard to its plane, in whatever manner you think proper. The best position for the eye, unless you intend to make a drawing somewhat fantastical by the position of the objects, will be in a line perpendicular to the plane of the frame, at a distance nearly equal to the breadth of the frame, and at the height of about two-thirds of that of the frame. This place must be marked by means of a sight or hole, about two lines in diameter, made in the middle of a square or circular vertical plane, of about an inch or two in breadth. Then divide the field of the frame into squares of an inch or two in size, by means of threads extended from the sides, and crossing each other at right angles.

Then provide a piece of paper, and divide it, by lines drawn with a black lead pencil, into the same number of squares as the frame. When these preparations have been made, nothing is necessary but to apply your eye to the sight above mentioned, and to draw in each square of the paper that part of the object observed in the corresponding square of the frame. By these means you will obtain an exact representation of the object in perspective; for it is evident that it will be delineated such as it appears to the eye, and perfectly similar to the figure which would

remain on any transparent substance extended on the frame, if the rays, proceeding from each point of the object to the eye, or the place of sight, should leave traces on this substance. The object, or assemblage of objects, will therefore be represented in perspective with great accuracy.

REMARK.—The same means may be employed to demonstrate, in a sensible manner, without the least knowledge of geometry, the truth of the greater part of the rules of perspective; for if a straight line be placed behind the frame, in a direction perpendicular to its plane, you will see its image pass through the point of sight, or through that point of the plane of the frame which corresponds to the perpendicular let fall from the eye on that plane. If the line be placed horizontally, and if you cause it to make an angle of 45 degrees with the plane of the picture, you will see the image of it pass through one of those points called the points of distance. If this line be placed in any direction whatever, you will see its image concur with one of the accidental points. It is in these three rules that the whole of perspective almost consists.

PROBLEM XIX.

Of the apparent magnitude of the heavenly bodies on the horizon.

It is a well known phenomenon that the moon and sun, when near the horizon, appear much larger than when they are at a mean altitude, or near the zenith. This phenomenon has been the subject of much research to philosophers; and some of them have given very bad explanations of it.

Those indeed who reason superficially, ascribe it to a very simple cause, viz, refraction; for if we look obliquely, say they, at a crown piece immersed in water, it appears to be sensibly magnified. But every body knows

that the rays, which proceed from the celestial bodies, experience a refraction when they enter the atmosphere of the earth. The sun and moon are then like the crown immersed in water.

But, those who reason in this manner do not remark that, if a crown piece immersed in a denser medium appears magnified to the eye situated in a rarer medium, the contrary ought to be the case when the eye is situated in a dense medium, while the crown piece is immersed in a rarer. A fish would see the crown piece out of the water much smaller than if it were in the water. But we are placed in the dense medium of the atmosphere, while the moon and sun are in a rarer. Instead therefore of appearing larger, they ought to appear smaller; and this indeed is the case, as is proved by the instruments employed to measure the apparent magnitude of the celestial bodies: these instruments show that the perpendicular diameter of the sun and moon, when on the horizon, is shortened about two minutes, which gives them that oval form, pretty apparent, under which they often appear.

The cause of this phenomenon must therefore be sought for in a mere optical illusion; and in our opinion the following explanation is the most probable.

When an object paints on the retina an image of a determinate size, the object appears to us larger, according as we judge it to be at a greater distance; and this is the consequence of a tacit reasoning pretty just; for an object which, at the distance of 600 feet, is painted in the eye under the diameter of a line, must be much larger than that which is painted under the same diameter, though only at the distance of 60 feet. But when the sun and moon are on the horizon, a multitude of intervening objects give us an idea of great distance; whereas when they are near the zenith, as no object intervenes, they appear to be nearer us. In the former situation then they must excite an idea of magnitude, quite different from what they do in the latter.

We must however confess that this explanation is attended with some difficulties.

1st. When we look at the moon on the horizon through a tube, or through the fingers bent into the form of one, the size of it appears to be much diminished, though the fingers conceal the intervening objects in a very imperfect manner. 2d. We often see the moon rising behind a hill at a small distance, and on such occasions she appears to be exceedingly large.

These facts, which seem to overturn the explanation before given, have induced other philosophers to endeavour to find out a different one. The following is that of Dr. Smith, a celebrated writer on optics.

The celestial arch does not exhibit to us the appearance of a hemisphere, but that of a very oblate surface, the elevation of which towards the zenith is much less than its extension towards the horizon. The sun and moon also appear under the same angle, whether at the horizon or near the zenith. But the intersection of a determinate angle, at a mean distance from the summit, is less than at a greater. The projection therefore of the sun and moon, or their perspective image on the celestial arch, is less at a great distance from the horizon than in the neighbourhood of it. Consequently, when at a distance from the horizon they must appear less than when they are near it.

This explanation of the phenomenon is very specious. But may it not here be asked, why these two images, though seen under the same angle, appear one greater than the other? Are we not still obliged to have recourse to the former explanation? But for the sake of brevity we shall leave the discussion of these two questions to the reader.

It is sufficient that it is fully demonstrated that this apparent magnification is not produced by a larger image painted on the retina. In regard to the moon, it is even somewhat less; since that luminary, when on the horizon,

is farther from us, by about a semi-diameter of the earth, or a 60th part, than when she is very much elevated above the horizon. In short, this phenomenon is merely an optical illusion, whatever may be the cause, which is still very obscure, but in our opinion it seems to depend chiefly on the idea of great distance excited by the intervening objects.

PROBLEM XX.

On the converging appearance of parallel rows of trees.

The phenomenon which is the subject of this problem, is well known. Every person must have observed, that when at the extremity of a very long walk, planted on each side with trees, the sides instead of appearing parallel, as they really are, seem to converge towards the other end. The case is the same with the ceiling of a long gallery; and indeed when it is necessary to represent these objects in perspective, the sides of the walk or ceiling must be represented by converging lines; for they are really so in the small image or picture painted at the bottom of the eye.

Other considerations however are necessary, in order to give a complete explanation of the phenomenon; for as the apparent magnitude of objects is not measured by the real magnitude of the images painted in the eye, but is always the result of the judgment formed of their distance by the mind, combined with the magnitude of the image present in the eye, the sides of a walk are far from appearing to converge with so much rapidity, as the lines which form the image of them in the perspective plane, or in the eye. M. Bouguer first gave a complete explanation of what takes place on this occasion: it is as follows:

As the ceiling of a long gallery appears to an eye, placed at one extremity of it, to become lower, the case is the same with a long level walk, the sides of which are parallel; the plane of that walk, instead of appearing horizontal,

seems still to rise. For the same reason, as when, on the sea-shore, the water appears like an inclined plane which threatens the earth with an inundation. Some superstitious persons, little acquainted with the principles of philosophy, have considered this inclination as real, and the apparent suspension of the waters as a visible and continued miracle. In like manner, in the middle of an immense plain we see it rise around us, as if we were at the bottom of a very broad and shallow funnel. M. Bouguer has taught us a very ingenious method of determining this apparent inclination; but it will be sufficient here to say that, to most men, it is about 2 or 3 degrees.

Let us then suppose two horizontal and parallel lines, and an inclined plane of 2 or 3 degrees passing below our feet: it is evident that these two horizontal lines will appear to our eye as if projected on that inclined plane. But their projection on that plane will be two lines concurring in one point, viz, that where the horizontal drawn from the eye would meet it. We must therefore see these lines as convergent.

It thence follows, that if, by any illusion peculiar to the sight, the plane where the parallel lines are situated, instead of appearing inclined upwards, should appear declined downwards, the sides of the walk would appear divergent. This Dr. Smith, in his Treatise of Optics, says, is the case with the avenue at the seat of Mr. North, in the county of Norfolk. But it is to be wished that Dr. Smith had described, in a more minute manner, the position of the places.

However, we shall solve according to these principles another curious problem, which has been much celebrated among opticians.

PROBLEM XXI.

In what manner must we proceed to trace out an avenue, the sides of which, when seen from one of its extremities, shall appear parallel?

Suppose an inclined plane of two degrees and a half, and that two parallel lines are traced out on it. From the eye, suppose two planes passing through these lines, and which being continued cut the horizontal plane in two other lines; these two lines will be convergent, and if continued backwards will meet behind the spectator.

Nothing then is necessary but to find this point of concurrence, which is very easy; for any one, in the least acquainted with geometry, must perceive that it is the point where a line drawn through the eye, parallel to the above inclined plane, and in the direction of the middle of the avenue, meets with the horizontal plane. Let a line then inclined to the horizon two or three degrees, be drawn through the eye of the spectator, and in the vertical plane passing through the middle of the avenue; the point where it meets the horizontal plane will be that where the two sides of the avenue must unite. If from this point therefore, two straight lines be drawn through the two extremities of the initial breadth of the avenue, they will trace out where all the trees ought to be planted, so as to appear to form parallel sides.

If the height of the eye be supposed equal to 5 feet, and the breadth of the commencement of the avenue to be 36, the point of concurrence will be found by calculation to be 102 backwards, and the angle formed by the sides of the avenue ought to be about 18 degrees. It is difficult however to believe, that lines which form so sensible an angle will ever appear parallel to an eye within them, in whatever point it may be placed.

PROBLEM XXII.

To form a picture which, according to the side on which it is viewed, shall exhibit two different subjects.

Provide a sufficient number of small equilateral prisms, a few lines only in breadth, and in length equal to the height of the painting which you intend to make, and place

them all close to each other on the ground to be occupied by the painting.

Then cut the painting into bands equal to each of the faces of the prisms, and cement them, in order, to the faces of the same side.

When this is done, take a painting quite different from the former, and having divided it into bands in the same manner, cement them to the faces of the opposite side.

It is hence evident, that when on one side you can see only the faces of the prism turned towards that side, one of the paintings will be seen; and if the picture be looked at on the opposite side, the first will disappear, and the second only will be seen.

A painting may even be made, which when seen in front, and on the two sides, shall exhibit three different subjects. For this purpose, the picture of the ground must be cut into bands, and be cemented to that ground in such a manner, that a space shall be left between them, equal to the thickness of a very fine card. On these intervals raise, in a direction perpendicular to the ground, bands of the same card, nearly equal in height to the interval between them; and on the right faces of these pieces of card cement the parts of a second painting, cut also into bands. In the last place, cement the parts of a third picture, cut in the same manner, on the left faces of the pieces of card. It is evident that when this picture is viewed in front, at a certain distance, the bottom painting only will be seen; but if you stand on one side, in such a manner that the height of the slips of card conceals from you the bottom, you will see only the picture cemented in detached portions to the faces turned towards that side: if you move to the other side, a third painting will be seen.

PROBLEM XXIII.

To describe on a plane a distorted figure, which when seen from a determinate point shall appear in its just proportions.

A figure, such as for example as a head, may be disguised, that is to say distorted, in such a manner, as to exhibit no proportion, when the plane on which it has been drawn is viewed in front; but when viewed from a certain point it shall appear beautiful, that is to say in its just proportions. This may be done in the following manner:

Having drawn on a piece of paper, in its just proportions, the figure you intend to disguise, describe a square around it, as ABCD (fig. 19 pl. 5), and divide it into several other small squares, which may be done by dividing the sides into equal parts, for example seven, and then drawing straight lines through the corresponding points of division, as the engravers do when they intend to make a reduced drawing from a picture.

Then describe, at pleasure, on the proposed plane, a parallelogram EBFG, and divide one of the two shorter sides, as EG, into as many equal parts as DC, one of the sides of the square ABCD, which in this case are seven. Divide the other side BF, into two equal parts, in the point H, and draw from it to the points of division of the opposite side EG, as many straight lines, the two last of which will be HE and HG.

Having then assumed at pleasure, in the side BF, the point I, above the point H, as the height of the eye above the plane of the picture, draw from I to the point E, the straight line EI, which will cut those lines proceeding from the point H, in the points 1, 2, 3, 4, 5, 6, 7. Through these points of intersection draw straight lines parallel to each other, and to the base EG of the triangle EGH, which will thus be divided into as many trapeziums as there are little squares in the square ABCD. Hence, if the figure in the square ABCD be transferred to the triangle EGH, by making those parts of the outline contained in the different natural squares of ABCD, to pass through the corresponding trapeziums or perspective squares, the figure will be found to be distorted. But it may be seen exactly like its

prototype, that is, as in the square ABCD, if it be viewed through a hole K, which ought to be small towards the eye and wide towards the object, made in a small board L, placed perpendicularly in H, so that the height LK shall be equal to HI, which must never be very great, in order that the figure may be more distorted in the picture.

In the convent of the Minimes de la Place Royale there is a Magdalen at prayers, distorted in the same manner, which has some celebrity. It is the work of Father Niceron of that order, who frequently employed himself on this kind of optical amusement.

Several other anamorphoses may be made in the same manner, by painting, for example, on a curved surface, either cylindric, or conical, or spherical, a certain figure, which when seen from a determinate point shall appear regular; but as this does not succeed so well in practice as in theory, we think it needless to say any thing further on the subject, while there are so many others much more curious. Those persons who are fond of such optical curiosities, may consult la Perspective Curieuse of Father Niceron, where they will find the subject treated of at full length.

PROBLEM XXIV.

Any quadrilateral figure being given; to find the different parallelograms or rectangles of which it may be the perspective representation. Or any parallelogram, whether right-angled or not, being given, to find its position, and that of the eye, which shall cause its perspective representation to be a given quadrilateral.

Let the given quadrilateral be the trapezium ABCD (fig. 20 pl. 6) which we shall suppose the most irregular possible, having none of its sides parallel. Continue the sides AB and CD, till they meet in F, and the sides AD and BC, till they meet in E; then draw EF, and through the point A, draw GH, parallel to it. Whatever be the position of the

eye, provided what is called the point of sight be in the line EF, not only in EF, but in the continuation of it on both sides; the object, of which the quadrilateral ABCD is the perspective representation, will be a parallelogram.

For, all persons, acquainted with the rules of perspective, know that lines parallel to each other on a horizontal plane, when represented in perspective meet in one point of the line parallel to the horizon, drawn through the point of sight. Thus, all the lines, perpendicular to the ground line, meet in the point of sight itself: all those which form with that line an angle of 45°, concur in what is called the points of distance; and those which form a greater or less angle, concur in other points, which are always determined by drawing from the eye to the picture a line parallel to those of which the perspective representation is required. All the lines then, which in the picture concur in points situated in the line of the point of sight, are images of horizontal and parallel lines. Thus, the lines on the horizontal plane, which have as representatives in the picture the lines BC and AD, are parallel; and the case is the same with those which give the lineal images AB and DC. But two pairs of parallel lines necessarily form, by their intersection, a parallelogram. The object then of which the quadrilateral ABCD is the image, to an eye situated in the line FE, wherever the point of sight may be, is a parallelogram.

This being demonstrated, we shall first suppose that the required object is a rectangle. To find in this case the place of the eye, divide the distance FE into two equal parts in I, and suppose the eye situated in such a manner that the perpendicular, drawn from its place to the painting, shall fall on the point I; and that the distance is equal to IE or IF: the points F and E will then be what, in the language of perspective, is called the points of distance. Continue the lines CB and CD to G and H in

the ground line: the lines HCF and ABF will be the images of the lines which form with the ground line angles of 45 degrees. The case will be the same with those of which GCE and ADE are the images. If the indefinite lines H*dc* and A*b*, inclined to the ground line at an angle of 45 degrees, be then drawn on the one side, and on the other, and in a contrary direction, the lines G*bc* and A*d*, inclined also at half a right angle, these lines will necessarily meet at right angles, and form the rectangle A*bcd*.

If the point of sight be supposed in another point, for example E, that is, if we suppose the eye to be directly opposite to the point E, and at a distance equal to EK, after drawing EL and FM perpendicular to the ground line in the plane of the picture, we must draw to the same ground line, in the horizontal plane, the perpendicular LN, equal to EK, and then the line NM, making with the ground line the angle LMN. If we then draw to the points G and A the indefinite perpendiculars AΔ and GK, and through the points A and H the indefinite lines HK and A*β*, making with the ground line angles equal to LMN, and in a contrary direction; these two pairs of lines will meet in *β*, K, Δ, and evidently form an oblique parallelogram, which will be the object of which BCDA is the representation, to an eye situated opposite to E, and at a distance from the picture equal to EK.

If the sides A*b* and *cd*, in the rectangle A*bcd*, were divided into equal parts by lines parallel to the other sides, it is evident that these parallels, being continued, would cut the line AG into as many equal parts. The case would be the same with lines parallel to A*b* and *cd* dividing into equal portions, the sides A*d* and *bc:* the line AH would likewise be divided by them into equal parts. Thus we have the means of dividing the trapezium ABCD, if necessary, into lozenges, which would be the representation of the squares into which A*bcd* might be divided.

We shall give hereafter the solution of a very curious problem, in regard to ornamental gardening, which is a consequence of the one here solved.

OF PLANE MIRRORS.

Plane mirrors are those the reflecting surface of which is plane; as is the case with the common glass mirrors used for decorating apartments. Plane mirrors may be made also of metal. Of this kind were those of the ancients; but since the invention of glass, metallic mirrors are never used, except small ones for certain optical instruments, where it is necessary to avoid the double reflection produced by glass, one from the anterior and the other from the posterior surface. It is the latter which gives the liveliest image; for if the silvering be scraped from the back of a mirror, you will see the bright image immediately disappear, almost entirely, and that which remains in its place will scarcely be equal to that produced by the nearer surface.

But in catoptrics, in general, the two surfaces of a mirror are supposed to be at such a small distance from each other, as to produce only one image; otherwise the determinations given by this science would require to be greatly modified.

PROBLEM XXV.

A point of the object B, *and the place of the eye* A, *being given; to find the point of reflection on the surface of a plane mirror*, (fig. 21 pl. 6).

Through B, the given point of the object, and A, the place of the eye, conceive a plane perpendicular to the mirror, and cutting it in the line CD: from the point B, draw BD perpendicular to CD, and continue it to F, so that DF and DB shall be equal: if through the points F and A, the line AF be drawn, intersecting CD in E, the point E will be the point of reflection: BE will be the incident

ray; EA the reflected ray; and BED, the angle of incidence, and AEC, the angle of reflection, will be equal.

For it is evident, by the construction, that the angles BED and DEF are equal; but the angles DEF and AEC are also equal, being vertical angles; therefore &c.

PROBLEM XXVI.

The same supposition being made as before; to find the place of the image of the point B.

The place of the image of the point B is exactly in the point F. But we shall not assign as the reason what is commonly given in books on catoptrics, viz, that in mirrors of every kind the place of the image is in the continuation of the reflected ray, where it is intersected by the perpendicular drawn from the point of the object to the reflecting surface: for what effect can this perpendicular, which is merely imaginary, have to fix the image, in this manner, in the point where it meets with the reflected ray continued, rather than in any other point? This principle then is ridiculous, and void of foundation.

It is however true that, in plane mirrors, the place where the object is perceived, is in the point where the above perpendicular meets with the reflected ray produced; but this is accidental, and the reason is as follows.

All the rays which emanate from the object B, and are reflected by the mirror, meet, if produced, in the point F: their arrangement then, in regard to the eye, is the same as if they proceeded from the point F. Consequently they must make the same impression on the eye, as if the object were in F; for the eye would not be otherwise affected if they really proceeded from that point.

Hence it may be concluded that, in a plane mirror, the object appears to be as far behind, as it is distant from the mirror.

It therefore follows, that AF, the distance of the image

F from the eye, is equal to the sum of BE, the ray of incidence, and EF the ray of reflection, since BE and EF are equal.

It thence follows also, that when the plane mirror is parallel to the horizon, as CD, a perpendicular object, such as BD, must appear inverted.

In the last place, when we look at ourselves in a mirror, the left seems to be on the right, and the right on the left.

PROBLEM XXVII.

Several plane mirrors being given, and the place of the eye, and of the object; to find the course of the ray proceeding from the object to the eye, when reflected two, three, or four times.

Let there be two mirrors, AB and CD, (fig. 22 pl. 6), and let OFE be the perpendicular, drawn from the object O to the mirror AB, and continued beyond it, so that FE be equal to OF; and let SHI be the perpendicular drawn from the eye to the mirror CD, and continued till HI be equal to HS; join the points I and E by the line EI, which will intersect the mirrors in G and K; and if the lines OG, GK, and KS be then drawn, they will represent the course of the ray, proceeding from the point O to the eye by two reflections.

Or, from the point E, the first part of the construction remaining the same, let fall, on the mirror CD, the perpendicular ELM, and continue it beyond it, till LM be equal to LE; draw the line SM, intersecting CD in K; and from the point K, the line KE, intersecting AB in G: if GO be also drawn, the lines OG, GK, and KS will represent the course of the ray proceeding from the point O, and conveyed to the eye by two reflections.

In this case, the point M will be the image of the point O, and the distance SM will be equal to the sum of the rays SK, KG, and GO.

If we suppose three mirrors, and three reflections, the

course which the incident ray must pursue, in order to reach the eye, may be found in the same manner. For this purpose, let OI (fig. 23 pl. 7) be the perpendicular drawn from the object to the mirror AB, and let HI be equal to HO. From the point I draw IK perpendicular to CB, produced if necessary, and make [illegible] equal to MI: from the point K let fall on DC produced the perpendicular KN, and continue it to L, so that LN shall be equal to KN: draw SL, which will intersect CD in G, and from the point G the line GK, which will intersect CB in F; if the line FI, intersecting AB in E, be then drawn from the point F, and also the line EO, then the line EO will be that according to which the incident ray must fall on the first mirror, to reach to the eye S, after three reflections at E, F, and G.

In this case the point L will be the apparent place of the image of the object, to an eye situated in S; and the distance SL will be equal to SG, GF, FE, and EO, taken all together.

The application of this problem is generally shown at the game of billiards; but as we have already treated that subject, under the head mechanics, the reader is referred to that article.

PROBLEM XXVIII.

Various properties of plane mirrors.

I. In plane mirrors, the image is always equal and similar to the object. For it may be easily demonstrated, that as each point of the image seems to be as far within the mirror as the object is distant from it, each point of the image is similarly situated, and at an equal distance in regard to all the rest, as in the object: the result must therefore necessarily be the equality and similarity of the image and object.

II. In a plane mirror, what is on the right appears in the object to be on the left, and vice versa. This may be easily proved in the following manner. If a piece of

common writing be held before a mirror, it cannot be read: as the word GENERAL, for example, will appear under this form, LARENEG*; but, on the other hand, if the latter word be presented to the mirror GENERAL will appear. This affords the means of forming a sort of secret writing; for if we write from right to left, it cannot be read by those ignorant of the artifice; but those acquainted with it by holding the writing before a mirror, will see it appear like common writing. This method however must not be employed for concealing secrets of great importance, as there are few persons to whom it is not known.

III. In a plane mirror, when you can see yourself at full length, at whatever distance you remove from it, you will always see your whole body; and the height of the mirror occupied by your image will always be equal to the half of your height.

IV. If one of the sun's rays be made to fall on a plane mirror, and if an angular motion be given to the mirror, the ray will be seen to move with a double angular motion; so that when the mirror has passed over 90°, the ray will have passed over 180°.

V. If a plane mirror be inclined to a horizontal surface, at an angle of 45°, its image will be vertical.

VI. If two plane mirrors be disposed parallel to each other; and if any object, such as a lighted taper, be placed between them; you will see in each a long series of tapers, which would be extended in infinitum, did not each image become fainter in proportion as the reflections, by which they are produced, become more numerous.

VII. When two mirrors are disposed in such a manner as to form an angle of at least 120°, several images will be

* This is a mistake; the letters individually will be inverted as well as the word, and instead of LARENEG it will be LAЯƎИƎƆ, as may be proved by trial: to use this artifice therefore for secret writing it would be necessary to invert the shape of each letter as well as its position in the word.

seen, according to the position of the eye. If the angle of the mirrors be diminished, without changing the place of the eye, these images will be seen to increase in number, as if they emerged from behind an opake body.

It must be observed, that all these images are in the circumference of a circle, described from the point where the mirrors meet, and passing through the place of the object.

Father Zacharias Traber, a Jesuit, in his Nervus Opticus, and Father Tacquet, in his Optics, have carefully examined all the cases resulting from the different angles of these mirrors, as well as from the different positions of the eye and the object. To these we refer the reader.

VIII. When a luminous object, such as the flame of a taper, is viewed in a plane glass mirror of some thickness, several images of that object are perceived; the first of which, or that nearest the surface of the glass, is less brilliant than the second; the latter is the most brilliant of the whole; and after it, a series of images less and less brilliant are observed, to the number sometimes of five or six.

The first of these images is produced by the anterior surface of the glass, and the second by the posterior, which being covered with tin-foil, must produce a more lively reflection: it is therefore the most brilliant of the whole. The rest are produced by the rays of the object, which reach the eye after being several times reflected from the anterior, as well as posterior, side of the mirror. This phenomenon may be explained as follows.

Let VX (fig. 24 pl. 7) be the thickness of the glass, A the object, and O the place of the eye, which we shall suppose to be both equally distant from the mirror. Of all the small bundles of incident rays, there is one AB, which being reflected by the anterior surface in B, is conveyed to the eye by the line BO, and forms at A′ the first image of the object. Another, as AC, penetrates the glass, and being refracted into the line CD, is wholly reflected into

DE, on account of the opacity of the posterior side of the mirror, and being again refracted at E proceeds to O, and forms at A″ the liveliest image of the point A.

Another small bundle AF penetrates also the glass, is refracted along the line FG, and reflected in the direction of GB, from which a part of it issues, but cannot reach the eye; the other part is reflected in the direction BH, and then into HI, from which a small part is still reflected, but the remainder issues from the glass and is refracted in the direction of the line IO, by which it reaches the eye: consequently it produces the third image, at A‴, weaker than the other two.

The fourth image is formed by a bundle of incident rays, which experience two refractions like the rest, and five reflections, viz, three from the posterior surface of the glass, and two from the anterior. In regard to the fifth, it requires two refractions, and seven reflections, viz, three from the anterior surface, and four from the posterior; and so of the rest. It may hence be easily conceived how much the brightness of the images must be diminished, and therefore it is very uncommon to see more than four or five.

PROBLEM XXIX.

To dispose several mirrors in such a manner, that you can see yourself in each of them, at the same time.

It may be readily conceived that, to produce this effect, nothing is necessary but to dispose the mirrors on the circumference of a circle, in such a manner, that they shall correspond with the chords of that circle; if you then place yourself in the centre, you will see your image in all the mirrors at the same time.

REMARK.—If these mirrors are disposed according to the sides of a regular polygon, of an equal number of sides, such as a hexagon or octagon, which seem to be fittest for the purpose, and if all the mirrors are perfectly

vertical and plane, they will form a kind of cabinet, which will appear of an immense extent, and in whatever part of it you place yourself, you will see your image, and immensely multiplied.

If this cabinet be illuminated in the inside, by a lustre placed in its centre, it will exhibit a very agreeable spectacle, as you will see long rows of lights towards whatever side your sight is directed.

PROBLEM XXX.

To measure, by means of reflection, a vertical height, the bottom of which is inaccessible.

We shall here suppose that AB (fig. 25 pl. 7) the vertical height to be measured, is that of a tower, steeple, or such like. Place a mirror at C, in a direction perfectly horizontal; or, because this is very difficult, and as the least aberration might produce a great error in the measurement, place in C a vessel containing water, which will reflect the light in the same manner as a mirror. The eye which receives the reflected ray being at O, measure with care the height OD above the horizontal plane of the mirror at C; measure also DC as well as CB, if the latter is accessible, and then say: As CD, is to DO, so is CB to a fourth proportional BA, which will be the height required.

But if the bottom of the tower be not accessible, to measure the height AB, we must proceed as follows:

Having performed every part of the preceding operation, except measuring CB, which by the supposition is impossible, take another station, as *c*, and place there a mirror, or vessel of water: then taking your station in *d*, from which you can see the point A, by means of the reflected ray *co*, measure *cd* and *do*. When this is done, you must employ the following proportion: As the difference between CD and *cd* is to CD, so is *cc*, the distance between the two points of reflection, to a fourth proportional, which will be the distance BC, before unknown.

When BC is known, nothing is necessary but to make use of the proportion indicated in the first case, to give the height AB.

We do not consider this operation as susceptible of much accuracy in practice. Methods purely geometrical, if good instruments are employed, ought always to be preferred; but we should perhaps have been considered as guilty of an omission had we taken no notice of this geometrico-catoptric speculation, though it has never perhaps been put in practice.

PROBLEM XXXI.

To measure an inaccessible height by means of its shadow.

Fix a stick in a perpendicular direction, on a plane perfectly horizontal, and measure the height of it above that plane, which we shall suppose to be exactly 6 feet. When the sun begins to sink towards the horizon in the afternoon, mark on the ground which is accessible the point C (fig. 26 pl. 8), where the shadow of the summit of the tower falls, and also the point *c* the extremity of the shadow of the stick erected perpendicularly on the same plane: at the end of 2 hours, more or less, mark, as speedily as possible, the two points D and *d*, which will be the summits of the shadows at that period; then join the two points of the shadow of the summit of the tower, by means of a straight line, and measure their distance; measure also, in like manner, the line which joins the two points *c* and *d* of the shadow of the stick; after which you will have nothing to do but to employ the following proportion: As the length of the line *cd* which joins the two points of the shadow of the stick, is to the height of the stick *ab*, so is the length of the line CD which joins the two points of the shadow of the tower, to the height of the tower AB.

It requires only an acquaintance with the first principles of geometry to be able to perceive, merely by inspecting fig.

26, that the pyramids BADC and *badc* are similar; consequently that *cd* is to *ab* as CD to AB, which is the height required.

PROBLEM XXXII.

Of some tricks or kinds of illusion, which may be performed by means of plane mirrors.

Many curious tricks, capable of astonishing those who have no idea of catoptrics, may be performed by the combination of several plane mirrors. Some of these we shall here describe.

1st. *To fire a pistol over your shoulder and hit a mark, with as much certainty as if you took aim at it in the usual manner.* Fig. 27 pl. 8.

To perform this trick, place before you a plane mirror, so disposed, that you can see in it the object you propose to hit; then rest the barrel of the pistol on your shoulder and take aim, looking at the image of the pistol in the glass as if it were the pistol itself; that is, in such a manner, that the image of the object may be concealed by the barrel of the pistol: it is evident that if the pistol be then fired, you will hit the mark.

2d. *To construct a box in which heavy bodies, such as a ball of lead, will appear to ascend contrary to their natural inclination.*

Construct a square box, as ABCD (fig. 28), where one of the sides is supposed to be taken off, in order to show the inside; and fix in it a board HGDC, so as to form a plane, somewhat inclined, with a serpentine groove in it of such a size, that a ball of lead can freely roll in it and descend. Then place the mirror HGFI in an inclined position, as seen in the figure, and make an aperture opposite to it at M, in the side of the box, but so disposed that the eye, when applied to it, can see only the mirror, and not the inclined plane HD. It may be easily perceived that the

image of this plane, viz. HLKG, will seem to be a plane almost vertical, and that a body which rolls from G to C, along the serpentine groove, will appear to ascend in a similar direction from G to L. Hence, if the mirror is very clean, so as not to be observed, or if only a faint light be admitted into the box, which will tend to conceal the artifice, the illusion will be greater, and those not acquainted with the deception will have a good deal of difficulty to discover it.

3d. *To construct a box in which objects shall be seen through one hole, different from what were seen through another, though in both cases they seem to occupy the whole box.*

Provide a square box, which, on account of its right angles, is the fittest for this purpose, and divide it into four parts, by partitions perpendicular to the bottom, crossing each other in the centre. To these partitions apply plane mirrors, and make a hole in each face of the box, to look through; but disposed in such a manner, that the eye can see only the mirrors applied to the partitions, and not the bottom of the box. In each right angle of division formed by the partitions, place some object, which, being repeated in the lateral mirrors, may form a regular representation, such as a parterre, a fortification or citadel, a pavement divided into compartments, &c. That the inside of the box may be sufficiently lighted, it ought to be covered with a piece of transparent parchment.

It is evident that, if the eye be applied to each of the small apertures formed in the sides of the box, it will perceive as many different objects, which however will seem to occupy the whole inside of it. The first will be a regular parterre, the second a fortification, the third a pavement in compartments, and the fourth some other object.

If several persons look at the same time through these holes, and then ask each other what they have seen, a scene highly comic to those acquainted with the secret may ensue, as each will assert that he saw a different object.

REMARK.—To render the parchment employed for covering optical machines, such as the above, more transparent, it ought to be repeatedly washed in a clear ley, which must be changed each time: it is then to be carefully extended, and exposed to the air to dry.

If you are desirous of giving it some colour, you may employ, for green, verdigrise diluted in vinegar, with the addition of a little dark green; for red, an infusion of Brasil wood; for yellow, an infusion of yellow berries: the parchment afterwards ought to be now and then varnished.

4th. *In a room on the first floor, to see those who approach the door of the house, without looking out at the window, and without being observed.*

Under the middle of the architrave of the window place a mirror, with its face downwards, and a little inclined towards the side of the apartment, so that it shall reflect to the distance of some feet from the bottom of the window, or on the bottom itself, objects placed before and near the door of the house. But as the objects by these means will be seen inverted, in which case it will be difficult to distinguish them, and as it is fatiguing and inconvenient to look upwards, fix another plane mirror in a horizontal position, in the place to which the image of objects is reflected by the first mirror. As this second mirror will exhibit the objects in their proper position, they can be better distinguished. They will appear however at a much greater distance, and as if placed perpendicularly on a plane, somewhat inclined, and almost in such a situation as they would be seen in if you looked downwards from the win-

dow; which will be sufficient in general to enable you to distinguish those with whom you are acquainted.

Two mirrors arranged in this manner are represented fig. 29 pl. 6.

Ozanam, and others before him, who published Mathematical Recreations, propose by way of problem, to show a jealous husband what his wife is doing in another apartment. To bore a hole near the ceiling in the partition wall which separates two apartments, and fix a horizontal mirror, half in the one room and half in the other, to reflect by means of another mirror placed opposite to it, the image of what might take place in one of these rooms, is certainly an ingenious idea; but there is reason to think that neither Ozanam nor his predecessors were jealous husbands, or that they had a singular dependance on the folly and stupidity of the two lovers.

PROBLEM XXXIII.

To inflame objects, at a considerable distance, by means of plane mirrors.

Arrange a great number of plane mirrors, each about six or eight inches square, in such a manner that the solar rays reflected from them may be united in one focus. It is evident, and has been proved by experience, that if there are a sufficient number of these mirrors, as 100 or 150 for example, they will produce in their common focus a heat capable of inflaming combustible bodies, and even at a very great distance.

This was, no doubt, the invention employed by Archimedes, if he really burnt the fleet of Marcellus by means of burning mirrors, as we are told in history; for Kircher, when at Syracuse, observed that the Roman ships could not have been at a less distance from the walls of the city than twenty-three paces. But it is well known that the focus of a concave spherical mirror is at the distance of half its radius; consequently the mirror employed by Ar-

chimedes must have been a portion of a sphere of at least 46 paces radius, the construction of which would be attended with insurmountable difficulties. Besides, can it be believed that the Romans, at so short a distance, would have suffered him to make use of his machine without interruption? On the contrary, would they not have destroyed it by a shower of missile weapons?

Anthemius of Tralles, the architect and engineer who lived under Justinian, is the first who, according to the account of Vitellio, conceived the idea of employing plane mirrors for burning*; but we are not told whether he ever carried this method into execution. It is to Buffon that we are indebted for a proof of its being practicable. In the year 1747, this eminent naturalist caused to be constructed a machine consisting of 360 plane mirrors, each 8 inches square, and all moveable on hinges in such a manner, that they could be made to assume any position at pleasure. By means of this machine he was able to burn wood at the distance of 200 feet. Buffon's curious paper on this subject may be seen in the Memoirs of the Academy of Sciences for the year 1748.

That the ancients made use of burning glasses is evident from a passage in a play of Aristophanes, called the Clouds, where Strepsiades tells Socrates, that he had found out an excellent method to defeat his creditors, if they should bring an action against him. His contrivance was, that he would get from the jewellers a certain transparent stone, which was used for kindling fire, and then, standing at a distance, he would hold it to the sun, and melt down the wax on which the action was written.

The astonishing philosophico-military exploit of Archimedes may deserve some farther notice. That exploit has been recorded by Diodorus Siculus, Lucian, Dion, Zonaras, Galen, Anthemius, Tzetzes, and other ancient writers. The

* Histoire des Mathematiques par Montucla, vol. I. p. 329.

account of Tzetzes is so particular, that it suggested to father Kircher the specific method by which Archimedes probably effected his purpose. "Archimedes," says that author, "set fire to the fleet of Marcellus by a burning glass, composed of small square mirrors, moving every way upon hinges; and which, when placed in the sun's rays, reflected them on the Roman fleet, so as to reduce it to ashes at the distance of a bow-shot." This account gained additional probability by the effect which Zonaras ascribes to the burning mirror of Proclus, by which he affirms, that the fleet of Vitellius, when besieging Byzantium, now Constantinople, was utterly consumed. But perhaps no historical testimony could have gained belief to such extraordinary facts, if similar ones had not been seen in modern times. In the Memoirs of the French Academy of Sciences for 1726, p. 172, we read of a plane mirror, of 12 inches square, reflecting the sun's rays to a concave mirror 16 inches in diameter, in the focus of which bodies were burnt at the distance of 600 paces. Speaking of this mirror, father Regnault asks, (in his Physics, vol. 3. disc. 10). "What would be the effect of a number of plane mirrors, placed in a hollow truncated pyramid, and directing the sun's rays to the same point? Throw the focus, said he, a little farther, and you re-discover or verify the secret of Archimedes." This was actually effected by M. Buffon: in the year 1747 he read to the Academy an account of a mirror, which he had composed of an assemblage of plane mirrors, which made the sun's rays converge to a point at a great distance.

Of Spherical Mirrors both Concave and Convex.

A spherical mirror is nothing else than a portion of a sphere, the surface of which is polished so as to reflect the light in a regular manner. If it be the convex surface that is polished, it will form a convex spherical mirror; if it be the concave surface, it will be a concave mirror.

We must here first observe, that when a ray of light falls on any curved surface whatever, it will be reflected in the same manner as from a plane touching the point of that surface where it falls. Thus, if a tangent be drawn at the point of reflection to the surface of a spherical mirror, in the plane of the incident ray and of the centre, the ray will be reflected, making with that tangent an angle of reflection equal to the angle of incidence.

PROBLEM XXXIV.

The place of an object, and that of the eye being given; to determine in a spherical mirror, the point of reflection, and the place of the image.

The solution of these two problems is not so easy in regard to spherical as to plane mirrors; for when the eye and the object are at unequal distances from the mirror, the determination of the point of reflection necessarily depends on principles which require the assistance of the higher geometry; and this point cannot be assigned in the circumference of the circle without employing one of the conic sections. For this reason, we shall omit the construction, and only observe that there is one extremely simple, in which two hyperbolas between their asymptotes are employed: one of these determines the point of reflection on the convex surface, and the second the point of reflection on the concave surface.

It will be sufficient for us here to take notice of one property belonging to this point. Let B be the object (fig. 30 pl. 9), A the place of the eye, E the point of reflection from the convex surface of the spherical mirror DI the centre of which is C; also let FG be a tangent to the point E, in the plane of the lines BC and AC, which it meets in I and G; and let the reflected ray AE, when produced, intersect the line BC in H: the points H and I will be so situated, that we shall have the following proportion: as BC is to CH, so is BI to IH.

In like manner, if BE be produced till it meet AC in h, we shall have, AC : ch :: Ai : ih; proportions which will be equally true in the case of reflection from a concave surface.

In regard to the place of the image, opticians have long admitted it as a principle that it is in the point H, where the reflected ray meets the perpendicular drawn from the object to the mirror. But this supposition, though it serves pretty well to show how the images of objects are less in convex, and larger in concave, than they are in plane mirrors, has no foundation in physics, and at present is considered as absolutely false.

A more philosophical principle advanced by Dr. Barrow is, that the eye perceives the image of the object in that point where the rays forming the small divergent bundle, which enters the pupil of the eye, meet together. It is indeed natural to think that this divergency, as it is greater when the object is near and less when it is distant, ought to enable the eye to judge of distance.

By this principle also we are enabled to assign a pretty plausible reason for the diminution of objects in convex, and their enlargement in concave mirrors; for the convexity of the former renders the rays, which compose each bundle that enters the eye, more divergent than if they fell on a plane mirror; consequently the point where they meet in the central ray produced, is much nearer. It may even be demonstrated, that in convex mirrors it is much nearer, and in concave much farther distant than the point H, considered by the ancients, and the greater part of the moderns, as the place of the image. In short, it is concluded that in convex mirrors this image will be still more contracted, and in concave ones more extended, than the ancients supposed; which will account for the apparent enlargement of objects in the latter, and their diminution in the former.

We must however allow that even this principle is at-

tended with difficulties, which Dr. Barrow, the author of it, does not conceal, and to which he confesses he never saw a satisfactory answer. This induced Dr. Smith, in his Treatise on Optics, to propose another; but we shall not here enter into a discussion on this subject, as it would be too dry and abstruse for the generality of readers.

PROBLEM XXXV.

The principal properties of spherical mirrors, both convex and concave.

1st. The first and principal property of convex mirrors is, that they represent objects less than they would be if seen in a plane mirror at the same distance. This may be demonstrated independently of the place of the image: for it can be shown that the extreme rays of an object, however placed, which enter the eye after being reflected by a convex mirror, form a less angle, and consequently paint a less image in the retina, than if they had been reflected by a plane mirror, which never changes that angle. But, the judgment which the eye in general forms respecting the magnitude of objects, depends on the magnitude of that angle, and that image, unless modified by some particular cause.

On the other hand, in concave mirrors it may be easily demonstrated, that the extreme rays of an object, in whatever manner situated, make a greater angle on arriving at the eye than they would do if reflected from a plane mirror; consequently the appearance of the object, for the above reason, must be much greater.

2d. In a convex mirror, however great be the distance of the object, its image is never farther from the surface than half the radius; so that a straight line perpendicular to the mirror, were it even infinite, would not appear to extend farther within the mirror, than the fourth part of the diameter of the circle of which it is a segment.

But in a concave mirror, the image of a line perpendi-

cular to the mirror is always longer than the line itself; and if this line be equal to half the radius, its image will appear to be infinitely produced.

3d. In convex mirrors, the appearance of a curved line, concentric to the mirror, is a circular line also concentric to the mirror; but the appearance of a straight line, or plane surface, presented to the mirror, is always convex on the outside, or towards the eye.

In a concave mirror, the contrary is the case: the image of a rectilineal or plane object appears concave towards the eye.

4th. A convex mirror disperses the rays; that is to say, if they fall on its surface parallel, it reflects them divergent; if they fall divergent, it reflects them still more divergent, according to circumstances.

On this property, of concave spherical mirrors, is founded the use made of them for collecting the sun's rays into a small space, where their heat, increased in the ratio of their condensation, produces astonishing effects. But this subject deserves to be treated of separately.

PROBLEM XXXVI.

Of Burning Mirrors.

The properties of burning mirrors may be deduced from the following proposition:

If a ray of light fall very near the axis of a concave spherical surface, and parallel to that axis, it will be reflected in such a manner, as to meet it at a distance from the mirror nearly equal to half the radius.

For let ABC (fig. 31 pl. 9) be the concave surface of a well polished spherical mirror, of which D is the centre, and DB the semi-diameter in the direction of the axis; if EF be a ray of light parallel to BD, it will be reflected in the direction of FG, which will intersect the diameter BD in a certain point G. But the point G will always be nearer

to the surface of the mirror than to the centre. For if the radius DF be drawn, we shall have the angles DFE and DFG equal, consequently the angles DFE and GDF will also be equal, since the latter, on account of the parallel lines FE and BD, is equal to DFE: the triangle DGF then is isosceles, and GD is equal to GF; but GF is always greater than GB; whence it follows that DG also is greater than GB; the point G therefore is nearer the surface of the mirror than the centre.

But when the arc BF is exceedingly small, it is well known that the difference between GF and GB will be insensible; consequently, in this case,* the point G will be nearly in the middle of the radius.

This is confirmed by trigonometry; for if the arc BF be only 5 degrees, and if we suppose the semi-diameter DB to be 100000 parts, the line BG will be 49809, which differs from half the radius but $\frac{191}{100000}$ part only, or less than $\frac{1}{523}$*. It is even found, that as long as the arc BF does not exceed 15 degrees, the distance of the point G from half the semi-diameter is scarcely a 56th part. Hence it appears, that all the rays which fall on a concave mirror, in a direction parallel to its axis, and at a distance from its summit not exceeding 15 degrees, unite at a distance from the mirror, nearly equal to half the semi-diameter. Thus, the solar rays, which are sensibly parallel when they fall on this concave surface, will be there condensed, if not into one point, at least into a very small space, where they will produce a powerful heat, so much the stronger as the breadth of the mirror is greater. For this reason the place where the rays meet is called the focus, or burning point.

The focus of a concave mirror then is not a point: it

* The calculation in this case is easy. For, the arc BF being given, we have given also the angle BDF, as well as GFD, which is equal to it, and consequently the angle DGF, which is the supplement of their sum to two right angles. In the triangle DGF then, we have given the three angles and a side, viz. DF, which is the radius; and therefore, by a very simple trigonometrical analogy, we can find the side DG or GF, which is equal to it.

has even a pretty sensible magnitude. Thus, for example, if a mirror be the portion of a sphere of 6 feet radius, and form an arc of 30 degrees, which gives a breadth of somewhat more than three feet, its focus ought to be about the 56th part of that size, or between 7 and 8 lines. The rays, therefore, which fall on a circle of 3 feet diameter, will for the most part be collected in a circle of a diameter 56 times less, and which consequently is only the 3136th part of the space or surface. It may hence be easily conceived what degree of heat such mirrors must produce, since the heat of boiling water is only triple that of the direct rays of the sun, on a fine summer's day.

Attempts however have been made to construct mirrors, to collect all the rays of the sun into one point. For this purpose it would be necessary to give to the polished surface a parabolic curve. For let CBD be a parabola (fig. 32 pl. 9), the axis of which is AB: we here suppose that the reader has some knowledge of conic sections. It is well known that in this axis there is a certain point F, so situated, that every ray, parallel to the axis of this parabola, will be reflected exactly to that point, which on this account has been called the *focus*. If the concave surface therefore of a parabolic spheroid be well polished, all the solar rays, parallel to each other, and to the axis, will be united in one point, and will produce there a heat much stronger than if the surface had been spherical.

REMARKS.—I. As the focus of a spherical mirror is at the distance of a 4th part of the diameter, the impossibility of Archimedes being able, with such a mirror, to burn the Roman ships, supposing their distance to have been only 30 paces, as Kircher says he remarked when at Syracuse, may be easily conceived; for it would have been necessary that the sphere, of which his mirror was a portion, should have had a radius of 60 paces; and to construct such a sphere, would be impossible. A parabolic mirror would be attended with the same inconvenience. Besides,

the Romans must have been wonderfully condescending, to suffer themselves to be burnt so near, without deranging the machine. If the mathematician of Syracuse therefore burnt the Roman ships by means of the solar rays, and if Proclus, as we are told, treated in the same manner the ships of Vitellius, which were besieging Byzantium, they must have employed mirrors of another kind, and could succeed only by an invention similar to that revived by Buffon, and of which he shewed the possibility. See Prob. 33.

The ancients made use of concave mirrors to rekindle the vestal fires. Plutarch, in his life of Numa, says that the instruments used for this purpose, were dishes, which were placed opposite to the sun, and the combustible matter placed in the centre; by which it is probable he meant the focus, conceiving that to be at the centre of the mirror's concavity.

II. We cannot here omit to mention some mirrors celebrated on account of their size, and the effects they produced; one of them was the work of Settala, a canon of Milan: it was parabolic, and, according to the account of father Schott, inflamed wood at the distance of 15 or 16 paces.

Villette, an artist and optician of Lyons, constructed three, about the year 1670, one of which was purchased by Tavernier, and presented to the king of Persia; the second was purchased by the king of Denmark, and the third by the king of France. The one last mentioned was 30 inches in diameter, and of about 3 feet focus. The rays of the sun were collected by it into the space of about half-a-guinea. It immediately set fire to the greenest wood; it fused silver and copper in a few seconds; and in one minute, more or less, vitrified brick, flint, and other vitrifiable substances.

These mirrors however were inferior to that constructed by baron von Tchirnhausen, about 1687, and of which a

description may be found in the Transactions of Leipsic for that year. This mirror consisted of a metal plate, twice as thick as the blade of a common knife; it was about 3 Leipsic ells, or 5 feet 3 inches, in breadth, and its focal distance was 2 of these ells, or 3 feet 6 inches: it produced the following effects:

Wood, exposed to its focus, immediately took fire; and the most violent wind was not able to extinguish it. Water, contained in an earthen vessel, was instantly thrown into a state of ebullition; so that eggs were boiled in it in a moment, and soon after the whole water was evaporated. Copper and silver passed into fusion in a few minutes, and slate was transformed into a kind of black glass, which, when laid hold of with a pair of pincers, could be drawn out into filaments. Brick was fused into a kind of yellow glass; pumice stone and fragments of crucibles, which had withstood the most violent furnaces, were also vitrified, &c.

Such were the effects of the celebrated mirror of baron von Tchirnhausen; which afterwards came into the possession of the king of France, and which was kept in the *Jardin du Roi*, exposed to the injuries of the air, which in a great measure destroyed its polish.

But metal is not the only substance of which burning mirrors have been made. We are told by Wolf, that an artist of Dresden, named Gærtner, constructed one in imitation of Tchirnhausen's mirror, composed only of wood, and which produced effects equally astonishing. But this author does not inform us in what manner Gærtner was able to give to the wood the necessary polish.

Father Zacharias Truber however seems to have supplied this deficiency, by informing us in what manner a burning mirror may be constructed with wood and leaf-gold: for nothing is necessary but to give to a piece of exceedingly dry and very hard wood, the form of the segment of a concave sphere, by means of a turning machine;

to cover it in a uniform manner with a mixture of pitch and wax, and then to apply bits of gold leaf, about three or four inches in breadth. Instead of gold leaf, small plane mirrors, he says, might be adapted to the concavity, and it will be seen with astonishment, that the effect of such a mirror is little inferior to that of a mirror made entirely of metal.

Father Zahn mentions something more singular than what is related by Wolf of the artist of Dresden, for he says that an engineer of Vienna, in the year 1699, made a mirror of pasteboard, covered on the inside with straw cemented to it, which was so powerful as to fuse all metals.

Concave mirrors of a considerable diameter, and which produce the same effect as the preceding, may be procured at present at much less expence. For this advantage we are indebted to M. de Bernieres, one of the controllers general of bridges and causeways, who discovered a method of giving the figure of any curve to glass mirrors; an invention which, besides its utility in optics, may be applied to various purposes in the arts. The concave mirrors which he constructed, were round pieces of glass bent into a spherical form; concave on one side and convex on the other, and silvered on the convex side. M. de Bernieres constructed one for the king of France, of 3 feet 6 inches in diameter, which was presented to his majesty in 1757. Forged iron exposed to its focus was fused in two seconds: silver ran in such a manner that when dropped into water it extended itself in the form of a spider's web; flint became vitrified, &c.

These mirrors have considerable advantage over those of metal. Their reflection from the posterior surface, notwithstanding the loss of rays, occasioned by their passage through the first surface is still more lively than that from the best polished metallic surface; besides, they are not subject, like metallic mirrors, to lose their polish by the

contact of the air, always charged with vapours which corrode metal, but which make no impression on glass: in short, nothing is necessary, but to preserve them from moisture, which destroys the silvering.

PROBLEM XXXVII.

Some properties of concave mirrors, in regard to vision, or the formation of images.

I. If an object be placed between a concave mirror and its focus, its image is seen within the mirror, and more magnified the nearer the object is to the focus; so that when the object is in the focus itself, it seems to occupy the whole capacity of the mirror, and nothing is seen distinct.

If the object, placed in the focus, be a luminous body, the rays which proceed from it, after being reflected by the mirror, proceed parallel to each other, so that they form a cylinder of light, extended to a very great distance, and almost without diminution. This column of light, if the observer stands on one side, will be easily perceived when it is dark; and at the distance of more than a hundred paces from the mirror, if a book be held before this light, it may be read.

II. If the object be placed between the focus and the centre, and if the eye be either beyond the centre, or between the centre and the focus, it cannot be distinctly perceived, as the rays reflected by the mirror are convergent. But if the object be strongly illuminated, or if it be a luminous body itself, such as a candle, by the union of its rays there will be formed, beyond the centre, an image in an inverted situation, which will be painted on a piece of paper, or cloth at the proper distance, or which, to an eye placed beyond it, will appear suspended in the air.

The case will be nearly the same when the object is beyond the centre, in regard to the mirror: an inverted

image of the object will be painted then between the focus and the centre; and this image will approach the centre in proportion as the object itself approaches it; or will approach the focus as the object removes from it.

In regard to the place where the image will be painted in both these cases, it may be found by the following rule.

Let ACS (fig. 33 pl. 9) be the axis of the mirror, indefinitely produced; F the focus, C the centre, and O the place of the object, between the centre and the focus. If F ω be taken a third proportional to FO and FC, it will represent the distance at which the image of the point placed in O will be painted.

If the object be in ω, by employing the same proportion, with the proper changes, that is, by making FO a third proportional to Fω and FC, as in o, the image of it will be found in o.

In the last place, if the object be between the focus and the glass, the place where it will be observed within the mirror, may be found by making Fω to FA, as FA to Fo.

REMARKS.—1st. This property which concave mirrors have, of forming between the centre and the focus, or beyond the centre, an image of the objects presented to them, is one of those which excite the greatest surprise in persons not acquainted with this theory. For if a man advance towards a large concave mirror, presenting a sword to it, when he comes to the proper distance, he will see a sword blade, with the point turned towards him, dart itself from the mirror; if he retires the image of the blade will retire; if he advances in such a manner that the point shall be between the centre and the focus, the image of the sword will cross the real sword as if two people were engaged in fighting.

2d. If, instead of a sword blade, the hand be presented at a certain distance; you will see a hand formed in the

are in an inverted situation; which will approach the real hand, when the latter approaches the centre, so that they will seem to meet each other.

3d. If you place yourself a little beyond the centre of the mirror, and then look directly into it, you will see beyond the centre the image of your face inverted. If you then continue to approach, this phantastic image will approach also, so that you can kiss it.

4th. If a nosegay be suspended in an inverted situation (fig. 34 pl. 9), between the centre and the focus, a little below the axis, and if it be concealed from the view of the spectator, by means of a piece of black pasteboard, an upright image of the nosegay will be formed above the pasteboard, and will excite the greater astonishment, as the object which produces it is not seen; for this reason those not acquainted with the deception will take it for a real object, and attempt to touch it *.

5th. If a concave mirror be placed at the end of a hall, at an inclination nearly equal to 45°, and if a print or drawing be laid on a table before the mirror, with the bottom part turned towards it, the figures in the print or drawing will be seen greatly magnified; and if a proper arrangement be made, so as to favour the illusion, that is if the mirror be concealed, and only a small hole left for looking through, you will imagine that you see the objects themselves.

On this principle are constructed what are called *Optical boxes*, which are now very common: the method of constructing them will be found in the following problem.

PROBLEM XXXVIII.

To construct an optical box or chamber, in which objects are seen much larger than the box itself.

Provide a square box, of a size proper to contain the

* Various spectres, and appearances, formed in this manner, have of late years been exhibited as shows to spectators in London.

concave mirror you intend to employ; that is to say, let each side be a little less than the focal distance of the mirror; and cover the top of it with transparent parchment, or white silk, or glass made smooth, but not polished.

Apply the mirror to one of the vertical sides of the box, and on the opposite side place a coloured print or drawing, representing a landscape, or seaport, or buildings, &c. The print ought to be introduced into the box by means of a slit, so that it can be drawn out, and another substituted in its place at pleasure.

At the top of the side opposite to the mirror, a round hole or aperture must be made, for the purpose of looking through; and if the eye be applied to this hole, the objects, represented in the print, will be seen very much magnified: those who look at them, will think they really behold buildings, trees, &c.

We have seen some of these machines, which by their construction, the size of the mirror, and the correctness of the colouring, exhibited a spectacle highly agreeable and amusing.

Of cylindric, or conical, &c mirrors, and the anamorphoses which may be performed by means of them.

There are other curved mirrors, besides those already mentioned; such as cylindric and conical mirrors, by means of which, effects very curious are produced. Thus, for example, a figure may be drawn on a plane so distorted, that it will be almost impossible to tell what it is; but by placing a cylindric or conical mirror, as well as the eye, in a certain position, the figure will appear in its just proportions. The method by which this is done, is as follows.

PROBLEM XXXIX.

To describe, on an horizontal plane, a distorted figure, which when seen from a given point, as reflected from the convex

surface of a right cylindric mirror, shall appear in its proper proportions,

Let ABC (plate 10 fig. 35 N°. 1 and 2) be the base of a portion of the cylindric polished surface, which is to serve as a mirror; and let AC be its chord. In the radius perpendicular to AC, and indefinitely produced, assume the point O, which corresponds perpendicularly with the place of the eye above it. This point O must be at a moderate distance from the mirror, and raised above the plane of the base only 3 or 4 times the diameter of the cylinder. It is proper that it should be at such a distance from the mirror, that the lines OA and OC, drawn from it, shall make with the cylindric surface a moderately acute angle; for if the lines OA and OC were tangents to the points A and C, the parts of the objects seen by these rays would be very much contracted, and appear confused.

The point O being thus determined, and having drawn the lines OA and OC, draw also AD and CE indefinitely, in such a manner, that they shall form, with the cylindric surface, or the circumference of the base, angles equal to those formed with them by the lines OA and OC; so that, if the lines OA and OC be considered as incident rays, AD and CE may represent the reflected rays.

Then divide AC into 4 equal parts, and form on it a square, which must be divided into 16 other small equal squares. To the points of division 2 and 4 draw the lines O2, O4, cutting the mirror in F and H; from which points draw indefinitely FG and HI, so that the latter lines shall be the reflected rays corresponding to the lines OF and OH, considered as incident rays.

When this is done, on the extremity O, of the indefinite line PO, (N°. 2) raise the perpendicular ON, equal to the height of the eye above the plane of the mirror: make OQ equal to OA; and from the point Q raise the perpendicular equal to AC, which must be divided into 4 equal parts: then draw, from the point N, through these points of divi-

sion, straight lines, which being produced will cut the line QOP in the point I, II, III, IV. Transfer these divisions, in the same order, to the radii AD and CE, so that A I, A II, A III, A IV, shall be respectively equal to Q I, Q II, Q III, Q IV.

Proceed in the same manner to divide the lines FG and HI into 4 unequal parts, as F I, F II, F III, F IV; H I, H II, H III, H IV; and divide, in the like manner, the line B IV; nothing then will be necessary, but to join by curved lines the similar points of division in these 5 lines; which may be easily done, by taking a very flexible rule, and bending it, so as to make it touch or bear on these points. But if these points be joined, three and three, by circular arcs, they will not deviate much from the truth. These circular or curved arcs, with the straight lines A IV, F IV, B IV, H IV, C IV, will form portions of circular rings, very irregular indeed, but which will correspond to the 16 squares into which AC was divided; so that the mixtilineal area *a* will correspond to the square *a;* the area *b* to the square *b*, *c* to *c*, *d* to *d*, and so of the rest.

If a regular figure then be described on the square AC, and if what is contained in the small square *a* be transferred into the area *a* of the base, lengthening or contracting the parts as may be necessary, you will obtain, if you proceed in the same manner with the rest, a figure exceedingly irregular and distorted, which, when seen in the cylindric mirror, by the eye placed properly above the point O, will appear regular and in its true proportions; for it is demonstrated, in the theory of cylindric mirrors, that all these irregular areas must appear to form the square of AC and its divisions, or nearly so. We say nearly, because this construction is not geometrically perfect, and cannot be, on account of the indecision in regard to the place of the image in mirrors of this kind. This construction however succeeds so well, that objects which cannot be distinguished on the base of the mirror, will be

pretty regular in their representation. But we must observe that, for this purpose, the eye ought to be applied to a sight or hole, of a few lines diameter, raised perpendicularly, above the point O, to a height equal to ON.

REMARK.—Instead of a cylindric mirror, one in the form of a right prism may be employed; but, to see a regular and well proportioned image, it must be transferred into parts of the base not contiguous, but parallelograms resting on the base, and arranged in the form of a fan, with triangular intervals between every two. Any particular subject may be painted in these intervals, so that when the mirror is placed in the proper position, some object different from what is represented shall be seen. But we shall not enter into farther details respecting this anamorphosis; as we mean to give that of the pyramidal mirror, which produces a similar effect. This is a problem on which beginners may exercise their ingenuity, and which can be solved without much difficulty.

PROBLEM XL.

To describe, on a horizontal plane, a distorted figure, which shall appear in its proper proportions, when seen as reflected by a conical mirror from a given point in the axis of that cone produced.

Around the place which you intend the distorted figure to occupy, describe a circle ABCD (pl. 10 fig. 36 N. 1 and 2), of any size at pleasure, and divide the circumference of it into any number of equal parts: from the centre E, draw, through the points of division, as many semi-diameters, one of which, as AE or DE, must also be divided into a certain number of equal parts. Then from E, as a centre, describe, through the points of division, as many circles, which with the preceding semi-diameters will divide the space terminated by the first and greatest circle ABCD, into several small spaces; and these will serve to contain the figure, and to distort it on the horizontal plane, around

the base* FGHI of the conical mirror, in the following manner.

Having assumed the circle FGHI (fig. 36 N°. 3), the centre of which is O, as the base of the cone, construct apart the right-angled triangle KLM (N°. 2), the base of which KL is equal to the semi-diameter OG of the base of the cone; and the height KM equal to the height of the cone. Continue this height to N, so that the part MN shall be equal to the distance of the eye from the point of the cone, or the whole line KN equal to the height of the eye above the base of the cone. Having divided the base KL into as many equal parts as the semi-diameter AE or DE contains of the prototype, draw from the point N, through the points of division P, Q, R, as many straight lines; which will give in the hypothenuse LM, representing the side of the cone, the points S, T, V; at the point V make the angle LV 1, equal to the angle LVR; at the point T the angle LT 2, equal to the angle LTQ; at the point S the angle LS 3, equal to the angle LSP; and at the point M, which represents the summit of the cone, the angle LM 4, equal to the angle LMK, in order to have, in the base KL produced, the points 1, 2, 3, 4.

Then from O, the centre of the base FGHI of the conical mirror, and with the distances K 1, K 2, K 3, K 4, describe circles representing those of the prototype ABCD, the largest of which must be divided into as many equal parts as the circumference ABCD; and from the centre O, draw semi-diameters through the points of division, which will give on the horizontal plane as many small distorted spaces as those in the prototype ABCD; and into these the figure of the prototype may consequently be transferred. This image will be very much distorted on the horizontal plane, yet by reflection from the surface of the conical mirror, placed on the circle FGHI, will appear in its just proportions, if the eye be situated perpendicularly above the centre O, and at a distance from it equal to the line KN.

REMARK.—In order to avoid deception in transferring what is in the prototype ABCD (pl. 10 fig. 36 N°. 1 & 2), to the horizontal plane, care must be taken that what is most distant from the centre E shall be nearest the base point of the conical mirror; as is seen by the same letters *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* of the horizontal plane and of the prototype. The distortion will be the more grotesque, if what in the regular image is contained in a sector *a*, (N°. 1) is contained in the distorted image in a portion of a circular ring.

PROBLEM XLI.

To perform the same thing by means of a pyramidal mirror.

It is well known, and may be easily conceived, that a square pyramidal mirror on the base ABCD (pl. 11 fig. 37 N° 1 & 2), reflects to the eye, placed above it in the axis, no more of the plane which surrounds the base, but the triangles BEC, CFD, DGA, and AHB; and that no ray proceeding from the intermediate spaces reaches the eye. It may also be readily seen that these four triangles occupy the whole surface of the mirror; and that to an eye raised above its summit, and looking through a small hole, they will appear together to fill the square of the base. In this case therefore the image to be distorted must be described in the square ABCD, equal to the plane of the base; and if through the centre *e* there be drawn two diagonals, and as many lines perpendicular to the sides, these with the small concentric squares, described in that of the base, will divide it into small triangular and trapezoidal portions.

Now the section of the mirror, through the axis and the line *e*L, being a right-angled triangle, it will be easy, by a method similar to that employed in the preceding problem, to find on the line *e*L produced, its image LE, and the points of division which are the image of those of the former. Let these points be I, III, II, E: if you draw through these points lines parallel to the base BC, and do

the same thing in regard to the other triangles HAB, &c, you will have the area of the image, to be painted, divided into parts corresponding to those of the base. By describing in each of these, in the proper situation, and with the proper degree of elongation or contraction, the parts of the figure contained in the corresponding parts of the base, you will have the distorted figure required, which, when seen from a certain point in the axis produced, will appear to be regular, and to occupy the whole base.

This kind of anamorphosis, on account of its singularity, is superior to any of the preceding; as the parts of the distorted figure are separated from each other, though they seem contiguous when seen in the mirror; other objects therefore may be painted in the intermediate spaces, which will mislead the spectators, and excite in them a greater degree of surprise.

Of Lenticular Glasses, or Lenses.

A lens is a bit of glass having a spherical form on both sides, or at least on one side. Some of them are convex on the one side and plane on the other; and others are convex on both sides: some are concave on one side, or on both; and others are convex on one side, and concave on the other. Those convex on both sides, as they resemble a lentil, are in general distinguished by the name of lenticular glasses, or lenses.

The uses to which these glasses are applied, are well known. Those which are convex magnify the appearance of objects, and aid the sight of old people; on the other hand, the concave glasses diminish objects, and assist those who are short sighted. The former collect the rays of the sun around one point called the *focus*; and, when of a considerable size, produce heat and combustion. The concave glasses, on the contrary, disperse the rays of the sun. Both kinds are employed in the construction of telescopes and microscopes.

PROBLEM XLII.

To find the focus of a glass globe.

As glass globes supply, on many occasions, the place of lenses, it is proper that we should here say a few words respecting their focal distance. The method of determining it is as follows.

Let BCD (pl. 11 fig. 38) be a glass sphere, the centre of which is F, and CD a diameter to which the incident ray AB is parallel. This ray, when it meets with the surface of the sphere in B, will not continue its course in a straight line, as would be the case if it did not enter a new medium, but will approach the perpendicular drawn from the centre F to B the point of incidence. Consequently, when it issues from the sphere at the point I, it would meet the diameter in a point E, if it did not deviate from the perpendicular FI, which makes it take the direction IO, and proceed to the point O, the focus required.

To determine the focus O, first find the point of meeting E, which may be easily done by observing that in the triangle FBE, the side FB is to FE as the sine of the angle FEB is to the sine of the angle FBE; or, on account of the smallness of these angles, as the angle FEB, or its equal GBE, is to the angle FBE; for we here suppose the incident ray to be very near the diameter CD; consequently the angle ABH is very small, as well as its equal FBG; and angles extremely small have the same ratio as their sines. But, by the laws of refraction, when a ray passes from air into glass, the ratio of the angle of incidence ABH, or GBF, to the angle of refraction FBI, if the angles be very small, is as 3 to 2, and therefore the angle FBE is nearly the double of EBG: it thence follows that the side FE, of the triangle FBE, is nearly the double of FB or equal to twice the radius; consequently DE is equal to the radius.

To find the point O, where the ray, when it issues from the sphere, and deviates from the perpendicular, ought to

meet the line DE, the like reasoning may be employed. In the triangle IOE, the side IO is to OE nearly as the angle IEO, or its equal IFE, is to the angle OIE. Now these two angles are equal; for the angle IFD is the one third of the angle of incidence FBG or ABH; but, by the law of refraction, the angle OIE is nearly the half of the angle of incidence EIK, or of its equal FIB, which is $\frac{2}{3}$ of the angle FBG: like the preceding it is therefore the third of FBG or HBA, and consequently the angles OIE and OEI are equal; whence it follows that OE is equal to OI, which is itself equal to DO, on account of their very great proximity. Therefore DO, or the distance of the focus of a glass globe from the surface, is equal to half the radius, or the fourth part of the diameter. Q.E.D.

PROBLEM XLIII.

To find the focus of any lens.

The same reasoning, as that employed to determine the course of a ray passing through a glass sphere, might be employed in the present case. But for the sake of brevity, we shall only give a general rule, demonstrated by opticians, which includes all the cases possible in regard to lenses, whatever combinations may be formed of convexities and concavities. We shall then show the application of it to a few of the principal cases. It is as follows.

As the sum of the semi-diameters of the two convexities is to one of them; so is the diameter of the other, to the focal distance.

In the use of this rule, one thing in particular is to be observed. When one of the faces of the glass is plane, the radius of its sphericity must be considered as infinite; and when concave, the radius of the sphere, of which this concavity forms a part, must be considered as negative. This will be easily understood by those who are in the least familiar with algebra.

CASE I. *When the lens is equally convex on both sides.*

Let the radius of the convexity of each of the faces be, for example, equal to 12 inches. By the general rule we shall have this proportion: as the sum of the radii, or 24 inches, is to one of them, or 12 inches, so is the diameter of the other, or 24 inches, to a fourth term, which will be 12 inches, the focal distance. Hence it appears that a lens equally convex on both sides unites the solar rays, or in general rays parallel to its axis, at the distance of the radius of one of the two sphericities.

CASE II. *When the lens is unequally convex on both sides.*

If the radii of the convexities be 12 and 24, for instance, the following proportion must be employed; as 12 + 24, or 36, is to 12, the radius of one of the convexities, so is 48, the diameter of the other, to 16; or as 12 + 24, or 36, is to 24, the radius of one of the convexities, so is 24, the diameter of the other, to 16. The distance of the focus therefore will be 16 inches.

CASE III. *When the lens has one side plane.*

If the sphericity on the one side be as in the preceding case, we must say, by applying the general rule: as the sum of the radii of the two sphericities, viz, 12 and an infinite quantity, is to one of them, or the infinite quantity, so is 24, the diameter of the other convexity, to a fourth term, which will be 24; for the two first terms are equal, because an infinite quantity increased or diminished by a finite quantity, is always the same: the two last terms therefore are equal; and it hence follows, that a plano-convex glass has its focus at the distance of the diameter from its convexity.

CASE IV. *When the lens is convex on the one side, and concave on the other.*

Let the radius of the convexity be still 12, and that of the concavity 27. As a concavity is a negative convexity,

this number 27 must be taken with the sign − prefixed. We shall therefore have this proportion.

As 12 inches − 27, or − 15 inches, is to the radius of the concavity − 27 (or as 15 is to 27, or as 5 is to 9), so is 24 inches, the diameter of the convexity, to 43$\frac{1}{5}$. This is the focal distance of the lens, and is positive or real; that is to say, the rays falling parallel to the axis, will really be united beyond the glass. The concavity indeed having a greater diameter than the convexity, this must cause the rays to diverge less than the convexity causes them to converge. But if the concavity be of a less diameter than the convexity, the rays, instead of converging when they issue from the glass, will be divergent, and the focus will be before the glass: in this case it is called *virtual*. Thus, if the radius of the concavity be 12, and that of the convexity 27, we shall have, by the general rule: as 27 − 12, or 15, is to 27, or as 5 is to 9, so is − 24 to − 43$\frac{1}{5}$. The last term being negative, it indicates that the focus is before the glass, and that the rays will issue from it divergent, as if they came from that point.

Case v. *When the lens is concave on both sides.*

If the radii of the two concavities be 12 and 27 inches, we shall have this proportion: as − 12 − 27 is to − 27, or as 39 is to 27, or as 13 to 9, so is − 24 to − 16$\frac{8}{13}$. The last term being negative, it shows that the focus is only virtual; and that the rays, when they issue from the glass, will proceed diverging, as if they came from a point situated at the distance of 16$\frac{8}{13}$ inches before the glass.

Case vi. *When the lens is concave on one side, and plane on the other.*

If the radius of the concavity be still 12, the above rule will give the following proportion: as − 12 + an infinite quantity, is to an infinite quantity, so is − 24 to − 24; for an infinite quantity, when it is diminished by a finite

quantity, remains still the same. Thus it is seen that, in this case, the virtual focus of a plano-concave glass, or the point where the rays after their refraction seem to diverge, is at a distance equal to the diameter of the concavity, as the point to which they converge is in the case of the plano-convex glass.

These are all the cases that can occur in regard to lenses: for that where the two concavities might be supposed equal, is comprehended in the fifth.

REMARK.—In all these calculations, we have supposed the thickness of the glass to be of no consequence in regard to the diameter of the sphericity, which is the most common case; but if the thickness of the glass were taken into consideration, the determinations would be different.

Of Burning Glasses.

Lenticular glasses furnish a third method of solving the problem, already solved by means of mirrors, viz, to unite the rays of the sun in such a manner, as to produce fire and inflammation: for a glass of a few inches diameter will produce a heat sufficiently strong to set fire to tinder, linen, black or grey paper, &c.

The ancients were acquainted with this property in glass globes, and they even sometimes employed them for the above purpose. It was probably by means of a glass globe that the vestal fire was kindled. Some indeed have endeavoured to prove, that they produced this effect by lenses: but de la Hire has shown, that this idea is entirely void of foundation, and that the burning glasses of the ancients were only glass globes, and consequently incapable of producing a very remarkable effect.

Baron von Tchirnhausen, who constructed the celebrated mirror already mentioned, made also a burning glass, the largest that had ever been seen. This mathematician, being near the Saxon glass manufactories, was enabled, about the year 1696, to procure plates of glass

sufficiently thick and broad, to be converted into lenses several feet in diameter. One of them, of this size, inflamed combustible substances at the distance of 12 feet. Its focus at this distance was about an inch and a half in diameter. But when it was required to make it produce its greatest effects, the focus was diminished by means of a second lens, placed parallel to the former, and at the distance of 4 feet. In this manner, the diameter of the focus was reduced to 8 lines, and it then fused metals, vitrified flint, tiles and slate, earthen ware, &c, in short, it produced the same effects as the burning mirrors of which we have already spoken.

Some years ago a lens, which might have been taken for that of Tchirnhausen, was exhibited at Paris. The glass of which it consisted was radiated and yellowish; and the person to whom it belonged asked no less for it than 500£. sterling.

For the means of obtaining, at a less expence, glasses capable of producing the same effects, we are indebted to M. de Bernieres, of whom we have already spoken. By his invention for bending glass, two round plates are bent into a spherical form, and being then applied to each other the interval between them is filled with distilled water, or spirit of wine. These glasses, or rather water lenses, have their focus a little farther distant, and *cæteris paribus* ought to produce a somewhat less effect; but the thinness of the glass and the transparency of the water occasion less loss in the rays, than in a lens of several inches in thickness. In short, it is far easier to procure a lens of this construction, than solid ones, like that of Tchirnhausen. M. de Trudaine, some years ago, caused to be constructed, by M. de Bernieres, one of these water lenses 4 feet in diameter, with which some philosophical experiments have been already made, in regard to the calcination of metals and other substances. The heat produced by this instrument, is much superior to that of all

the burning glasses and mirrors hitherto known, and even to that of all furnaces. We have reason to expect from it new discoveries in chemistry. We shall here add that with water lenses, of a much smaller size, M. de Bernieres has fused metals, vitrifiable stones, &c.

PROBLEM XLIV.

Of some other properties of lenticular glasses.

1st. If an object be exceedingly remote, so that there is no proportion between its distance and the focal distance of the glass, there is painted in the focus of the lens an image of the object in an inverted situation. This experiment serves as the basis of the construction of the camera obscura. In this manner the rays of the sun, or of the moon, unite in the focus of a glass lens, and form a small circle, which is nothing else than the image of the sun or moon, as may be easily perceived.

2d. In proportion as the object approaches the glass, the image formed by the rays proceeding from the object, recedes from the glass; so that when the distance of the object is double that of the focus, the image is painted exactly at the double of that distance; if the object continues to approach, the image recedes more and more; and when the object is in the focus, no image is formed; for it is at an infinite distance that it is supposed to form itself. In this case therefore the rays which fall on the glass, diverging from each point of the object, are refracted in such a manner, as to proceed parallel to each other.

The method of determining, in general, the distance from the lens at which the image of the object is formed, is as follows. Let OC be the object (fig. 39 pl. 11), DE its distance from the glass, and EF the focal distance of the glass; if we make use of this proportion: as FD is to FE, so is EF to EG, taking EG on the other side of the glass when ED is greater than EF, the point G will be that of the

axis to which the point D of the object, situated in the axis, will correspond.

Hence it may be easily seen, that when the distance of the object from the focus is equal to nothing, the distance EG must be infinite, that is to say there can be no image.

It must also be observed that when EF is greater than EB, or when the object is between the glass and the focus, the distance EG must be taken in a contrary direction, or on this side of the glass, as E*g*; which indicates that the rays proceeding from the object, instead of forming an image beyond the glass, diverge as if they proceeded from an object placed at *g*.

Of Telescopes, both Refracting and Reflecting.

Of all optical inventions, none is equal to that of the telescope: for, without mentioning the numerous advantages derived from the common use of this wonderful instrument, it is to it we are indebted for the most interesting discoveries in astronomy. It is by its means that the human mind has been able to soar to those regions otherwise inaccessible to man, and to examine the principal facts which serve as the foundation of our knowledge respecting the heavenly bodies.

The first telescope was constructed in Holland, about the year 1609; but there is much uncertainty in regard to the name of the inventor, and the means he employed in the formation of his instrument. A dissertation on this subject may be seen in Montucla's History of the Mathematics. We shall confine ourselves at present to a description of the different kinds of telescopes, both refracting and reflecting, and of the manner in which they produce their effect.

Of Refracting Telescopes.

1st. The first kind of telescope, and that most commonly used, is composed of a convex glass, called the

object glass, because it is that nearest the objects, and a concave one, called the *eye-glass*, because it is nearest the eye. These glasses must be disposed in such a manner, that the posterior focus of the object glass shall coincide with the posterior focus of the concave glass. By means of this disposition, the object appears magnified in the ratio of the focal distance of the object glass, to that of the eye-glass. Thus, if the focal distance of the object glass be 10 inches, and that of the eye-glass 1 inch, the instrument will be 9 inches in length and will magnify objects 10 times.

This kind of telescope is called the *Batavian*, on account of the place where it was invented. It is known also by the name of the Galilean, because Galileo, having heard of it, constructed one of the same kind, and by its means was enabled to make those discoveries in the heavens which have immortalized his name. At present, very short telescopes only are made according to this principle; because they are attended with one defect, which is, that when of a considerable length they have a very confined field.

2d. The second kind of telescope is called the *astronomical*, because employed chiefly by astronomers. It is composed of two convex glasses, disposed in such a manner, that the posterior focus of the object glass and the anterior focus of the eye-glass coincide together, or very nearly so. The eye must be applied to a small aperture, at a distance from the eye-glass equal to that of its focus. It will then have a field of large extent, and it will show the objects inverted, and magnified in the ratio of the focal distances of the object glass and eye-glass. If we take, by way of example, the proportions already employed, the astronomical telescope will be 12 inches in length, and will magnify 10 times.

Telescopes of very great length may be constructed

according to this combination. It is common for astronomers to have them of 12, 15, 20 and 30 feet. Huygens constructed one for himself of 123 feet, and Hevelius employed one of 140. But the inconvenience which attends the use of such long telescopes, in consequence of their weight, and the bending of the tubes, has made them be laid aside, and another instrument more commodious has been substituted in their stead. Hartsoecker made an object glass of 600 feet focus, which would have produced an extraordinary effect had it been possible to use it.

3d. The inconvenience of the Batavian telescopes, which suffer only a small quantity of objects to be seen at once, and that of the astronomical telescope, which represents them inverted, have induced opticians to devise a third arrangement of glasses, all convex, which represents the objects upright, gives the same field as the astronomical telescope, and which is therefore proper for terrestrial objects: on this account it is called the terrestrial telescope. It consists of a convex object glass, and three equal eye-glasses. The posterior focus of the object glass generally coincides with the anterior one of the first eye-glass; the posterior focus of the latter coincides also with the anterior focus of the second, and in like manner the posterior focus of the second with the anterior one of the third, at the posterior focus of which the eye ought to be placed. This instrument always magnifies in the ratio of the focal distances of the object glass and one of the eye-glasses. But it may be readily seen that the length is increased 4 times the focal distance of the eye-glass.

4th. The image of objects might be made to appear upright by employing only two eye-glasses: for this purpose it would be necessary that the first should be at a distance from the focus of the object glass equal to twice its own focal distance; and that the anterior focus of the second should be at twice that distance. Such is the ter-

restrial telescope with three glasses; but experience has shown that, by this arrangement, the objects are somewhat deformed, for which reason it is no longer used.

5th. Telescopes with 5 glasses have also been proposed, in order to bend the rays gradually, as we may say, and to obviate the inconveniences of the too strong refraction, which suddenly takes place at the first eye-glass; and also to increase the field of vision. We have even heard of some telescopes of this kind which were attended with great success; but we do not find that this combination of glasses has been adopted.

6th. Some years ago, a new kind of telescope was invented, under the name of the *achromatic*, because it is free from those faults occasioned by the different refrangibility of light, which in other telescopes produces colours and indistinctness. The only difference between this and other telescopes is, that the object glass, instead of being formed of one lens, is composed of two or three made of different kinds of glass, which have been found by experience to disperse unequally the different coloured rays of which light is composed. One of these glasses is of crown-glass, and the other of flint glass. An object glass of this kind, constructed according to certain dimensions determined by geometricians, produces in its focus an image far more distinct than the common ones; on which account much smaller eye-glasses may be employed without affecting the distinctness, as is confirmed by experience. These telescopes are called also *Dollond's telescopes*, after the name of the English artist who invented them. By the above means, the English opticians construct telescopes of a moderate length, which are equal to others of a far greater size; and small ones, not much longer than opera-glasses, with which the satellites of Jupiter may be seen, are sold under Dollond's name at Paris. M. Antheaume, according to the dimensions given by M. Clairault, made, in that capital, an achromatic telescope of 7 feet focal

distance, which when compared with a common one of 30 or 35 feet, was found to produce the same effect.

This invention gives us reason to hope that discoveries will be made in the heavens, which a few years ago would have appeared altogether impossible. It is not improbable even that astronomers will be able to discover in the moon habitations and animals, spots in Saturn and Mercury, and the satellite of Venus, so often seen and so often lost.

To give an accurate idea of the manner in which telescopes magnify the appearance of objects, we shall take, by way of example, that called the astronomical telescope, as being the simplest. If it be recollected that a convex lens produces in its focus an inverted image of objects which are at a very great distance, it will not be difficult to conceive, that the object glass of this telescope will form behind it, at its focal distance, an inverted image of any object towards which it is directed. But, by the construction of the instrument, this image is in the anterior focus of the eye-glass, to which the eye is applied; consequently the eye will perceive it distinctly; for it is well known, that when an object is placed in the focus of a lens, or a little on this side of it, it will be seen distinctly through the glass, and in the same direction. The image of the object, which here supplies its place, being then inverted, the eye-glass, through which it is viewed, will not make it appear upright, and consequently the object will be seen inverted.

In regard to the size, it is demonstrated, that the angle under which the image is seen, is to that under which the object is seen, from the same place, as the focal distance of the object glass, is to that of the eye-glass: hence the magnified appearance of the object.

In terrestrial telescopes, the two first eye-glasses only invert the image; and this telescope therefore must represent objects upright. But having said enough

ng refracting telescopes, we shall now proceed to reflecting ones.

Of Reflecting Telescopes.

Those who are well acquainted with the manner in which objects are represented by common telescopes, will readily conceive that the same effect may be produced by reflection; for a concave mirror, like a lens, paints on its focus an image of distant objects. If means then are found to reflect the image on one side, or backwards, in such a manner as to be made to fall in the focus of a convex glass, and to view it through this glass, we shall have a reflecting telescope. It need therefore excite no surprise that before Newton, and in the time of Descartes and Mersenne, telescopes on this principle were proposed.

Newton was led to this invention while endeavouring to discover some method of remedying the want of distinctness in the images formed by glasses; a fault which arises from the different refrangibility of the rays of light that are decomposed. Every ray, of whatever colour, being reflected under an angle equal to the angle of incidence, the image is much more distinct, and better terminated in all its parts, as may be easily proved by means of a concave mirror. On this account he was able to apply an object glass much smaller, which would produce a greater magnifying power; and this reasoning was confirmed by experience.

Newton never constructed telescopes of more than 15 inches in length. According to his method, the mirror was placed in the bottom of the tube, and reflected the image of the object towards its aperture: near this aperture was placed a plane mirror, that is, the base of a small isosceles rectangular prism, silvered at the back, and inclined at an angle of 45 degrees. This small mirror reflected the image towards the side of the tube, where there was a hole, into which was fitted a lens of a very short

focal distance, to serve as the eye-glass. The object then was viewed from the side, a method, in many cases, exceedingly convenient. Mr. Hadley, a fellow of the Royal Society, constructed, in the year 1723, a telescope of this kind, 5 feet in length, which was found to produce the same effect as the telescope of 123 feet, presented to the Royal Society by Huygens.

The reflecting telescopes, used at present, are constructed in a manner somewhat different. The concave mirror, at the bottom of the tube, has a round hole in the middle, and towards the other end is a mirror, sometimes plane, turned directly towards the other one, which, receiving the image near the middle of the focal distance, reflects it towards the hole in the other mirror. Against this hole is applied a lens of a short focal distance, which serves as an eye-glass, or for viewing terrestrial objects, in order that they may appear upright; and three eye-glasses are used, arranged in the same manner as in terrestrial telescopes.

A telescope however may be made to magnify much more by the following construction. The large mirror, as in all the others, is placed at the bottom, and has a hole in the centre, before which the eye-glass is applied. At the other end of the tube is another concave mirror, of a less focal distance than the former, and so disposed that the image reflected by the former is painted very near its focus, but at a little farther distance than the focus, from its surface. This produces another image beyond the centre, which is greater as the first one is nearer the focus: this image is formed very near the hole in the centre of the large mirror, opposite to which the eye-glass is in general placed.

This kind of reflecting telescope is called the Gregorian, because proposed by Mr. James Gregory, even before Newton conceived the idea of his; and it is this kind which is at present most in use.

There is also the telescope of Cassegrain, who employs a convex mirror to magnify the image formed by the first concave one. Dr. Smith thought it attended with so many advantages, that he was induced to analyse it in his Treatise on Optics. Cassegrain was a French artist, who proposed this method of construction about the year 1665, and nearly at the same time that Gregory proposed his. It is certain that the length of the telescope is by these means considerably diminished.

The English, for a long time, have enjoyed a superiority in works of this kind. The art of casting and polishing the metallic mirrors, necessary for these instruments, is indeed exceedingly difficult. M. Passement, a celebrated French artist, and the brothers Paris and Gonichon, opticians at Paris, are the first who attempted to vie with them in this branch of manufacture; and both have constructed a great number of reflecting telescopes, some of which are 5 or 6 feet in length. Among the English, no artist distinguished himself more in this respect than Short, though his telescopes were not of great length: besides some of 4, 5 and 6 feet, he made one of 12, which belonged some years ago to the physician of Lord Macclesfield. By applying a lens of the shortest focal distance which it could bear, it magnified about 1200 times. The satellites of Jupiter therefore, seen through this telescope, are said to have had a sensible apparent diameter. But this telescope, as we have heard, is no longer in existence, the large mirror being lost.

The longest of all the reflecting telescopes ever yet constructed, if we except that lately made by Herschel, is one in the king's collection of philosophical and optical instruments at la Meute; it is the work of dom Noel, a Benedictine, the keeper of the collection, and was begun several years before he was placed at the head of that establishment, where he finished it, and where the curious were allowed to see it, and to contemplate with it the

heavens. It is mounted on a kind of moveable pedestal, and, notwithstanding its enormous weight, can be moved in every direction, along with the observer, by a very simple mechanism. But what would be most interesting, is to ascertain the degree of its power, and whether it produces an effect proportioned to its length, or at least considerably greater than the largest and best reflecting telescopes constructed before; for we know that the effects of these instruments, supposing the same excellence in the workmanship, do not increase in proportion to the length.

Huygens' telescope of 123 feet, which he presented to the Royal Society, did not produce an effect quadruple that of a good telescope of 30 feet; and the case must be the same in regard to reflecting telescopes, where the difficulties of the labour are still greater; so that if a telescope of 24 feet produce one half more effect than another of 12, or only the double of one of 6 feet, it ought, in our opinion, to be considered as a good instrument.

We have heard that dom Noel was desirous of making this comparison, and the method he proposed was rational. We have long considered it as the only one proper for comparing such instruments. It is to place at the distance of several hundred feet printed characters of every size, composing barbarous words without any meaning, in order that those who make the experiment may not be assisted by one or two words to guess the rest. The telescope by means of which the smallest characters are read, will undoubtedly be the best. We have seen stuck up, on the dome of the Hospital of Invalids, pieces of paper of this kind, which dom Noel had placed there for the purpose of making this comparison; but unfortunately such instruments cannot be brought to one place. Printed characters, such as above described, might therefore be fixed up at a convenient distance from each, without removing the instruments, and persons, appointed for the purpose, ought

to go to the different observatories, at times when the weather is exactly similar, and examine what characters can be read by each telescope. By this method a positive answer to the above question would be obtained.

But the largest, and the most powerful, of all the reflecting telescopes, have been lately made by Dr. Herschel, under the auspices of the British monarch; a consequence of which was the discovery of his new primary planet, and of many additional satellites. After a long perseverance in a series of improvements of reflecting telescopes, of the Newtonian form, making them successively larger and more accurate, this gentleman came at length to make one of the amazing size of 40 feet in length. This telescope was begun in the year 1785, and completed in 1789. The length of the sheet iron tube is 40 feet, and diameter 4 feet 10 inches. The great mirror is 49½ inches in diameter, 3½ inches thick, and weighs 2118*lb.* The whole is managed by a large apparatus of machinery, of wheels and pulleys, by means of which it is easily moved in any direction, vertically and sideways. The observer looks in at the outer or object end; from whence proceeds a pipe to a small house near the instrument, for conveying information by sound, backward and forward to an assistant, who thus under cover sets down the time and observations made by the principal observer. The consequences of this, and the other powerful machines of this gentleman, have been new discoveries in the heavens of the most important nature.

PROBLEM XLV.

Method of constructing a telescope, by means of which an object may be seen, even when the instrument appears to be directed towards another.

As it is not polite to gaze at any one, a sort of glass has been invented in England, by means of which, when the person who uses it seems to be viewing one object, he is

really looking at another. The construction of this instrument is very simple.

Adapt to the end of an opera glass (fig. 40 pl. 11), the object glass of which in this case becomes useless, a tube with a lateral aperture as large as the diameter of the tube will admit, and opposite to this aperture place a small mirror inclined to the axis of the tube at an angle of 45 degrees, and having its reflecting surface turned towards the object glass. It is evident that when this telescope is directed straight forwards, you will see only some of the lateral objects, viz, those situated near the line drawn from the eye in the direction of the axis of the telescope and reflected by the mirror. These objects will appear upright, but transposed from right to left. To conceal the artifice better, the fore part of the telescope may be furnished with a plane glass, which will have the appearance of an object glass placed in the usual manner.

This instrument, which is not very common in France, is exceedingly convenient for gratifying one's curiosity in the playhouse, and other places of public amusement, especially if the mirror be so fixed, as to be susceptible of being more or less inclined, for those who use it, while they seem to look at the stage and the performers, may without affectation, and without violating the rules of politeness, examine an interesting figure in the boxes.

We must however observe that the first idea of this instrument is not very new, for the celebrated Hevelius, who it seems was afraid of being shot, proposed many years ago his *polemoscope*, or telescope for viewing under cover, and without danger, warlike operations, and those in particular which take place during the time of a siege. It consisted of a tube bent in such a manner as to form two elbows, in each of which was a plane mirror inclined at an angle of 45 degrees. The first part of the tube was made to rest on the parapet towards the enemy; the image reflected by the first inclined mirror passed through the tube

in a perpendicular direction, and meeting with the second mirror was reflected horizontally towards the eye glass, where the eye was applied: by these means a person behind a strong parapet could see what the enemy were doing without the walls. The chief thing to be apprehended in regard to this instrument was, that the object glass might be broken by a ball; but this was certainly a trifling misfortune, and not very likely to happen.

Of Microscopes.

What the telescope has performed in the philosophy of the heavenly bodies, the microscope has done in regard to that of the terrestrial: for by the assistance of the latter we have been able to discover an order of beings which would otherwise have escaped our notice; to examine the texture of the smallest of the productions of nature, and to observe phenomena which take place only among the most minute parts of matter. Nothing can be more curious than the facts which have been ascertained by the assistance of the microscope: but in this part of science much still remains to be done.

There are two kinds of microscopes; simple and compound; we shall speak of both, and begin with the former.

PROBLEM XLVI.

Method of constructing a single microscope.

I. Every convex lens of a short focal distance is a microscope; for it is shown that a lens magnifies in the ratio of the focal distance to the least distance at which the object can be placed to be distinctly seen; which, in regard to most men who are not short-sighted, is about 8 inches. Thus a lens, the focal distance of which is 6 lines, will magnify the dimensions of the object 16 times; if its focal distance be only one line it will magnify 96 times.

II. It is difficult to construct a lens of so short a focus; as it is necessary that the radius of each of its convexities

should be only a line; for this reason small glass globes, fused at an enameller's lamp, or the flame of a taper, are employed in their stead. The method by which this is done, is as follows.

Break off a piece of very pure transparent glass, either by means of an instrument made for that purpose, or the wards of a key; then take up one of these fragments by applying to it the point of a needle a little moistened with saliva, which will make it adhere, and present it to the blue flame of a taper, which must be kept somewhat inclined that the fragment of glass may not fall upon the wax. As soon almost as it is held to the flame it will be fused into a round globule, and drop down: a piece of paper therefore, with a turned up border, must be placed below, in order to receive it.

It is here to be observed that there are some kinds of glass which it is difficult to fuse: in this case it will be necessary to employ another kind.

Of these globules select the brightest and roundest; then take a plate of copper, 5 or 6 inches in length, and about 6 lines in breadth, and having folded it double, make a hole in it somewhat less in diameter than the globule, and raise up the edges. If you then fix one of these globules in this hole, between the two plates, and bind them firmly together, you will have a single microscope.

As it is easy to obtain globules of $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of a line in diameter, and as the focus of a glass globule is at the distance of a quarter of its diameter without it, we are enabled by this process to magnify objects in a very high degree; for if the diameter of the globule be only $\frac{1}{2}$ line, by employing this proportion: as $\frac{1}{4}$ of half a line, or $\frac{1}{8}$, are to 96 lines, so is 1 to a fourth term, we shall have as fourth term the number 153, which will express the increase of the diameter of the object. The object, therefore, in regard to surface will be magnified 23409 times, and in regard to solidity 3581677 times.

The celebrated Lewenhoeck, so well known on account of his microscopical observations, never employed microscopes of any other kind. It is however certain that they are attended with many inconveniences, and can be used only for objects which are transparent, or at least semi-transparent, as it may be readily conceived that it is not possible to illuminate a surface which is viewed in any other way than from behind. By means of these microscopes Lewenhoeck made a great number of curious observations, an account of which will be found hereafter, under the head Microscopical Observations.

III. The water microscope of Grav, which is much simpler, may be constructed in the following manner.

Provide a plate of lead, $\frac{1}{3}$ of a line in thickness at most, and make a round hole in it with a needle or a large pin; pare the edges of this hole, and put into it, with the point of a feather, a small drop of water: the anterior and posterior surfaces of the water will assume a convex spherical form, and thus you will have a microscope.

The focus of such a globule is at a distance somewhat greater than that of a glass globule of equal size; for the focus of a globule of water is at the distance of the radius from its surface. A globule of water therefore, $\frac{1}{4}$ a line in diameter, will magnify only 128 times; but this deficiency is fully compensated by the ease with which a globule of any diameter, however small, may be obtained.

If water be employed in which leaves, wood, pepper, or flour has been infused, in the open air, the microscope will be both object and instrument; for by this means the small microscopic animals which the water contains will be seen. Mr. Gray was very much astonished, the first time he observed this phenomenon; but it afterwards occurred to him that the posterior surface of the drop produced, in regard to those animals placed between it and its focus, the same effects as a concave mirror, and magnified their images.

which was still farther enlarged by the kind of convex lens of the anterior surface.

IV. Another kind of microscope may be also procured at a very small expence, by making a hole of about the fourth or the fifth part of a line in diameter, in a card or very thin plate of metal. If very small objects be viewed through this hole, they will appear magnified in the ratio of their distance from the eye, to that at which an object can be distinctly seen by the naked eye.—This kind of microscope is much extolled in the *Journal de Trevoux*; but we must confess that we never could see small objects distinctly through such holes, unless at the distance of an inch or half an inch; and even then they did not appear to be much magnified.

PROBLEM XLVII.

Of Compound Microscopes.

The compound microscope consists of an object glass, which is a lens of a very short focus, such for example as 4 or 6 lines, and an eye-glass of 2 inches focus, at the distance from it of about 6 or 8 inches. The object must be placed a little beyond the focus of the object glass, and the distance of the eye from the eye-glass ought to be equal to the focal distance of the latter. Having formed such a combination of glasses, if the object be made to approach gently to the object glass, there will be a certain point at which it will appear to be considerably magnified.

If the focal distance of the object glass be 4 lines, for example, and if the object be $4\frac{1}{4}$ lines from it, the image will be formed at the distance of 64 lines, or 5 inches 4 lines: it will therefore be 14 times as large as the object, for 64 is to $4\frac{1}{4}$ nearly as 14 to 1. If the focal distance of the eye-glass, in the focus of which this image is formed, be 2 inches, it will magnify about 4 times more: but 14×4

=56, which expresses the number of times that the diameter of the object will appear to be magnified.

If you are desirous that it should not be magnified so much, remove gradually the object from the object glass, and bring the eye-glass nearer; the image will then be seen not so large, but more distinct.

On the other hand, if you wish it to be magnified more, move the object gradually towards the object glass, or move the latter towards the object, and remove the eye-glass: the object will then appear much larger; but there are certain limits beyond which every thing seems confused.

Instead of one eye-glass, two are sometimes used to increase the field of vision; the first of which has a focal distance of 4 or 5 inches, while that of the second is much less; but this is still the same thing. The image of the small object must be placed, in regard to this compound eye-glass, in the same point where an object ought to be, to be seen distinctly when viewed through it.

A concave object glass might be employed, by making its posterior focus coincide with the image: this would form a kind of microscope similar to the Batavian telescope; but it would be attended with the same inconvenience, that of having too contracted a field.

There are also reflecting microscopes as well as telescopes: the principle of both is the same, a minute object placed very near the focus of a concave mirror, and on this side of it, in regard to the centre, reflects an image of it beyond the centre; and this image will be larger the nearer it is to the focus. The image is viewed through a convex lens, and in this kind of microscope an object glass of a much shorter focus may be employed, which will contribute to the amplification of the object.

Every thing relating to this subject may be found in a very curious work by Baker, entitled the *Microscope made easy*. The reader may consult also Smith's Optics, Part 4. These works, and particularly the first, contain a great

variety of curious details respecting the method of employing microscopes, and the observations made by means of them. See also *Essais de Physique de Muschenbroeck.*

We intend to give an account of the most curious observations which have been made by the assistance of the microscope; but to avoid confusion we shall reserve that article for the end of this part of our work.

PROBLEM XLVIII.

A very simple method of ascertaining the real magnitude of objects, seen through a microscope.

It is often useful, and may sometimes gratify curiosity, to be able to determine the real magnitude of certain objects examined by means of the microscope: the following very simple and ingenious method for this purpose was invented by Dr. Jurin, a celebrated philosopher, and a fellow of the Royal Society of London.

Take a piece of the finest silver wire possible to be obtained, and roll it as close as you can around an iron cylinder, a few inches in length. It will be necessary to examine it with a microscope, in order to discover whether there be any vacuity or opening between the folds: by these means you will ascertain, with great precision, the diameter of the silver wire. For if we suppose that there are 520 turns in the space of an inch, it is evident that the diameter of the wire will be the 520th part of an inch; a measure which cannot be obtained in any other manner.

Then cut this silver wire into very small bits, and scatter a certain quantity of them over the small plate on which the objects, submitted to examination, are placed: if you look at these bits of wire along with the objects, you will be enabled, by comparing them together, to judge of the size of the latter.

It was by a similar process that Dr. Jurin determined the size of the globules which give to blood its red colour. He first found that the diameter of his silver wire was the

485th part of an inch, and then judged by comparison that the diameter of a red globule of the blood was a 4th part of that of the wire; from which he concluded that the diameter of the globule was the 1940th part of an inch.

PROBLEM XLIX.

To construct a Magic Picture, which being seen in a certain point, through a glass, shall exhibit an object different from that seen with the naked eye.

As this optical problem is solved by means of a glass cut into facets, or what is called a multiplying glass, we shall first explain the nature of such glasses.

Multiplying glasses are generally lenses, plane on one side, and on the other cut into several facets in the form of a polyedron; of this kind is the glass represented fig 41 and 42, plate 12, where it is seen in front, and also edgewise. It consists of a plane hexagonal facet in the centre, and six trapeziums arranged round the circumference.

These glasses have the property of representing the object as many times as there are facets; for if we suppose the object to be O, the rays which proceed from it fall upon all the facets of the glass AD, DC, &c. Those which traverse the facet DC, pass through it as through a plane glass interposed between the eye and the object; but the rays that proceed from O, to the inclined facet AD, experience a double refraction, which makes them converge towards the axis OE, nearly as they would do if they fell upon the spherical surface, in which the glass polyedron might be inscribed. The eye, being placed in the common point of concurrence, sees the point O, at ω, in the continuation of the radius EF; consequently an image of the point O, different from the former, will be observed. As the same thing takes place in regard to each facet, the object will be seen as many times as there are facets on the glass, and in different places.

Now if we suppose a luminous point in the axis of the

glass, and at a proper distance, all the rays which fall on one facet will, after a double refraction, proceed to a piece of white paper placed perpendicular to the axis continued, and paint on it an image of that facet of a greater or less size, and which at a certain distance will be inverted. Consequently, if we suppose the eye to be substituted instead of the luminous point, and that the image itself is luminous or coloured, the rays which proceed from that image, or part of the paper, will terminate at the eye; and they will be the only ones that reach it after experiencing a double refraction on the same facet: If the like reasoning be employed in regard to the rest, it may be easily seen that, when the eye is placed in a fixed point, it will observe through each facet only a certain portion of the paper, and that the whole together will fill the field of vision, though detached on the paper; so that if a certain part of a regular and continued picture, be painted on each, they will all together represent that picture.

The artifice then of the proposed magic picture, after having fixed the place of the eye, that of the glass and the field of the picture, is to determine those portions of the picture which shall alone be seen through the glass; to paint upon each the determinate portion, according to a given subject, such as a portrait, so that when united together they may produce the painting itself; and in the last place to fill up the remainder of the field of the picture with any thing at pleasure; but arranging the whole in such a manner as to form a regular subject.

Having thus explained the principle of this optical amusement, we shall now show how it is to be put in practice.

Let ABCD, fig. 43 pl. 12, represent a board, at the extremity of which is fixed another in a perpendicular direction, having at its edges two pieces of wood with grooves, to receive a piece of pasteboard, covered with white paper or canvas. This pasteboard, which may be pushed in or

drawn out at pleasure, is the field of the intended picture: EDH is a vertical board, the bottom part of which must be contrived in such a manner, that it can be brought nearer to or farther from the painting; and towards the upper part it is furnished with a tube, having at its anterior extremity a glass cut into facets, and at the other a piece of card, in which is a small hole made by means of a needle, and to which the eye is applied We shall here suppose the glass to be plane on one side, and on the other to consist of six rhomboidal facets, placed around the centre, and of six triangular ones which occupy the remainder of the hexagon.

When every thing is thus prepared, fix the support EDH at a certain distance from the field of the picture, according as you are desirous that the parts to be delineated should be nearer to or farther from each other. But this distance ought, at least, to be 4 times the diameter of the sphere in which the polyedron of the glass could be inscribed; and the distance of the eye from the glass may be equal to twice that diameter. Then place the eye at the hole K, the distance of which has been thus determined, and with a stick, having a pencil at the end of it, if the hand cannot reach the pasteboard, trace out, in as light a manner as possible, the outline of the space observed through one facet, and do the same thing in regard to the rest. This operation will require a great deal of accuracy and patience; for, to render the work perfect, no perceptible interval must be left between the two spaces seen through two contiguous facets: it will be better on the whole if they rather encroach a little on each other. Care must also be taken to mark each space with the same number as that assigned to each facet, in order that they may be again known. This however will be easy, by observing that the space corresponding to each facet is always transferred parallel to itself from top to bottom, or from right to left, on the other side of the centre.

The next thing is to delineate the regular picture intended to be seen, and to transpose it into the spaces where it appears distorted. According to mathematical accuracy, it would be necessary for this purpose to form a projection of the glass cut into facets, supposing the eye at the distance at which it is really placed; but as we suppose it a little more remote, we may without any sensible error assume, as the field of the regular picture, the vertical projection, as seen fig. 44 nº. 1, where it is represented such as it would appear to the eye placed perpendicularly above its centre, and at a very considerable distance.

Delineate in the field, which in this case will be hexagonal, and composed of 6 rhomboids and 6 triangles, any figure whatever, as a portrait for example, and then, considering that the space *abcd* is that where the portion of the picture marked 1 ought to appear, it must be transferred thither with as much care as possible; do the same thing in regard to the rest; and by these means the principal part of the picture will be completed. But as it is intended to shew something else beside what ought to be seen, it must be disguised by means of some other objects painted in the remaining part of the field, making them to harmonize with what is already painted, in such a manner, that the whole shall appear to form one regular and connected subject. All this however must depend on the taste and genius of the artist.

In the *Perspective curieuse* of father Niceron, a much more minute explanation of the whole process may be found. Those to whom what is here said does not seem sufficient, must consult that work. Niceron tells us that he executed, at Paris, and deposited in the library of the Minimes of the Place Royale, a picture of this kind, which when seen with the naked eye represented fifteen portraits of Turkish Sultans; but when viewed through the glass was the portrait of Louis the 13th.

A picture by Amadeus Vanloo, much more ingenious,

was shown in the year 1759, in the exhibition room of the Royal Academy of Painting. To the naked eye, it was an allegorical picture, which represented the Virtues, with their attributes, properly grouped; but when seen through the glass, it exhibited the portrait of Louis the 15th.

REMARKS.—1st. It is necessary to observe that the place of the glass, when once fixed, must be invariable; for as glasses perfectly regular cannot be obtained, if they are moved it will be almost impossible to replace them in the proper point; hence it will be necessary to be assured that the glass is of a good quality; for if it be too alkaline, and happen to lose its polish by the contact of the air, another capable of producing the same effect cannot be substituted in its stead. This is an accident which, according to what we have heard, happened to the glass of Vanloo's picture.

2d. Instead of a glass, like that employed in the above example, or of one more compounded, a plain pyramidal glass might be employed, by which the problem would be greatly simplified.

3d. A glass, the portion of a prism, cut into a great number of planes parallel to its axis, might also be employed; in this case the painting to be viewed through the glass ought to be delineated on parallel bands.

4th. A glass might be formed of several concentric conical surfaces, or of several spherical surfaces of different diameters, likewise concentric: in this case the picture to be viewed through the glass ought to be distributed in different concentric rings.

5th. A magic picture might be formed by reflection. For this purpose, provide a metal mirror with facets well polished, and having very sharp edges; place before it, in a direction parallel to its axis, a piece of white paper or card, and by means of the principles above explained, delineate a picture, which when viewed in front by the

naked eye shall represent a certain subject; if you then make a hole in the middle of the picture, and look through this hole at the image of it formed by the mirror, it will appear to be entirely different.

PROBLEM L.

To construct a Lantern, by means of which a book can be read at a great distance, at night.

Construct a lantern of a cylindric form, or shaped like a small cask placed lengthwise, so that its axis shall be horizontal; and in one end of it fix a parabolic mirror, or merely a spherical one, the focus of which falls about the middle of the length of the cylinder: if a taper or lamp be then placed in this focus, the light will be reflected through the open end, and will be so strong that very small print may be read by it at a great distance, if looked at through a telescope. Those who see this light at a distance, if standing in the axis of the lantern continued, will imagine that they see a large fire.

PROBLEM LI.

To construct a Magic Lantern.

The name of *magic lantern*, as is well known, is given to an optical instrument, by means of which figures greatly magnified may be represented on a white wall or cloth. This instrument, invented, we believe, by Father Kircher, a jesuit, has become a useful resource to a great number of people, who gain their livelihood by exhibiting this spectacle to the populace. But though it has fallen into vulgar hands, it is nevertheless ingenious, and deserves a place in this work. We shall therefore describe the method of constructing it, and add a few observations, which may tend to improve it, and to render it more interesting.

First, provide a box about a foot square (fig. 45 pl. 13) of tin-plate, or copper or wood, and make a hole towards

the middle of the fore-part of it, about 3 inches in diameter: into this hole let there be soldered a tube, the interior aperture of which must be furnished with a very transparent lens, having its focus within the box, and at the distance of two thirds or three fourths of the breadth of the box. In this focus place a lamp with a large wick, in order that it may produce a strong light; and that the machine may be more perfect, the lamp ought to be moveable, so that it can be placed exactly in the focus of the lens. To avoid the aberration of sphericity, the lens in question may be formed of two lenses, each of a double focus. This, in our opinion, would greatly contribute to the distinctness of the picture.

At a small distance from the aperture of the box, let there be a slit in the tube, for which purpose this part of it must be square, capable of receiving a slip of glass surrounded by a frame, 4 inches in breadth, and of any length at pleasure. Various objects according to fancy are painted on this slip of glass, with transparent colours; but in general the subjects chosen are of the comic and grotesque kind (fig. 46 pl. 13).

Another tube, furnished with a lens of about 3 inches focal distance, must be fitted into the former one, and in such a manner, that it can be drawn out or pushed in as may be found necessary.

Having thus given a description of the machine, we shall now explain its effect. The lamp being lighted, and the machine placed on the table opposite to a white wall, if it be exhibited in the day time, shut the windows of the apartment, and introduce into the slit above mentioned one of the painted slips of glass, but in such a manner that the figures may be inverted: if the moveable tube be then pushed in or drawn out, till the proper focus is obtained, the figures on the glass will be seen painted on the wall in their proper colours, and greatly magnified.

If the other end of the moveable tube be furnished with

a lens of a much greater focal distance, the luminous field will be increased, and the figures will be magnified in proportion. It will be of advantage to place a diaphragm in this moveable tube, at nearly the focal distance of the first lens, as it will exclude the rays of the lateral objects, and thereby contribute to render the painting much more distinct.

We have already said that the small figures on the glass must be painted with transparent colours. The colours for this purpose may be made in the following manner: red by a strong infusion of Brasil wood, or cochineal, or carmine, according to the tint required; green by a solution of verdigris; or for dark greens, of martial vitriol (sulphate of iron); yellow, by an infusion of yellow berries; blue, by a solution of vitriol of copper (sulphate of copper); these three or four colours, as is well known, will be sufficient to form all the rest: they may be mixed up and rendered tenacious by means of very pure and transparent gum-water, after which they will be fit for painting on glass. In most machines of this kind, the paintings are so coarsely executed, that they cannot fail to excite disgust; but if they are neatly designed, and well finished, this small optical exhibition must afford a considerable degree of pleasure.

PROBLEM LII.

Method of constructing a Solar Microscope.

The solar microscope, for the invention of which we are indebted to Mr. Lieberkun, is nothing else properly speaking than a kind of magic lantern, where the sun performs the part of the lamp, and the small objects exposed on a glass or the point of a pin, that of the figures painted on the glass slips of the latter. But the following is a more minute description of it.

Make a round hole in the window shutter, about 3 inches in diameter, and place in it a glass lens of about 12 inches

focal distance. To the inside of the hole adapt a tube having, at a small distance from the lens, a slit or aperture, capable of receiving one or two very thin plates of glass, to which the objects to be viewed must be affixed by means of a little gum water exceedingly transparent. Into this tube fit another, furnished at its anterior extremity with a lens of a short focal distance, such for example as half an inch. If a mirror be then placed before the hole in the window shutter on the outside, in such a manner as to throw the light of the sun into the tube, you will have a solar microscope. The method of employing it is as follows.

Having darkened the room, and by means of the mirror reflected the sun's rays on the glasses in a direction parallel to their axes, place some small object between the two moveable plates of glass, or affix it to one of them with very transparent gum water, and bring it exactly into the axis of the tube: if the moveable tube be then pushed in or drawn out, till the object be a little beyond the focus, it will be seen painted very distinctly on a card or piece of white paper, held at a proper distance; and will appear to be greatly magnified. A small insect, such as a flea for example, may be made to appear as large as a sheep, or a hair as large as a walking stick: by means of this instrument the eels in vinegar, or flour paste, will have the appearance of small serpents

REMARK.—As the sun is not stationary, this instrument is attended with one inconvenience, which is, that as this luminary moves with great rapidity, the mirror on the outside requires to be continually adjusted. This defect however s'Gravesande remedied by means of a very ingenious machine, which moves the mirror in such a manner, that it always throws the sun's rays into the tube. This machine, therefore, has been distinguished by the name of the *sol-sta*.

Some curious details respecting the solar microscope

may be seen in the French Translation of Smith's Optics, where several useful inventions for improving it, and for which we are indebted to Euler, are explained. A method, invented by Æpinus, of rendering it proper for representing opake objects, will be found there also. It consists in reflecting, by means of a large lens and a mirror, the condensed light of the sun on the surface of the object, presented to the object glass of the microscope. M. Mumenthaler, a Swiss optician, proposed a different expedient. But solar microscopes are still attended with another inconvenience: as the objects are very near the focus of the first lens, they are subjected to a heat which soon destroys or disfigures them. Dr. Hill, who made great use of this microscope, proposed therefore to employ several lamps, the light of which united into one focus is exceedingly bright and free from the above inconvenience; but we do not know whether he ever carried this idea into practice, and with what success.

PROBLEM LIII.

Of Colours, and the different Refrangibility of Light.

One of the noblest discoveries of the 17th century, is that made by the celebrated Newton, in 1666, respecting the composition of light, and the cause of colours. Who could have believed that white, which appears to be a colour so pure, is the result of the seven primitive unalterable colours mixed together in a certain proportion! This however has been proved by his experiments.

The instrument which he employed for decomposing light in this manner, was the prism, now well known, but at that time a mere object of curiosity on account of the colours, with which every thing viewed through it seems to be bordered. But on this subject we shall confine ourselves to two of Newton's experiments, and a deduction of the consequences which result from them.

If a ray of solar light, an inch or half an inch in diameter

(fig. 47, pl. 13), be admitted into a darkened room, so as to fall on a prism placed horizontally, with a piece of white paper behind it, and if the prism be turned in such a manner, that the image seems to stop; instead of an image of the sun nearly round, you will observe a long perpendicular band, consisting of seven colours, in this invariable order, red, orange, yellow, green, blue, indigo, violet. When the angle of the prism is turned downwards, the red will be at the bottom, and vice versa; but the order will be always the same.

From this, and various other experiments of a similar kind, Newton concludes,

1st. That the light of the sun contains these 7 primitive colours.

2d. That these colours are formed by the rays experiencing different refractions; and the red, in particular, is that which is the least broken or refracted; the next is the orange, &c; in the last place, that the violet is that which, under the same inclination, suffers the greatest refraction. The truth of these consequences cannot be denied by those who are in the least acquainted with geometry.

But the nicest experiment is that by which Newton proved, that these differently coloured rays are afterwards unalterable. To make this experiment in a proper manner, it will be necessary to proceed as follows:

In the first place, the hole in the window shutter of the darkened room must be reduced to the diameter of a line at most; and the light every where else must be carefully excluded. When this is done, receive the solar rays on a large lens, of 7 or 8 feet focus, placed at the distance of 15 feet from the hole, and a little beyond the lens place a prism, in such a manner, that the stream of light may fall upon it. Then hold a piece of white card at such a distance that the image of the sun would be painted upon it without the interposition of the prism, and you will see painted

on the card, instead of a round image, a very narrow coloured band, containing the seven primitive colours.

Then pierce a hole in the card, about a line in diameter, and suffer any one of the colours to pass through it, taking care that it shall do so in the middle of the space which it occupies, and receive it on a second card placed behind the former. If intercepted by another prism, it will be found that it no longer produces a lengthened, but a round image, and all of the same colour. Besides, if you hold in that colour any object whatever, it will be tinged by it; and if you look at the object with a third prism, it will be seen of no other colour but that in which it is immersed, and without any elongation, as when it is immersed in light susceptible of decomposition.

This experiment, which is now easy to those tolerably well versed in philosophy, proves the third of the principal facts advanced by Newton.

3d. That when a colour is freed from the mixture of others, it is unalterable; that a red ray, whatever refraction it may be made to experience, will always remain red, and so of the rest.

It does no great honour to the French philosophers of the 17th century to have disputed, and even declared false, this assertion of the English philosopher, especially on no better foundation than an experiment so badly performed, and so incomplete as that of Mariotte. We even cannot help accusing that philosopher, who in other respects deserves great praise, of too much precipitation; for his experiment was not the same as that described by Newton in the *Philosophical Transactions*, for 1666; and it may be readily seen that, if performed according to Mariotte's manner, it is impossible it should succeed.

However, it is at present certain, notwithstanding the remonstrances of Father Castel and the Sieur Gautier*,

* The Sieur Gautier, who pretended to be the inventor of the method of engraving in colours, opposed with great violence, in the year 1750, the

that there are in nature seven primitive, homogeneous colours, unequally refrangible, unalterable, and which are the cause of the different colours of bodies; that white contains them all, and that all of them together compose white; that what makes a body be of one colour rather than another, is the configuration of its minute parts, which causes it to reflect in greater number the rays of that particular colour; and in the last place that black is the privation of all reflection; but this is understood of perfect black, for the material and common black is only an exceedingly dark blue.

Some people, such as Father Castel, have admitted only three primitive colours, viz red, yellow and blue, because red and yellow form orange; yellow and blue green, and blue and red violet or indigo, according as the former or the latter predominates. But this is another error. It is very true that with two rays, one yellow and the other blue, green can be formed; and this holds good also in regard to material colours, but the green of the coloured image of the prism is totally different: it is primitive, and stands the same proof as red, yellow or blue, without being decomposed. The case is the same with orange, indigo, and violet.

PROBLEM LIV.

Of the Rainbow, how formed; method of making an artificial one.

Of all the phenomena of nature, none has excited more the admiration of mankind, in all ages, than the rainbow; but there is none perhaps at present which philosophy can explain in a more satisfactory manner.

theory of Newton, both in regard to colours and to the system of the universe. His reasoning and experiments, however, are as conclusive as experiments made with a faulty air-pump would be against the gravity of the atmosphere. For this reason, he never had any partizans but a few of his own countrymen, one of whom was a poet, who had found out that objects are not painted on the retina in an inverted position.

The rainbow is formed by the solar rays being decomposed into their principal colours, in the small drops of rain, by means of two refractions, which they experience in entering them and issuing from them. In the interior rainbow, which often appears alone, the solar ray enters at the upper part of the drop, is reflected against the bottom, and issues at the lower side. This decomposition may be seen fig. 48. In the exterior rainbow, the rays enter at the bottom of the drop, experience two reflections, and issue at the upper part. Their progress and decomposition, which produces colours in an order contrary to the former, are represented fig. 49. Hence the colours of the exterior rainbow appear to be inverted, in regard to those of the first.

The manner in which the eye perceives this double series of colours is seen fig. 50.

But the explanation would be incomplete if we did not show that there is a certain determinate inclination, under which the red rays issue parallel, and as close to each other as possible, while all the rest are divergent; that there is another under which the green rays issue in this manner: and so of the rest. It is by this alone that they can produce an effect on a distant eye.

This explanation of the rainbow is confirmed by a simple experiment. When the sun is very near the horizon, suspend in an apartment a glass globe filled with water, in such a manner as to be illuminated by the sun; and place yourself with your back to that luminary, so that the globe shall be elevated, in regard to your eye, about 42 degrees above the horizon. By advancing or retiring a little, you will not fail to meet with the coloured rays, and it will be easily seen that they issue from the bottom of the globe; it will be seen also that the red ray issues from it under the greatest angle with the horizon, and the violet, which is the lowest one, under the least, so that the red must be without the axis, and the violet within it.

Then raise the globe, in regard to your eye, to 54 degrees, or continue to approach it till it be elevated at that angle, and you will meet with the coloured rays issuing from the top of it; first the violet, and then the blue, green and red, in an order altogether contrary to the preceding. If you cover, in the first case, the upper part of the globe, and in the second the lower part, no colours will be produced; which is a proof of the manner in which they enter it, and issue from it.

The spectacle of an artificial rainbow may be easily obtained; it is seen in the vapour of a jet of water, when the wind disperses it in minute drops. For this purpose, place yourself in a line between the jet of water and the sun, with your back turned towards the latter. If the sun be at a moderate elevation above the horizon, by advancing towards the jet of water or receding from it, you will soon find a point from which a rainbow will be seen in the drops that fall down in fine light-rain.

If there be not a jet of water in the neighbourhood, you may make one at a very small expence. Nothing will be necessary but to fill your mouth with water, and having turned your back to the sun when at a moderate elevation, to spirt the water into the air as high as possible, and in a direction somewhat oblique to the horizon. The imitation of this phenomenon may be greatly facilitated by employing a syringe, which will scatter the water in very small drops.

If you are desirous of performing the experiment in a manner still easier, fill a very transparent cylindric glass bottle with water, and place it on a table in an upright position; place a lighted candle at the same height, and at the distance from it of 10 or 12 feet, and then walk in a transversal direction between the light and the bottle, keeping your eye at the same elevation. When you have reached a certain point, you will see bundles of coloured rays issuing from one of the sides of the bottle, in the fol-

lowing order: violet, blue, yellow, red; and if you continue to walk transversely, you will meet with a second series, in a contrary order, viz, red, yellow, blue and violet, proceeding from the other side of the bottle. This is exactly what takes place in regard to the drops of rain; and to imitate the phenomenon completely, seven similar bottles might be arranged in such a manner, that the eye being placed in the proper point, one of the seven colours should be seen in each; and seven others might be arranged at some distance, so as to exhibit the same colours in an inverted order.

Two rainbows would still be produced, even if the solar rays were not differently refrangible; but they would be destitute of colour, and would consist only of two circular bands of white or yellowish light.

The rainbow always forms a portion of a circle around the line drawn from the sun and passing through the eye of the spectator; for this reason, when the sun is elevated above the horizon, the rainbow is less than a semicircle; but when the sun is in the horizon, it is equal to a semicircle.

A rainbow however has been seen larger than a semicircle, and which intersected the common rainbow; but this phenomenon was produced by the image of the sun reflected from the calm, smooth surface of a river. The image of the sun, in this case, produced the same effect as if that luminary had been below the horizon.

Dr. Halley has calculated, from the ratio of the different refrangibilities of the sun's rays, that the semi-diameter of the interior rainbow, taken in the middle of its extent, ought to be 41° 10′; and that its breadth, which would be only 1° 45′ if the sun were a point, ought to be 2° 15′ on account of the apparent diameter of that luminary. This apparent diameter is the cause why the colours are not separated from each other with the same distinctness as they would be, if the sun were a luminous point: the radius

of the exterior rainbow, taken in the same manner, that is to say in the middle of its extent, is 52° 30′.

This geometrician and astronomer not only calculated the dimensions of that rainbow which actually appears to us in the heavens, but of those also which would be produced if the light of the sun did not issue from the drop of water till after 3, 4, 5, &c, reflections; whereas, in the principal and interior rainbow, it issues after one, and in the second or exterior one, after two. By these calculations it is found that the semi-diameter of the third rainbow, counted from the place of the sun, would be 41°; that of the fourth, 43° 50′; &c. But geometry here goes much farther than nature: for besides the continued weakness of the rays, which would render these rainbows scarcely perceptible, being towards the sun, they would be lost amidst the splendour of that luminary. If the drops which form the rainbow, instead of being water, were glass, the mean semi-diameter of the interior rainbow would be 22° 52′, and that of the exterior 9° 30′, towards the side opposite to the sun.

PROBLEM IV.

Analogy between Colours and the Tones of Music. Of the Ocular Harpsichord of Father Castel.

As soon as it had been observed that there were seven primitive colours in nature, there was some reason to conceive that there might be an analogy between these colours and the tones of music; for the latter form a series of seven in the whole extent of the octave. This observation did not escape Newton, who remarked also that, in the coloured spectrum, the spaces occupied by the violet, indigo, blue, &c., correspond to the divisions of the monochord, which gives the sounds *re*, *mi*, *fa*, *sol*, *la*, *si*, *ut*, *re*.

Newton on this subject proceeded no farther. But Father Castel, whose visionary scheme is well known, enlarged this idea; and on the above analogy of sounds

founded a system, in consequence of which he promised to the eyes, but unfortunately without success, a new pleasure similar to that which the ears experience from a concert.

Father Castel, for reasons of analogy, first changes the order of the colours into the following, viz, blue, green, yellow, orange, red, violet, indigo, and in the last place blue, which forms as it were the octave of the first. These, according to his system, are the colours which correspond to the diatonic octave of our modern music, *ut*, *re*, *mi*, *fa*, *sol*, *la*, *si*, *ut*. The flats and the sharps gave him no embarrassment; and the chromatic octave divided into its twelve colours, was blue, sea-green, olive-green, yellow, apricot, orange, red, crimson, violet, agate, indigo, blue, which corresponded to *ut*, *ut**, *re*, *re**, *mi*, *fa*, *fa**, *sol*, *sol**, *la*, *la**, *si*, *ut*.

Now if a harpsichord be constructed in such a manner, says Father Castel, that on striking the key *ut*, instead of hearing a sound, a blue band shall appear; that on striking *re*, a green one shall be seen, and so on, you will have the required instrument; provided that for the first octave of *ut* a different blue be employed. But what are we to understand by a blue an octave to another? We do not find that Father Castel ever explained himself on this subject in a manner sufficiently clear. He only says that as there are reckoned to be twelve octaves appreciable by the ear, from the lowest sound to the most acute, there are in like manner twelve octaves of colours, from the darkest blue, to the lightest; which gives us reason to believe that since the darkest blue is that which ought to represent the lowest key, the blue corresponding to the octave must be formed of eleven parts of pure blue, and one of white; that the lightest must be formed of one part of blue and eleven parts of white, and so of the rest.

However, Father Castel did not despair of producing by these means an ocular music, as interesting to the eyes as

the common music is to well organised ears; and he even thought that a piece of music might be translated into colours for the use of the deaf and dumb. "You may conceive," says he, "what spectacle will be exhibited by a room covered with rigadoons and minuets, sarabands and passcailles, sonatas and cantatas, and if you choose with the complete representation of an opera? Have your colours well diapasoned, and arrange them on a piece of canvas according to the exact series, combination and mixture of the tones, the parts and concords of the piece of music which you are desirous to paint, observing all the different values of the notes, minims, crotchets, quavers, syncopes, rests, &c; and disposing all the parts according to the order of counter-point. It may be readily seen that this is not impossible, nor even difficult, to any person who has studied the elements of painting, and at any rate that a piece of tapestry of this kind would be equal to those where the colours are applied as it were at hazard in the same manner as they are in marble.

"Such a harpsichord," continues he, "would be an excellent school for painters, who might find in it all the secrets and combinations of the colours, and of that which is called claro-obscuro. But even our harmonical tapestry would be attended with its advantages; for one might contemplate there at leisure what hitherto could be heard only in passing with rapidity, so as to leave little time for reflection. And what pleasure to behold the colours in a disposition truly harmonical, and in that infinite variety of combinations which harmony furnishes! The design alone of a painting excites pleasure. There is certainly a design in a piece of music; but it is not so sensible when the piece is played with rapidity. Here the eye will contemplate it at leisure; it will see the concert, the contrast of all the parts, the effect of the one in opposition to the other, the fugues, imitations, expression, concatenation of the ca-

dences, and progress of the modulation. And can it be believed that those pathetic passages, those grand traits of harmony, those unexpected changes of tone, that always cause suspension, languor, emotions, and a thousand unexpected changes in the soul which abandons itself to them, will lose any of their energy in passing from the ears to the eyes, &c? It will be curious to see the deaf applauding the same passages, as the blind, &c. Green, which corresponds to *re*, will no doubt show that the tone *re* is rural, agreeable and pastoral; red, which corresponds to *sol*, will excite the idea of a warlike and terrific tone; blue, which corresponds to *ut*, of a noble, majestic and celestial tone; &c. It is singular that the colours should have the proper characters ascribed by the ancients to the exact tones which correspond to them, but a great deal might be said, &c.

"A spectacle might be exhibited of all forms human and angelical, animals, birds, reptiles, fishes, quadrupedes, and even geometric figures. By a simple game the whole series of Euclid's Elements might be demonstrated." Father Castel's imagination seems here to conduct him in the straight road to Bedlam.

These passages of Father Castel are so singular, that we could not help quoting them; but unfortunately all his fine promises came to nothing. He had constructed a model of his harpsichord, as he tells us himself, so early as the end of the year 1734, and he spent almost the remainder of his life, till the time of his death, which took place in 1757, in completing his instrument, but without success. This harpsichord, constructed at a great expence, as we are told by the author of his life, neither answered the author's intention, nor the expectation of the public. And indeed if there be any analogy between colours and sounds, they differ in so many other points, that it need excite no wonder that this project should miscarry.

PROBLEM LVI.

To compose a Table representing all the Colours; and to determine their number.

Though Newton has proved the homogeneity of the colours into which the solar rays are decomposed, and the orange, green and purple produced by this decomposition are no less unalterable, by farther refraction, than the red, yellow and blue, it is however well known that with the three latter, the three former, and all the other colours of nature, can be imitated: for red combined with yellow, in different proportions, gives all the shades of orange; yellow and blue produce pure greens; red and blue violets, purples and indigoes; in short, the different combinations of these compound colours, give birth to all the rest. On these principles is founded the invention of the chromatic triangle, which serves to represent them.

Construct an equilateral triangle, as seen Plate xv fig. 51, and divide the two sides adjacent to the vertical angle into 13 equal parts: if parallel lines be then drawn through the points of division, in each side, they will form 91 equal rhombuses

In the three angular rhombs place the three primitive colours, red, yellow, and blue, having an equal degree of strength, and as we may say of concentration; consequently, between the yellow and blue, there will be left 11 rhomboidal cells, which must be filled up in the following manner: in that nearest the yellow put 11 parts of yellow and 1 of red; in the next, 10 parts of yellow and 2 of red, &c; so that in the cell nearest the red there will be 1 part of yellow and 11 of red: by these means we shall have all the shades of orange, from the one nearest red to that nearest yellow By filling up, in like manner, the intermediate cells, between red and blue, and between blue and yellow, the result will be all the shades of purple, and all those of green, in a similar gradation.

To fill up the other cells, let us take for example those of the third row below red, where there are three cells. The two extreme cells being filled up on the one side with a combination of 10 parts of red and 2 of yellow, and on the other with a combination of 10 parts of red and 2 of blue, the middle cell will be composed of 10 parts of red, 1 of blue and 1 of yellow.

In the band immediately below, we shall have, for the same reason, in the first cell towards the yellow, 9 parts of red and 3 of yellow; in the next, 9 parts of red, 2 of yellow and 1 of blue; in the third, 9 parts of red, 1 of yellow and 2 of blue; in the fourth, 9 parts of red, and 3 of blue; and the case will be similar in regard to the lower bands; but we shall here content ourselves with detailing the colours in the last except one, or that above the band containing the greens, the cells of which must be filled up as follows: In

The 1st on the left, 11 parts yellow and 1 of red.
The 2d, 10 parts yellow, 1 red, 1 blue.
The 3d, 9 parts yellow, 1 red, 2 blue.
The 4th, 8 parts yellow, 1 red, 3 blue.
The 5th, 7 parts yellow, 1 red, 4 blue.
The 6th, 6 parts yellow, 1 red, 5 blue.
The 7th, 5 parts yellow, 1 red, 6 blue.
The 8th, 4 parts yellow, 1 red, 7 blue.
The 9th, 3 parts yellow, 1 red, 8 blue.
The 10th, 2 parts yellow, 1 red, 9 blue.
The 11th, 1 part yellow, 1 red, 10 blue.
The 12th, 0 part yellow, 1 red, 11 blue.

This band, as may be seen, contains all the greens of the lowest band into which one part of red has been thrown. In like manner, there will be found in the band parallel to the purples all the purples with which 1 part of yellow has been mixed; and in the band parallel and contiguous to the oranges, all the orange colours with one part of blue.

In the central cell of the triangle there are 4 parts of red, 4 of blue, and 4 of yellow.

All these mixtures might be easily made with colours ground exceedingly fine; and if the proper quantities were employed we have no doubt that they would produce all the shades of the different colours. But if all the colours of nature, from the lightest to the darkest, that is from black to white, be required, we shall find for each cell 12 degrees of gradation to white, and 12 others to black. If 91 therefore be multiplied by 24, we shall have 2184 perceptible colours; to which if we add 24 grays, formed by the combination of pure black and white, and white and black, the number of compound colours, which we believe to be distinguishable by the senses, will amount to 2218. But we ought not perhaps to consider as real colours those formed by the pure colours with black; for black only obscures, but does not colour. In this case the real colours with their shades, from the darkest to the lightest, ought to be reduced to 1092, which with white, a black, and 12 grays, will form 1106 colours.

PROBLEM LVII.

On the Cause of the Blue Colour of the Sky.

This is a very remarkable phenomenon, though little attention is paid to it, as our eyes are so much accustomed to it from our infancy; and it would be difficult to explain it had not Newton's theory respecting light, by teaching us that it is decomposed into seven colours of different degrees of refrangibility and reflexibility, afforded us the means of discovering the cause.

To explain this phenomenon, we shall observe then, that according to Newton's theory, so well proved by experience, of the seven colours which the solar light produces when decomposed by the prism, the blue indigo, and violet, are those easiest reflected, when they meet with a

medium of a different density. But whatever may be the transparency of the air, that which surrounds our earth, and which constitutes our atmosphere, contains always a mixture of vapours more or less combined with it: hence it happens that the light of the sun and stars, sent back in a hundred different ways into the atmosphere, must experience in it numberless inflections and reflections. But as the blue, indigo and violet rays are those chiefly sent back to us, at each of these reflections, from the minute particles of the vapours which they are obliged to pass through, it is necessary that the medium which sends them back should appear to assume a blue tint. This must even be the case if we suppose a perfect homogeneity in the atmosphere: for however homogeneous a transparent medium may be, it necessarily reflects a part of the rays of light which pass through it. But of all these rays, the blue are reflected with the greatest facility; consequently the air, even supposing it homogeneous, would assume a blue, or perhaps a violet colour.

It is for the same reason that the water of the sea appears of a blue colour when very pure, as is the case at a distance from the coasts. When illuminated by the sun a part of the rays enters the water, and another part is reflected; but the latter is composed chiefly of blue rays, and consequently it must appear blue.

This explanation is confirmed by a curious observation of Dr. Halley. This celebrated philosopher having descended in a diving bell to a considerable depth in the sea, while it was illuminated by the sun, was much surprised to see the back of his hand, which received the direct rays, of a beautiful rose colour, while the lower part, which received the reflected rays, was blue. This indeed is what ought to take place, if we suppose that the rays reflected by the surface of the sea, as well as by the minute parts of the middle of it, are blue rays. In proportion as the light

penetrates to a greater depth, it must be more and more deprived of the blue rays, and consequently the remainder must incline to red.

PROBLEM LVIII.

Why the Shadows of bodies are sometimes Blue, or Azure coloured, instead of being Black.

It is often observed at sun-rise, during very serene mornings, that the shadows of bodies projected on a white ground, at a small distance, are blue or azure coloured. This phenomenon appears to us to be sufficiently curious to deserve here a place, as well as an explanation.

If the shadow of a body exposed to the sun were absolute, it would be perfectly black, since it would be a complete privation of light; but this does not really take place: for to be so, the field of the heavens ought to be absolutely black; whereas it is blue, or azure coloured, and it is so only because it sends back to us chiefly blue rays, as already observed.

The shadow therefore projected by bodies exposed to the sun, is not a pure shadow, but is itself illuminated by that whole part of the sky not occupied by the luminous body. This part of the heaven being blue, the shadow is softened by the blue or azure coloured rays, and consequently must appear of that colour. It is exactly in the same manner that in painting, reflections are tinted with the colour of the surrounding bodies. The shadow which we here examine, is nothing else than a shadow mixed with the reflection of a blue body, and therefore it must participate in that colour.

It is well known that this phenomenon was first observed and explained by Buffon.

But it may here be asked, why are not all shadows blue? In reply to this question, we shall observe, that to produce this effect, the concurrence of several circumstances are necessary: 1st, a very pure sky, and of a very dark blue

colour; for if the heavens be interspersed with light clouds, the rays reflected from them, falling on the blueish shadow, will destroy its effect; if the blue be weak, as is often the case, the quantity of the blue rays will not be sufficient to enlighten the shadow. 2d, The light of the sun must be livelier than it usually is when that luminary is near the horizon, in order that the shadows may be full and strong. But these circumstances are rarely united. Besides, the sun must be only at a small elevation above the horizon; for even when at a moderate altitude there is too much splendour in the atmosphere, to allow the blue rays to be sensible. This light renders the shadow less strong, but does not tinge it blue.

PROBLEM LIX.

Experiment on Colours.

Hold before your eyes two glasses of different colours, the one blue suppose, and the other red; and having placed yourself at a proper distance from a candle, if you shut one of your eyes, and look at the light with the other, that for example before which the blue glass is held, the light will appear blue. If you next shut this eye and open the other, the flame will appear red; and if you then open them both, you will see it of a bright violet colour.

Every person almost, in our opinion, must have foreseen the success of this experiment; which we have mentioned, merely because an oculist of Lyons, M. Janin, thought he could deduce from it a particular consequence; which is, that the retina may perform the part of a concave mirror, and reflect the rays of light, so that each eye forms at a certain distance an aerial image of the object. Both eyes forming each an image afterwards in the same place, the result is a double image, one blue and the other red, which by their union produce a violet image, in the same manner as when red and blue rays are mixed together. But this explanation will certainly not bear to be examined ac-

cording to the true principles of optics. How is it possible to conceive that such an image can be formed by the retina? Is it not more probable, and more agreeable to the well known phenomena of vision, that from the two impressions received by the two eyes, there is produced in the *common sensorium*, or in the place where the optic nerves are united in the brain, one compound impression? In this experiment therefore the same thing must take place, as when a person looks at a candle with one eye, through two glasses, the one red and the other blue. In this case the flame will be seen of a violet colour, and consequently it must have the same appearance in the former.

PROBLEM LX.

Method of constructing a Photophorus, very convenient to illuminate a table where a person is reading or writing.

Construct a cone of tin-plate, $4\frac{1}{2}$ inches in diameter at the base, and $7\frac{2}{3}$ inches in height, measured on the slant side; which may be easily done by cutting from a circle, of $7\frac{2}{3}$ inches radius, a sector of $105\frac{1}{2}$ degrees, and bending it into the form of a cone. Then through a point in the axis, $2\frac{1}{5}$ inches distant from the summit, cut off the upper part of the cone by a plane inclined to one of its sides at an angle of 45°. The result will be an elongated elliptical section, which must be placed before a candle or other light, as near to it as possible, the plane of the section being vertical, and the greatest diameter in a perpendicular direction. When disposed in this manner, if the flame of the candle or lamp be raised 12 or 13 inches above the plane of the table, you will be astonished to see the vivacity and uniformity of the light which it will project over an extent of 4 or 5 feet in length.

M. Lambert, the inventor of this apparatus, observes that it may be used with great advantage to give light to those who read in bed; for by placing a lamp or taper furnished with this photophorus upon a pretty high stand,

at the distance of 5, 6, or 8 feet, from the bed, it will afford a sufficiency of light without any danger. He says he tried this apparatus also in the street, by placing a lamp furnished with it in a window raised 15 feet above the pavement, and that its effect was so great, that at the distance of 60 feet, a bit of straw could be distinguished much better than by moon-light, and that writing could be read at the distance of 35 or 40 feet. A few of these machines, placed on each side of a street, and arranged in a diagonal form, would consequently light it much better than any of the means hitherto employed. See *Mémoires de l'Academie de Berlin, ann.* 1770.

PROBLEM LXI.

The place of an object, such for example as of a piece of paper on a table, being given; and that of a candle destined to throw light upon it; to determine the height at which the candle must be placed, in order that the object may be illuminated the most possible.

That we may exclude from this problem several considerations, which would render the solution of it very difficult, we shall suppose the object destined to be illuminated to be very small, or that it is only required that the middle of it shall be illuminated as much as possible. We shall suppose also that the light is entirely concentrated into one point, where the splendor of all its different parts is united.

But, it is well known that the light diffused by a luminous point, over any surface which it illuminates, decreases, the angle being the same, in the inverse ratio of the square of the distance; and that when the angle of inclination varies, it is as the sine of that angle. Hence it follows that it decreases in the compound ratio of the square of the distance taken inversely, and the sine of the angle of inclination taken directly. To solve this problem then, we must find that height of the luminous point in the given

perpendicular, which will render this ratio the greatest possible.

But it will be found that this ratio is greatest when the perpendicular height, and the distance of the object to be illuminated from the bottom of the candle, are to each other as the side of a square is to the diagonal. On this given and invariable distance as hypothenuse, if a right-angled isosceles triangle be therefore described, the side of this triangle will be the height at which, if the flame of the candle be placed, the given point or centre of the paper will be illuminated in the highest degree possible.

On this subject, the following problem, which is of a similar nature, might also be proposed:

Two candles of unequal height, placed at the extremities of a horizontal line, being given; to find in that line a point so situated, that the object placed in it shall be illuminated the most possible?

But we shall not give the solution, that our readers may exercise their own ingenuity in discovering it.

PROBLEM LXII.

On the Proportion which the Light of the Moon bears to that of the Sun.

This is a very curious problem: but it was only within these few years that philosophers began to turn their attention to the principles, and means, which can lead to the solution of it. We are indebted for them to M. Bouguer, who has explained them in his *Treatise on the Gradation of Light*; a work that contains many curious particulars, a few of which we shall here extract.

To obtain this measure of the intensity of light, M. Bouguer sets out with a fact, founded on experience, which is, that the eye judges pretty exactly by habit, whether two similar and equal surfaces are equally illuminated. Nothing then is necessary, but to place at unequal distances two unequal lights, or by means of concave

glasses, the focal distances of which are unequal, to make them be unequally dilated, so that the surfaces which are illuminated by them shall appear to be so in an equal degree. The rest depends merely on calculation: for if two lights, one of which is four times nearer than the other, illuminate equally two similar surfaces, it is evident that, as the degrees of the illumination of the same light decrease in the inverse ratio of the squares of the distances, we ought to conclude that the splendour of the first light is sixteen times as great as that of the second. In like manner, if a light dilated into a circular space, double in diameter, illuminates as much as another direct light, there is reason to conclude that the former is quadruple the second.

By employing these means, M. Bouguer found that the light of the sun diminished 11664 times was equal to that of a flambeau, which illuminates a surface at the distance of 16 inches; and that the same flambeau illuminating a similar surface, at the distance of 50 feet, gave it the same light as that of the moon, when diminished 64 times. By compounding these two ratios he concludes, that the light of the sun is to that of the moon, at their mean distances and at the same altitude, as 256289 to 1; that is to say more than 250 thousand times greater. From some other experiments he is even inclined to think, that the light of the moon is only equal to the 300 thousandth part of that of the sun.

The result of a celebrated experiment, made by Couplet and La Hire, two academicians of Paris, need therefore excite no surprise. These two philosophers collected the lunar rays by means of the burning mirror at the Observatory, which is 35 inches in diameter, and made the focus fall on the bulb of a thermometer, but no motion was produced in the liquor. And indeed this ought to be the case, for if we suppose a mirror like the above, which collects the rays that fall on its surface into a space 1200

or 1400 times less, the heat thence resulting will be 1200 or 1400 times greater; but, on account of the dispersion of the rays, it will be sufficient to suppose this light to be a thousand times denser than the direct light, and the heat in proportion. A mirror of this kind then, by collecting the lunar rays, would produce in its focus a heat 1000 times greater than that of the moon. If 300000 therefore be divided by 1000, we shall have for quotient 300, which expresses the ratio of the direct solar heat to that of the moon thus condensed. But a heat 300 times less than the direct heat of the sun is not capable of producing any effect on the liquor in the thermometer. This fact then is far from being inexplicable, as we are told by the author of the *History of the Progress of the human Mind in Philosophy**, for it is a necessary consequence of Bouguer's calculations, which this writer no doubt overlooked.

We shall, in the last place, observe, that Bouguer found, by a mean calculation, that the splendour of the sun, when on the horizon, supposing the sky to be free from clouds or fog, is about 2000 times less than when elevated 66°. The case ought to be the same also with the light of the moon.

PROBLEM LXIII.

Of certain Optical Illusions.

I. If you take a seal with a cypher engraved on it, and view it through a convex glass of an inch or more focal distance, the cypher or engraving will be seen sunk in the stone as it really is; but if you continue to look at it, without changing your situation, you will soon see it in relief; and by still continuing to look at it, you will see it once more sunk, and then again in relief. Sometimes, after having discontinued to look at it, instead of seeing it sunk, it will appear in relief; it will then appear sunk,

* *Saverien Histoire du Progrès de l'Esprit humain dans les Sciences physiques.*

and so on. When the side of the light is changed, this generally causes a change in the appearance.

Some have taken a good deal of pains to discover the cause of this illusion; which in our opinion may be explained without much difficulty. When an object is viewed with a lens of a short focal distance, and consequently with one eye, we judge very imperfectly of the distance, and the imagination has a great share in that assigned to the image which we perceive. On the other hand, the position of the shadow can never serve to rectify the judgment formed of it: for if the engraving is hollow, and if the light comes from the right, the shadow is on the right; it is also on the right if the engraving is in relief, and if the light comes from the left. But when an engraved stone is attentively viewed with a magnifying glass, we do not pay attention to the side from which the light proceeds. Here then every thing, as we may say, is ambiguous and uncertain; consequently it is not surprising that the organ of sight should form an undecisive and continually variable judgment; but we are fully persuaded that an experienced eye will not fall into these variations.

The same phenomenon is not observed when the experiment is performed with a piece of money. The reason of this probably is, that we are accustomed to handle such pieces, and to see the figures on them in relief, which does not permit the mind to form, in consequence of the image painted in the eye, any other idea than that which it always has had on seeing a piece of money, viz, that of figures in relief.

II. If a glass decanter, half filled with water, be presented to a concave mirror, and at a proper distance, that is to say between the centre and the focus, it will be seen inverted before the mirror. But it is very singular, in regard to many persons, though it is not general, that they imagine they see the water in the half of the bottle next the

neck, which is turned downwards. For our part, we are of opinion that the cause why some think they see the water in this situation, arises from their knowing by experience that if a bottle half filled with water be inverted, the fluid will descend into the lower part, or that next the neck.

Another cause concurs to make us judge in this manner. When a decanter is half full of very pure water, each half is as transparent as the other, and the presence of the water is perceived only by the reflection of the light which takes place at its surface; but in the inverted image this surface reflects the light below, and even with the same force; by which means we are led to conclude that the fluid is at the bottom.

This subject may be farther exemplified as follows.

Take a glass bottle; fill it partly with water, and cork it in the usual way: place this bottle opposite a concave mirror, and beyond its focus, that it may appear reversed: then place yourself still farther distant than the bottle, and this will be seen inverted in the air, and the water, which is really in the lower part of the bottle, will appear to be in the upper. See fig. 52, 53, pl. 17.—If the bottle be inverted while it is before the mirror, the image will appear in its natural erect position, and the water will appear in the lower part of the bottle. While it is in this inverted state, uncork the bottle, then while the water is running out, the image is filling. But as soon as the bottle is empty, the illusion ceases. The illusion also ceases when the bottle is quite full.

The remarkable circumstances in this experiment are, 1st. Not only to see an object where it is not, but also where its image is not. 2d. That of two objects which are really in the same place, as the surface of the bottle and the water it contains, the one is seen in one place, and the other in another, &c.—It is conceived that this illusion arises, partly from our not being accustomed to see water

suspended in a bottle with the neck downward, and partly from the resemblance there is between the colour of water and the air.

Additional Amusements and Experiments with Concave Mirrors, &c, as described by Mr. Adams, Mr. Jones, &c.

1. Placing yourself before a concave mirror, but farther from it than the centre, you will see an inverted image of yourself, but smaller, in the air between you and the mirror: holding out your hand towards the mirror, the hand of the image will come out towards your hand, and when at the centre of concavity, be of an equal size with it; and you may as it were shake hands with this aërial image. On advancing your hand farther, the hand of the image passes by your hand, and comes between it and your body: on moving your hand towards one side, the hand of the image moves towards the other, the image moving always contrariwise to the object. All this while, the by-standers see nothing of the image, because none of the reflected rays which form it enter their eyes. To render this effect more surprizing, and more vivid, the mirror is often concealed in a box, after the manner as we shall shew presently. In fact, the appearance of the image in the air, between the object and the mirror, has been productive of many agreeable deceptions, which, when exhibited with art and an air of mystery, have been very successful, and the source of emolument to many of our public showmen. In this manner they have exhibited the images of animated and other objects, in such a way, as to surprize the ignorant, and please the scientific, or better informed.

2. Mr. Ferguson mentions two pleasing experiments to be made with a concave mirror, which may be easily tried. If a fire be made in a large room, and a smooth, well-polished mahogany table be placed at a good distance near the wall, before a large concave mirror, so that the light

of the fire may be reflected from the mirror to its focus on the table; if you stand by the table, you will see nothing but a long beam of light; but if you stand at some distance, as towards the fire, you will see, on the table, an image of the fire, large and erect: and if another person, who knows nothing of the matter beforehand, should chance to enter the room, he will be startled at the appearance, for the table will seem to be on fire, and, being near the wainscot, to endanger the whole house. For the better deception, there ought to be no light in the room but what proceeds from the fire.

3. If the fire be darkened by a screen, and a large candle be placed at the back of the screen; then a person standing by the candle will see the appearance of a fine large star, or rather planet, on the table, as large as Jupiter or Venus; and if a small wax taper be placed near the candle, it will appear as a satellite to the planet; if the taper be moved round the candle, the satellite will be seen to go round the planet.

4. *The Simple Camera Obscura.*

A camera obscura, very useful to painters, artists, &c, may be easily constructed by means of a single lens only. When landscapes or distant objects are to be represented, a lens from 4 to 12 feet focus may be used, according to the size of the room and the picture. But for representing smaller figures, models, or pictures, a lens from about 9 to 12 inches focus, and about 2 inches and a half in diameter, will be best. It must be fixed in a ball and socket, or, simply in the window-shutter of the darkened room. See fig. 54, pl. 17. A small temporary wooden frame or stage, B, may be attached to the shutter, on which must be steadily fixed the figure or picture C, inverted. To suit lenses of different foci, the stage may be conveniently from 12 to 24 inches in length.

A white paper screen, D, being brought by trial to a

suitable distance within the darkened room, will receive a beautiful representation of the external objects, and from which the artist may readily copy, or trace the various parts of them.

By means of a plane reflecting glass mirror, placed obliquely, the images may be reflected down on a white painted table, or one covered with white paper, within the room: but in this case they will not appear bright or distinct, owing to the light being somewhat diminished by this reflection.

5. *The Dioptrical Paradox*, *or Optical Deception*.

Plate 17, fig. 55, represents the dioptrical paradox. It consists of a mahogany base ABCD, about 8 inches square, with a groove, in which slides various coloured prints, or ornamental drawings: and connected with the base are, a pillar E, a horizontal bar F, with a perspective G, which is placed exactly over the centre of the base, and containing a glass of a particular form. The curious and surprizing effect of this instrument is, that an ace of diamonds, in the centre of one of the drawings, when placed on the base, shall through the perspective G be actually represented as the ace of clubs; a figure of a cat in another, seen as an owl; a letter A, as an O; and a variety of others equally astonishing.

The principle of this machine is very simple, and is as follows. The glass in the tube G, which produces this change, is somewhat on the principle of the common multiplying glass, and is represented at fig. 56. The only difference is, that its sides are flat, and diverging from its hexagonal base upwards, to a point in the axis of the glass, like a pyramid, each side forming an isosceles triangle. Its distance from the eye is to be so adjusted, that each angular side, by its refractive power on the rays of light coming from the border of the print, and such a portion designedly there placed, will refract to the eye the various parts as

one entire figure to be represented; the shape of the glass preventing any appearance of the original figure in the centre, such as the ace of diamonds, being seen: so that the ace of clubs being previously and mechanically drawn on the circle of refraction, at six different parts of the border, 1, 2, 3, 4, 5, 6, fig. 57, and artfully disguised there by blending them with it; then the glass in the tube G will change, in appearance, the ace of diamonds into the ace of clubs. And in like manner for the other prints.

6. *The Optical Paradox.*

Plate 17, fig. 58, is a representation of the double perspective, or optical paradox. One of the perspectives of the instrument being placed before the eye, an object will be seen directly through both. A board A, or any opake object, being interposed, will not make the least obstruction to the rays; and the observer will be surprized that he sees through a perspective having the property of penetrating as it were either solid metal or wood.

The artifice in this instrument consists chiefly in four small plane mirrors, *a*, *b*, *c*, *d*, of which *a* and *d* are placed at an angle of 45 degrees in the two perspectives, and *b* and *c* parallel to them in the trunk below; this being so formed as to appear only as a solid handle to the two perspectives. It is hence obvious that, on the principle of catoptrics, the object T, falling on the first mirror *d*, will be reflected down to *c*, thence to *b*, then up to *a*, and so out to the eye, giving the appearance of the straight lineal direction *ad*.

7. *The Endless Gallery.*

Fig. 59, pl. 17, represents a box, about 18 inches in length, 12 in width, and 9 deep, or any others that are nearly in the same proportions. Against each of its opposite ends A and B within place a plane true-ground glass mirror, as free from veins as possible, of the dimensions nearly equal to the faces, only allowing a small space for a trans-

parent paper, or other cover, at top. From the middle of the mirror C, placed at B, take off neatly a round surface of the silvering, about an inch and half in diameter, against which, in the end of the box, must be cut a hole of the same size, or less. The top of the box should be of glass, covered with gauze, or of oiled transparent paper, to admit as much light as possible into the box. On the two longer sides within must be cut or placed two grooves, at E and F, to receive various drawings or paintings. Indeed many grooves may be cut in the sides, for the reception of a variety of objects. Two paintings or good drawings, of any perspective subject, must be made on the two opposite faces of a pasteboard, as at fig. 60, 61, such as a forest, gardens, colonnades, &c: after having cut the blank parts neatly out, place them in the two grooves, E, F, of the box. Take also two other boards, of the same dimensions, painted on one side only with similar subjects, to be placed at the opposite ends C and D; observing that the one which is to be placed against C should have nothing drawn there to prevent the sight, and that the other, for the opposite end D, should also not be very full of figures, that after being neatly cut out, and placed against the glass, it may cover but a small part of it. The top being then closed over with its transparent cover, the instrument is ready for use.

The effect is very striking and entertaining. The eye, being applied to the hole C, will see the various objects drawn on the scenes reflected in a successive and endless manner, by being reflected alternately from each mirror to that which is opposite. As for instance, if they be trees, they will appear an entire grove, very long, seemingly without end; each mirror repeating the objects more faintly, as the reflections are more numerous, and so contributing still more to the illusion.

Ingenuity will suggest a variety of amusing figures, of men, women, &c, to increase the effect. And two mirrors

may also be placed on the longer sides, to convey an idea of great breadth, as well as length.

8. *The Real Apparition.*

Behind a partition AB, (fig. 62, pl. 18), place somewhat inclined a concave mirror EF, which must be at least 10 inches in diameter, and its distance equal to three-fourths from its centre. In the partition is cut a square or circular opening, of 7 or 8 inches in diameter, directly opposite to the mirror. Behind this a strong light is so disposed as to illuminate strongly an object placed at C, without ...ing on the mirror, and without being seen at the open-... Beneath the aperture, and behind the screen, is ...ced any object at C, which is intended to be represented, but in an inverted position, which may be either a flower, or figure, or picture, &c. Before the partition, and below the aperture, place a flowerpot D, or other pedestal suitable to the object C, so as the top may be even with the bottom of the aperture, and that the eye placed at G may see the flower in the same position as if its stalk came out of the pot. The space between the mirror and the back part of the partition being painted black, to prevent any extraneous light being reflected on the mirror; and indeed the whole disposed so as to be as little enlightened as possible.—Then a person placed at G will perceive the flower, or other object, placed behind the partition, as if standing in the flowerpot, or pedestal: but on putting forth his hand to pluck it, he will find that he grasps only at a phantom.

Fig. 63, pl. 18, represents a different position of the mirror and partition, and better adapted for exhibiting effect by various objects. ABC is a thin partition of a room, down to the floor, with an aperture for a good convex lens turned outwards into the ...om, nearly in a horizontal direction, proper for viewi... ...y the eye of a person standing upright from the floor ... footstool. D is

a large concave mirror, supported at a proper angle, to reflect upwards through the glass in the partition B, images of objects at E, presented towards the mirror below. A strong light from a lamp &c, being directed on the object E, and no where else; then to the eye of a spectator at F, in a darkened room, it is truly surprizing and admirable to what effect the images are reflected up into the air at G.

It is from this arrangement that a showman, both in London and the country, excited the people to the surprize of wonderful apparitions of various kinds of objects, such as a relative's features for his own, paintings of portraits, plaster figures, flowers, fruit, a sword, dagger, death's head, &c.

The phenomena to be produced by concave mirrors are endless; what have been just described will be a sufficient specimen of what might be exhibited to elucidate the principles of that curious machine.

PROBLEM LXIV.

Is it true that Light is reflected with more Vivacity from Air, than from Water?

This assertion is certainly true, provided it be understood in a proper sense, that is as follows: when light tends to pass from air into water, under a certain obliquity, such as 30° for example, the latter reflects fewer rays, than when the light tends, under the same inclination, to pass from water into air. But what is very singular, if the air were entirely removed, so as to leave a perfect vacuum in its stead, the light, so far from passing with more facility through this vacuum, which could oppose no resistance, would experience more difficulty, and more rays would be reflected in the passage.

We do not know why this has been given in the *Philosophical Transactions* as a paradoxical novelty; for this kind of phenomenon is a necessary consequence of the law of refraction. When light indeed passes from a rare

medium into a denser, as from air into water, the passage is always possible; because the sine of the angle of refraction is less than that of the angle of incidence, that is to say, these sines, in the present case, are in the ratio of 3 to 4. But, on the other hand, when light tends to pass obliquely from water into air, the passage under a certain degree of obliquity is impossible, because the sine of the angle of refraction is always much greater than that of the angle of incidence, their sines, in this case, being in the ratio of 4 to 3. There is therefore a certain obliquity of such a nature, that the sine of the angle of refraction would be much greater than the radius; and this will always happen when the sine of the angle of incidence is greater, however small the excess, than $\frac{3}{4}$ of the radius, which corresponds to an angle of 48° 36′. But a sine can never exceed radius, consequently it is impossible, in this case, that the ray of light should penetrate the new medium. Thus while light passes from a rare medium into a denser, from air into water for example, under every degree of inclination, there are some rays, viz, all those which form with the refracting substance an angle less than 41° 24′, that will not admit of the passage of light from water into air: it is then under the necessity of being reflected, and refraction is changed into reflection. But though light may pass from water into air, under greater angles of inclination, this tendency to be reflected, or this difficulty of proceeding from one medium into another, is continued at all these angles, in such a manner, that fewer rays are reflected when they tend to pass from air into water under an angle of 60°, than when they tend to pass from water into air under the same angle. In the last place, when light tends in a perpendicular direction from water into air, it is more reflected than when it tends to pass in the same direction from air into water.

This truth may be proved by a very simple experiment.

Fill a bottle nearly two thirds with quicksilver, and fill up the other third with water; by which means you will have two parallel surfaces, one of water, and the other of quicksilver. If you then place a luminous object at a mean height between these two surfaces, and the eye on the opposite side at the same height as the object, you will see the object through the bottle, and reflected almost with equal vivacity from the surface of the quicksilver, and from that which separates the water and the air. The air then, in this case, reflects the light with almost as much vivacity as the quicksilver.

REMARKS.—1st. We have reason therefore to conclude, that the surface of the water, to beings immersed in that fluid, is a much stronger reflecting mirror, than it is to those beings which are in the air. Fishes see themselves much more distinctly, and clearly, when they swim near the surface of the water, than we see ourselves in the same surface.

2d. Nothing is better calculated than this phenomenon to prove the truth of the reasons assigned by Newton for reflection and refraction. Light passing from a dense fluid into a rarer, is, according to Newton, exactly in the same case as a stone thrown obliquely into the air; if we suppose that the power of gravitation does not act beyond a determinate distance, such for example as 24 feet; for it may be demonstrated that, in this case, the deviation of the stone would be exactly the same, and subject to the same law, as that followed by light in refraction. There would also be certain inclinations under which the stone could not pass from this atmosphere of gravity, though there were nothing beyond it capable of resisting it, and even though there were a perfect vacuum.

In this case however we must not say as a certain celebrated man, when explaining the Newtonian philosophy, that a vacuum reflects light: this is only a mode of speak-

ing. To express our ideas correctly, we ought to say that light is sent back with greater force to the dense medium, as the medium beyond it is rarer.

We are far from being satisfied with what is said on this subject in the *Dictionnaire d'Industrie*, into which one may be surprized to see optical phenomena introduced; for it is there asserted, that this phenomenon depends on the impenetrability of matter, and the high polish of the reflecting surface. But when light is strongly reflected, during its passage from water into a vacuum, or a space almost free from air, where is the impenetrability of the reflecting substance, since such a space has less impenetrability than air or water? In regard to the polish of the reflecting surface, it is the same, both for the ray which passes from air into water, and that which passes from water into air.

PROBLEM LXV.

Account of a Phenomenon, either not observed, or hitherto neglected by Philosophers.

If you hold your finger in a perpendicular direction very near your eye, that is to say, at the distance of a few inches at most, and look at a candle in such a manner, that the edge of your finger shall appear to be very near the flame, you will see the border of the flame coloured red. If you then move the edge of your finger before the flame, so as to suffer only the other border of it to be seen, this border will appear tinged with blue, while the edge of your finger will be coloured red.

If the same experiment be tried with an opake body surrounded by a luminous medium, such for example as the upright bar of a sash window, the colours will appear in a contrary order. When a thread of light only remains between your finger and the bar, the edge of the finger will be tinged red, and the edge next the bar will be bordered with blue; but when you bring the edge of your

finger near the second edge of the bar, so that it shall be entirely concealed, this second edge will be tinged red, and the edge of the finger would doubtless appear to be coloured blue, were it possible that this dark colour could be seen on an obscure and brown ground.

This phenomenon depends no doubt on the different refrangibility of light; but a proper explanation of it has never yet been given.

PROBLEM LXVI.

Of some other Curious Phenomena in regard to Colours and Vision.

I. When the window is strongly illuminated by the light of the day, look at it steadily and with attention for some minutes, or until your eyes become a little fatigued; if you then shut your eyes, you will see in your eye a representation of the squares which you looked at; but the place of the bars will be luminous and white, while that of the panes will be black and obscure. If you then place your hand before your eyes, in such a manner as absolutely to intercept the remainder of the light which the eyelids suffer to pass, the phenomenon will be changed; for the squares will then appear luminous, and the bars black: if you remove your hand, the panes will be black again, and the place of the bars luminous.

II. If you look steadily and with attention for some time at a luminous body, such as the sun, when you direct your sight to other objects in a place very much illuminated, you will observe there a black spot: a little less light will make the spot appear blue, and a degree still less will make it become purple: in a place absolutely dark, this spot, which you have at the bottom of your eye, will become luminous.

III. If you look for a long time, and till you are somewhat fatigued, at a printed book through green glasses; on removing the glasses, the paper of the book will appear

reddish: but if you look at a book in the same manner, through red glasses; when you lay aside the glasses, the paper of the book will appear greenish.

IV. If you look with attention at a bright red spot on a white ground, as a red wafer on a piece of white paper, you will see, after some time, a blue border around the wafer; if you then turn your eye from the wafer to the white paper, you will see a round spot of delicate green, inclining to blue, which will continue longer according to the time you have looked at the red object, and according as its splendour and brightness have been greater. On directing your eyes to other objects, this impression will gradually become weaker, and at length disappear.

If, instead of a red wafer, you look at a yellow one; on turning your eye to the white ground you will observe a blue spot.—A green wafer on a white ground, viewed in the same manner, will produce in the eye a spot of a pale purple colour: a blue wafer will produce a spot of a pale red.

In the last place, if a black wafer on a white ground be viewed in the same manner, after looking at it for some time with attention, you will observe a white border form itself around the wafer; and if you then turn your eye to the white ground, you will observe a spot of a brighter white than the ground, and well defined. When you look at a white spot on a black ground, the case will be reversed.

In these experiments, red is opposed to green, and produces it, as green produces red; blue and yellow are also opposed, and produce each other; and the case is the same with black and white, which evidently indicates a constant effect depending on the organization of the eye.

This is what is called the *Accidental colours*, an object first considered by Dr. Jurin, which Buffon afterwards extended, and respecting which he transmitted a memoir in 1743 to the Royal Academy of Sciences. This celebrated man gave no explanation of these phenomena, and

only observed that, though certain in regard to the correctness of his experiments, the consequences did not appear to be so well established as to admit of his forming an opinion on the production of these colours. There is reason however to believe that he would have explained the cause, had he not been prevented by other occupations. But this deficiency has been supplied by Dr. Godard of Montpellier; for the explanation which he has given of these phenomena, and several others of the same kind, in the *Journal de Physique* for May and July 1776, seems to be perfectly satisfactory.

PROBLEM LXVII.

To determine how long the Sensation of Light remains in the Eye.

The following phenomenon, which depends on this duration, is well known. If a fiery stick be moved round in a circular manner, with a motion sufficiently rapid, you will perceive a circle of fire. It is evident that this appearance arises merely from the vibration impressed on the fibres of the retina not being obliterated, when the image of the fiery end of the stick again passes over the same fibres; and therefore, though it is probable that there is only one point of light on the retina, you every moment receive the same sensation as if the luminous point left a continued trace.

But it has been found by calculating the velocity of a luminous body put in motion, that when it makes its revolution in more than 8 thirds, the string of fire is interrupted; and hence there is reason to conclude, that the impression made on the fibre continues during that interval of time. But it may be asked, whether this time is the same for every kind of light, whatever be its intensity? We do not think it is; for a brighter light must excite a livelier and more durable impression.

SUPPLEMENT,

Containing a short Account of the Most Curious Microscopical Observations.

PHILOSOPHERS were no sooner in possession of the microscope, than they began to employ this wonderful instrument in examining the structure of bodies, which, in consequence of their minuteness, had before eluded their observation. There is scarcely an object in nature to which the microscope has not been applied; and several have exhibited such a spectacle as no one could have ever imagined. What indeed could be more unexpected than the animals or moleculæ (for philosophers are not yet agreed in regard to their animality) which are seen swimming in vinegar, in the infusions of plants, and in the semen of animals? What can be more curious than the mechanism in the organs of the greater part of insects, and particularly those which in general escape our notice; such as the eyes, trunks, feelers, terebræ or augres, &c.? What more worthy of admiration than the composition of the blood, the elements of which we are enabled to perceive by means of the microscope; the texture of the epidermis, the structure of the lichen, that of mouldiness, &c? We shall here take a view of the principal of these phenomena, and give a short account of the most curious observations of this kind.

§ I.

Of the Animals, or Pretended Animals, in Vinegar, and the infusions of plants.

1st. Leave vinegar exposed for some days to the air, and then place a drop of it on the transparent object-plate of the microscope, whether single or compound: if the object-plate be illuminated from below, you will observe in this drop of liquor, animals resembling small eels, which are in continual motion. On account of the circumvolu-

tions which they make with their long, slender bodies, they may be justly compared to small serpents.

But it would be wrong, as many simple people have done, to ascribe the acidity of vinegar to the action of these animalcules, whether real or supposed, on the tongue and the organs of taste; for vinegar deprived of them is equally acid, if not more so. These eels indeed, or serpents, are never seen but in vinegar which, having been for some time exposed to the air, is beginning to pass from acidity to putrefaction.

2d. If you infuse pepper, slightly bruised, in pure water for some days, and then expose a drop of it to the microscope, you will behold small animals of another kind, almost without number. They are of a moderately oblong, elliptical form, and are seen in continual motion, going backwards and forwards in all directions; turning aside when they meet each other, or when their passage is stopped by any immoveable mass. Some of them are observed sometimes to lengthen themselves, in order to pass through a narrow space. Certain authors of a lively imagination, it would appear, even pretend to have seen them copulate, and bring forth; but this assertion we are not bound to believe.

If other vegetable bodies be infused in water, you will see animalcules of a different shape. In certain infusions they are of an oval form, with a small bill, and a long tail: in others they have a lengthened shape like lizards: in some they exhibit the appearance of certain caterpillars, or worms, armed with long bristles; and some devour, or seem to devour their companions.

When the drop in which they swim about, and which to them is like a capacious bason, becomes diminished by the effect of evaporation, they gradually retire towards the middle, where they accumulate themselves, and at length perish when entirely deprived of moisture. They then appear to be in great distress; writhe their bodies, and

endeavour to escape from death, or that state of uneasiness which they experience. In general, they a have strong aversion to saline or acid liquors. If a small quantity of vitriolic acid be put into a drop of infusion which swarms with these insects, they immediately throw themselves on their backs and expire; sometimes losing their skin, which bursts, and suffers to escape a quantity of small globules that may be often seen through their transparent skin. The case is the same if a little urine be thrown into the infusion.

A question here naturally arises: ought these moveable moleculæ to be considered as animals? On this subject opinions are divided. Buffon thinks they are not animals; and consigns them, as well as spermatic animals, to the class of certain bodies which he calls *organic moleculæ.* But what is meant by the expression organic moleculæ? As this question would require too long discussion, we must refer the reader to the Natural History of that learned and celebrated writer.

Needham also contests the animality of these small bodies, that is to say perfect animality, which consists in feeding, increasing in size, multiplying, and being endowed with spontaneous motion; but he allows them a sort of obscure vitality, and from all his observations he deduces consequences on which he has founded a very singular system. He is of opinion that vegetable matter tends to animalise itself. As the eels produced in flour paste act a conspicuous part in the system of this naturalist, a celebrated writer has omitted no opportunity of ridiculing his ideas, by calling these animals the eels of the jesuit Needham, and representing him as a partisan of spontaneous generation, which has been justly exploded by all the modern philosophers. But ridicule is not reasoning: we are so little acquainted with the boundaries between the vegetable and animal kingdoms, that it would be presuming too much to fix them. But it must be allowed that

Needham's ideas on this subject are so obscure, that in our opinion few have been able to comprehend them.

Other naturalists and observers assert the animality of these small beings: for they ask, by what can an animal be better characterized than spontaneity of motion? But these moleculæ, when they meet each other in the course of their movements, retire backwards, not by the effect of a shock as two elastic bodies would do, but the part which is generally foremost turns aside on the approach of the body that meets it: and sometimes both move a little from their direction, in order to avoid running against each other. They have never yet indeed been seen for certain to copulate, to produce eggs, or even to feed; but the last mentioned function they may perform without any apparent act like the greater part of the other animalcules. The smallness and strange form of these moleculæ can afford no argument against their animality. That of the water polypes is at present no longer doubted, though their form is very extraordinary, and perhaps more so than that of the moving moleculæ of infusions. Why then should animality be refused to the latter?

It might however be replied, in opposition to this supposed similarity, that the polype is seen to increase in size, to regenerate itself, in a way indeed very different from that of the generality of animals, and in particular to feed. The pretended microscopic animals do nothing of the kind, and consequently ought not to be ranked in the same class. But it must be allowed that this subject is still involved in very great obscurity; and therefore prudence requires that we should suspend our opinion respecting it.

§ II.

Of Spermatic Animals.

Of the microscopic discoveries of the last century, none has made a greater noise than that of the moving moleculæ observed in the semen of animals, and which are called

Spermatic animalcules. This singular discovery was first made and announced by the celebrated Lewenhoek, who observed in the human semen a multitude of small bodies, most of them with very long slender tails, and in continual motion. In size they were much less than the smallest grain of sand, and even so minute in some seminal liquors that a hundred thousand, and even a million of them were not equal to a poppy seed. By another calculation Lewenhoek has shown, that in the milt of a cod-fish, there are more animals of this species, than human beings on the whole surface of the earth.

Lewenhoek examined also the prolific liquor of a great many animals, both quadrupeds and birds, and that even of some insects. In all these he observed nearly the same phenomenon; and these researches since repeated by many other observers, have given rise to a system in regard to generation, which it is unnecessary here to explain.

No one however has made more careful, or more correct observations on this subject, than Buffon; and for this reason we shall give a short view of them.

This celebrated naturalist, having procured a considerable quantity of semen, extracted from the seminal vessels of a man who had perished by a violent death, observed in it, when viewed through an excellent microscope, longish filaments, which had a kind of vibratory motion, and which appeared to contain in the inside small bodies. The semen having assumed a little more fluidity, he saw these filaments swell up in some points, and oblong elliptical bodies issue from them; a part of which remained at first attached to the filaments by a very slender long tail. Some time after, when the semen had acquired a still greater degree of fluidity, the filaments disappeared, and nothing remained in the liquor but these oval bodies with tails, by the extremity of which they seemed attached to the fluid, and on which they balanced themselves like a pendulum, having however a progressive motion, though

slow, and as it were embarrassed by the adhesion of their tails to the fluid; they exhibited also a sort of heaving motion, which seems to prove that they had not a flat base, but that their transverse section was nearly round. In about twelve or fifteen hours after, the liquor having acquired a still greater degree of fluidity, the small moving moleculæ had lost their tails, and appeared as elliptic bodies, moving with great vivacity. In short, as the matter became attenuated in a greater degree, they divided themselves more and more so as at length to disappear, or they were precipitated to the bottom of the liquor, and seemed to lose their vitality.

Buffon, while viewing these moving moleculæ, once happened to see them file off like a regiment, seven by seven, or eight by eight, proceeding always in very close bodies towards the same side. Having endeavoured to discover the cause of this appearance, he found that they all proceeded from a mass of filaments accumulated in one corner of the spermatic drop, and which resolved itself successively in this manner into small enlongated globules, all without tails. This circumstance reminds us of the singular idea of a naturalist, who observing a similar phenomenon in the semen of a ram, thought he could there see the reason of the peculiar propensity which sheep have to follow each other, when they march together in a flock.

Buffon examined, in like manner, the spermatic liquor of various other animals, such as the bull, the ram, &c, and always discovered the same moleculæ, which at first had tails, and then gradually lost them as the liquor assumed more fluidity. Sometimes they seemed to have no tails, even on their first appearance and formation. In this respect, Buffon's observations differ from those of Lewenhoek, who always describes these animalcules as having tails, with which he says they seem to assist themselves in their movements; and he adds, that they are seen to twist themselves in different directions. Buffon's ob-

servations differ also from those of the Dutch naturalist in another respect, as the latter says that he never could discover any trace of these animalcules in the semen or liquor extracted from the ovaria of females; whereas Buffon saw the same moving moleculæ in that liquor, but not so often, and only under certain circumstances.

It appears from what has been said, that many researches still remain to be made in regard to the nature of these moving moleculæ; since two observers so celebrated do not agree in all the circumstances of the same fact.

Nothing of this kind is observed in the other animal fluids, such as the blood, lymph, milk, saliva, urine, gall, and chyle; which seems to indicate that these animalcules, or living moleculæ, act a part in generation.

§ III.

Of the Animals or Moving Moleculæ in Spoilt Corn.

This is another microscopic observation, which may justly be considered as one of the most singular; for if we deduce from it all those consequences which some authors do, it exhibits an instance of a resurrection, repeated, as we may say, at pleasure.

The disease of corn which produces this phenomenon is neither smut nor blight, as some authors for want of a sufficient knowledge in regard to the specific differences of the maladies of grain, have asserted, but what ought properly to be called *abortion* or *rachitis*. If a grain of corn, in this state, be opened with caution, it will be found filled with a white substance, which readily divides itself into a multitude of small, white elongated bodies, like small eels, swelled up in the middle. While these moleculæ, for we must be allowed as yet to remain neuter in regard to their pretended animality, are in this state of dryness, they exhibit no signs of life; but if moistened with very pure water, they immediately put themselves in motion, and show every mark of animality. If the fluid drop, in which

they are placed, be suffered to dry, they lose their motion; but it may be restored to them at pleasure, even some months after their apparent death, by immersing them in water. Fontana, an Italian naturalist, does not hesitate to consider this phenomenon as a real resurrection. If this circumstance should be verified by repeated observations, and that also of the Peruvian serpent, which may be restored to life by plunging it in the mud, its natural element, several months after it has been suffered to dry at the end of a rope, our ideas respecting animality may be strangely changed. But we must confess that we give very little credit to the latter fact; though Bouguer, who relates it on the authority of Father Gumilla a Jesuit and a French Surgeon, does not entirely disbelieve it. Some other observers, such as Roffredi, pretend to have distinguished, in these eel-formed moleculæ, the aperture of the mouth; that of the female parts of sex, &c. They assert also that they have perceived the motion of the young ones contained in the belly of the mother eel, and that having opened the body, the young were seen to disperse themselves all over the object-plate of the microscope. These observations deserve to be further examined, as a confirmation of them would throw great light on animality.

§ IV.

On the Movements of the Tremella.

The tremella is that gelatinous, green plant, which forms itself in stagnant water, and which is known to naturalists by the name of, *conferva gelatinosa omnium tenerrima et minima, aquarum limo innascens.* It consists of a number of filaments interwoven through each other, which when considered singly are composed of small parts, about a line in length, united by articulations.

This natural production, when viewed with the naked eye, exhibits nothing remarkable or uncommon; but by

means of microscopic observations, two very extraordinary properties have been discovered in it. One is, the spontaneous motion with which these filaments are endowed. If a single one sufficiently moistened, be placed on the object-plate of the microscope, its extremities are seen to rise and fall alternately, and to move sometimes to the right and sometimes to the left: at the same time, it twists itself in various directions, and without receiving any external impression. Sometimes, instead of appearing extended like a straight line, it forms itself into an oval or irregular curve. If two of them are placed side by side, they become twisted and twined together, and by a sort of imperceptible motion, the one from one side, and the other from the other. This motion has been estimated by Adanson to be about the 400th part of a line, per minute.

The other property of this plant is, that it dies and revives, as we may say, several times; for if several filaments, or a mass of tremella, be dried, it entirely loses the faculty above mentioned. It will remain several months in that state of death or sleep; but when immersed in the necessary moisture, it revives, recovers its power of motion, and multiplies as usual.

The abbé Fontana, a celebrated observer of Parma, does not hesitate, in consequence of these facts, to class the tremella among the number of the Zoophytes; and to consider it as the link which connects the vegetable with the animal kingdom, or the animal with the vegetable; in a word as an animal or a vegetable endowed with the singular property of being able to die and to revive alternately. But is this a real death, or only a kind of sleep, a suspension of all the faculties in which the life of the plant consists? To answer this question it would be necessary to know exactly what is the nature of death; a great deal might be said on this subject, were not such disquisitions foreign to the present work.

§ V.

Of the Circulation of the Blood.

Those who are desirous to observe the circulation of the blood by means of the microscope, may easily obtain that satisfaction. The objects employed chiefly for this purpose, are the delicate, transparent member which unites the toes of the frog, and the tail of the tadpole. If this membrane be extended, and fixed on a piece of glass illuminated below, you will observe with great satisfaction the motion of the blood in the vessels with which it is interspersed: you will imagine that you see an archipelago of islands with a rapid current flowing between them.

Take a tadpole, and having wrapped up its body in a piece of thin, moist cloth, place its tail on the object-plate of the microscope, and enlighten it below: you will then see very distinctly the circulation of the blood; which in certain vessels proceeds by a kind of undulations, and in others with a uniform motion. The former are the arteries, in which the blood moves in consequence of the alternate pulsation of the heart; the latter are the veins.

The circulation of the blood may be seen also in the legs and tails of shrimps, by putting these fish into water with a little salt, but their blood is not red. The wings of the locust are also proper for this purpose: in these the observer will see, not without satisfaction, the green globules of their blood carried away by the serosity in which they float. The transparent legs of small spiders, and those of small bugs, will also afford the means of observing the circulation of their blood. The latter exhibit an extraordinary vibration of the vessels, which Mr. Baker says he never saw any where else.

But the most curious of all the spectacles of this kind, is that exhibited by the mesentery of a living frog, applied in particular to the solar microscope, which Mr. Baker tells us he did in company with Dr. Alexander Stuard,

physician to the queen. It is impossible to express, says he, the wonderful scene which presented itself to our eyes. We saw at the same moment the blood, which flowed in a prodigious number of vessels, moving in some to one side, and in others to the opposite side. Several of these vessels were magnified to the size of an inch diameter; and the globules of blood seemed almost as large as grains of pepper, while in some of the vessels, which were much smaller, they could pass only one by one, and were obliged to change their figure into that of an oblong spheroid.

§ VI.

Composition of the Blood.

With the end of a quill, or a very soft brush, take up a small drop of blood just drawn from a vein, and spread it as thin as possible over a bit of talc; if you then apply to your microscope one of the strongest magnifiers, you will distinctly see its globules.

By these means it has been found, that the red globules of the human blood are each composed of six smaller globules, united together; and that when disunited by any cause whatever, they are no longer of a red colour. These red globules are so exceedingly minute, that their diameter is only the 160th part of a line, so that a sphere of a line in diameter would contain 4096000 of them.

§ VII.

Of the Skin; its Pores, and Scales.

If you cut off a small bit of the epidermis by means of a very sharp razor, and place it on the object-plate of the microscope; you will see it covered with a multitude of small scales, so exceedingly minute, that, according to Lewenhoek, a grain of sand would cover two hundred of them: that is to say, in the diameter of a grain of sand there are 14 or 15. These scales are arranged like those

on the back of fishes, or like the tiles of a house; that is, each covering the other.

If you are desirous of viewing their form with more convenience, scrape the epidermis with a penknife, and put the dust obtained by these means into a drop of water: you will then observe that these scales, in general, have five planes, and that each consists of several strata.

Below these scales are the pores of the epidermis, which when the former are removed may be distinctly perceived, like small holes pierced with an exceedingly fine needle. Lewenhoek counted 120 in the length of a line; so that a line square, 10 of which form an inch, would contain 14400; consequently a square foot would contain 144000000, and as the surface of the human body may be estimated at 14 square feet, it must contain 2016 millions.

Each of these pores corresponds in the skin to an excretory tube, the edge of which is lined with the epidermis. When the epidermis has been detached from the skin, these internal prolongations of the epidermis may be observed in the same manner as we see in the reverse of a piece of paper, pierced with a blunt needle, the rough edge formed by the surface, which has been torn and turned inwards.

The pores of the skin are more particularly remarkable in the hands and the feet.* If you wash your hands well with soap, and look at the palm with a common magnifier, you will see a multitude of furrows, between which the pores are situated. If the body be in a state of perspiration at the time, you will see issuing from these pores a small drop of liquor, which gives to each the appearance of a fountain.

§ VIII.

Of the Hair of Animals.

The hairs of animals, seen through the microscope, ap-

pear to be organized bodies, like the other parts; and, by the variety of their texture and conformation, they afford much subject of agreeable observation. In general, they appear to be composed of long, slender, hollow tubes, or of several small hairs covered with a common bark; others, such as those of the Indian deer, are hollow quite through. The bristles of a cat's whiskers, when cut transversely, exhibit the appearance of a medullary part, which occupies the middle, like the pith in a twig of the elder-tree. Those of the hedge-hog contain a real marrow, which is whitish, and formed of radii.

As yet however we are not perfectly certain in regard to the organization of the human hair. Some observers, seeing a white line in the middle, have concluded that it is a vessel which conveys the nutritive juice to the extemity. Others contest this observation, and maintain that it is merely an optical illusion, produced by the convexity of the hair. It appears however that some vessel must be extended lengthwise in the hair, if it be true that blood has been seen to issue at the extremity of the hair cut from persons attacked with that disease called the *Plica Polonica*. But quere, is this observation certain?

§ IX.

Singularities in regard to the Eyes of most Insects.

The greater part of insects have not moveable eyes, which they can cover with eye-lids at pleasure, like other animals. These organs, in the former, are absolutely immoveable; and as they are deprived of that useful covering assigned to others for defending them, nature has supplied this deficiency by forming them of a kind of corneous substance, proper for resisting the shocks to which they might be exposed.

But it is not in this that the great singularity of the eyes of insects consists. We discover by the microscope that these eyes are themselves divided into a prodigious multi-

tude of others much smaller. If we take a common fly, for example, and examine its eyes by the microscope, we shall find that it has on each side of its head a large excrescence, like a flattened hemisphere. This may be perceived without a microscope; but by means of this instrument these hemispheric excrescences will be seen divided into a great number of rhomboids, having in the middle a lenticular convexity, which performs the part of the crystalline humour. Hodierna counted more than 3000 of these rhomboids on one of the eyes of a common fly; M. Puget reckoned 8000 on each eye of another kind of fly, so that there are some of these insects which have 16000 eyes; and there are some which even have a much greater number, for Lewenhoek counted 14000 on each eye of another insect.

These eyes however are not all disposed in the same manner: the dragon-fly, for example, besides the two hemispherical excrescences on the sides, has between these two other eminences, the upper and convex surface of which is furnished with a multitude of eyes, directed towards the heavens. The same insect has three also in front, in the form of an obtuse and rounded cone. The case is the same with the fly, but its eyes are less elevated.

It is an agreeable spectacle, says Lewenhoek, to consider this multitude of eyes in insects; for if the observer is placed in a certain manner, the neighbouring objects appear painted on these spherical eminences of a diameter exceedingly small, and by means of the microscope they are seen multiplied, almost as many times as there are eyes, and in such a distinct manner as never can be attained to by art.

A great many more observations might be made in regard to the organs of insects, and their wonderful variety and conformation, but these we shall reserve for another place.

§ X.

Of the Mites in Cheese, and other Insects of the same kind.

If you place on the object-plate of the microscope some of the dust which is formed on the rind and other neighbouring parts of old cheese, it will be seen to swarm with a multitude of small transparent animals, of an oval figure, terminating in a point, and in the form of a snout. These insects are furnished with eight scaly, articulated legs, by means of which they move themselves heavily along, rolling from one side to the other; their head is terminated by an obtuse body in the form of a truncated cone, where the organ through which they feed is apparently situated. Their bodies, particularly the lateral parts, are covered with several long sharp-pointed hairs, and the anus bordered with hair, as seen in the lower part of the belly.

There are mites of another kind which have only six legs, and which consequently are of a different species.

Others are of a vagabond nature, as the observer calls them, and are found in all places where there are matters proper for their nourishment.

This animal is extremely vivacious, for Lewenhoek says that some of them, which he had attached to a pin before his microscope, lived in that manner eleven weeks.

§ XI.

Of the Louse and Flea.

Both these animals are exceedingly disagreeable, particularly the latter, and do not seem proper for being the subject of microscopic observation; but to the philosopher no object in nature is disagreeable, because deformity is merely relative, and the most hideous animal often exhibits singularities, which serve to make us better acquainted with the infinite variety of the works of the Creator.

If you make a louse fast for a couple of days, and then

place it on your hand, you will see it soon attach itself to it, and plunge its trunk into the skin. If viewed in this state by means of a microscope, you will see, through its skin, your blood flowing under the form of a small stream, into its ventricle, or the vessel that supplies its place, and thence distributing itself to the other parts, which will become distended by it.

This animal is one of the most hideous in nature: its head is triangular, and terminates in a sharp point, to which is united its proboscis or sucker. On each side of the head, and at a small distance from its anterior point, are placed two large antennæ, covered with hair; and behind these, towards the two other obtuse angles of the triangle, are the animal's two eyes. The head is united by a short neck to the corslet, which has six legs furnished with hair at the articulations, and with two hooks each at the extremity. The lower part of the belly is almost transparent, and on the sides has a kind of tubercles, the last of which are furnished with two hooks. Dr. Hook, in his *Micrographia*, has given the figure of one of these animals, about half a foot in length. Those who see the representation of this insect will not be surprised at the itching on the skin, which it occasions to persons, who in consequence of dirtiness are infested with it.

The flea has a great resemblance to the shrimp, as its back is arched in the same manner as the back of that animal. It is covered as it were with a coat of mail, consisting of large scales laid over each other; the hind part is round, and very large in regard to the rest of the body; its head is covered by a single scale, and at the extremity has a kind of three terebræ, by means of which the insect sucks the blood of animals. Six legs, with thighs exceedingly thick, and of which the first pair are remarkably long, enable it to perform all its movements. The great size of the thighs is destined, no doubt, to contain the powerful muscles which are necessary to carry the insect to a height

or distance equal to several hundred times its length. Being destined to make such large leaps, it was also necessary that it should be strongly secured against falls to which it might be exposed, and nature has made ample provision against accidents of this kind, by supplying it with scaly armour. Figures of the flea and louse, highly magnified, will be found in the works of Hook and Joblot.

§ XII.

Mouldiness.

Nothing can be more curious than the appearance exhibited by mouldiness, when viewed through the microscope. When seen by the naked eye, one is almost induced to consider it as an irregular tissue of filaments; but the microscope shows that it is nothing else than a small forest of plants, which derive their nourishment from the moist substance, tending towards decomposition, which serves them as a base. The stems of these plants may be plainly distinguished; and sometimes their buds, some shut and others open. Baron de Munchausen has even done more: when carefully examining these small plants, he observed that they had a great similarity to mushrooms. They are nothing, therefore, but microscopic mushrooms, the tops of which, when they come to maturity, emit an exceedingly fine kind of dust, which is their seed. It is well known that mushrooms spring up in the course of one night; but those of which we here speak, being more rapid, almost in the inverse ratio of their size, grow up in a few hours. Hence the extraordinary progress which mouldiness makes in a very short time.

Another very curious observation of the same kind, made by M. Ahlefeld of Giessen, is as follows: Having seen some stones covered with a sort of dust, he had the curiosity to examine it with a microscope, and found to his great astonishment, that it consisted of small microscopic mushrooms, raised on very short pedicles, the heads of

which, round in the middle, were turned up at the edges: they were striated also from the centre to the circumference, as certain kinds of mushrooms are. He remarked likewise, that they contained, above their upper covering, a multitude of small grains, shaped like cherries, somewhat flattened; which in all probability were the seeds. In the last place, he observed, in this forest of mushrooms, several small red insects, which no doubt fed upon them. See *Act. Leips.* for the year 1739.

§ XIII.

Dust of the Lycoperdon.

The lycoperdon, or puff-ball, is a plant of the fungus kind, which grows in the form of a tubercle, covered with small grains like shagreen. If pressed with the foot, it bursts, and emits an exceedingly fine kind of dust, which flies off under the appearance of smoke; but commonly a pretty large quantity remains in the half opened cavity of the plant. If some of this dust be placed on the object-plate of the microscope, it appears to consist of perfectly round globules, of an orange colour, the diameter of which is only about the 50th part of a hair, so that each grain of this dust is but the 125000th part of a globule equal in diameter to the breadth of a hair. Some lycoperdons contain browner spherules, attached to a small pedicle. This dust no doubt is the seed of this anomalous plant.

§ XIV.

Of the Farina of Flowers.

It is not long since the utility of this farina in the vegetable economy was known. Before this discovery, it was thought to be nothing else than the excrement of the juices of the flower; but it is shown by the microscope that this dust is regularly and uniformly organised in each kind of plant. In the mallow, for example, each grain is an opake ball, entirely covered with points. The farina

of the tulip, and of most of the lily kind of flowers, has a resemblance to the seeds of cucumbers and melons. That of the poppy resembles a grain of barley, with a longitudinal groove in it.

But we are taught by observation still more; for it is found that this dust or farina is only a capsule, which contains another far more minute; and it is the latter which is the real fecundating dust of plants.

§ XV.

Of the Apparent holes in the Leaves of some Plants.

There are certain plants the leaves of which appear to be pierced with a multitude of small holes. Of this kind, in particular, is that called by botanists *hypericum*, and by the vulgar St. John's wort. But if a fragment of one of these leaves be viewed through a microscope, the supposed holes are found to be vesicles, contained in the thickness of the leaf, and covered with an exceedingly thin membrane; in fact, they are the receptacles which contain the essential and aromatic oil peculiar to that plant.

§ XVI.

Of the Down of Plants.

The spectacle exhibited by those plants which have down, such as borage, nettles, &c, is exceedingly curious. When viewed through the microscope, they appear to be so covered with spikes as to excite horror. Those of borage are for the most part bent so as to form an elbow; and, though really very close, they appear by the microscope to be at a considerable distance from each other. Persons who are not previously told what substance they are looking at, will almost be induced to believe that they see the skin of a porcupine.

§ XVII.

Of the Spark struck from a piece of Steel by means of a Flint.

If sparks struck from a piece of steel by a flint be made to fall on a leaf of paper, they will be found, for the most part, to be globules, formed of small particles of steel, detached by the shock, and fused by the friction. Dr. Hook observed some which were perfectly smooth, and reflected with vivacity the image of a neighbouring window. When in this state, they are susceptible of being attracted by the magnet; but very often they are reduced by the fusion to a kind of scoria, and in that case the magnet has no power over them. The cause of this we shall explain hereafter. This fusion will excite no surprise when it is known that the bodies most difficult to be liquefied need only, for that purpose, to be reduced to very minute particles.

§ XVIII.

Of the Asperities of certain bodies, which appear to be exceedingly Sharp and highly Polished.

If a needle, apparently very sharp, be viewed through the microscope, it will seem to have a very blunt, irregular point, much resembling that of a peg broken at the end.

The case is the same with the edge of the best set razor. When viewed through the microscope, it will appear like the back of a penknife, and at certain distances exhibit indentations like the teeth of a saw, but irregular.

If a piece of the highest polished glass be exposed to the microscope, you will be much astonished at its appearance. It will be seen furrowed, and filled with asperities, which reflect the light in an irregular manner, making it assume different colours. The case is the same with the best polished steel.

Art, in this respect, is far inferior to nature; for if

works which have been made and polished, as we may say, by the latter, are exposed to the microscope, instead of losing their polish, they appear with greater lustre. When the eyes of a fly, if illuminated by means of a lamp or taper, are viewed through this instrument, each of them exhibits an image of the taper with a precision and vivacity which nothing can equal.

§ XIX

Of Sand seen through the Microscope.

It is well known that there are some kinds of sand calcareous, and others vitrifiable. The former, seen through the microscope, resemble in a great measure large irregular fragments of rock. The most curious spectacle however is exhibited by the vitreous kind; when it consists of rolled sand, it appears like so many rough diamonds, and sometimes like polished ones. One kind of sand, when seen through the microscope, appears to be an assemblage of diamonds, rubies and emeralds; another presents the embryos of shells exceedingly small.

[illegible] XX

Of the Pores of C[illegible]coal.

Dr. Hook had the curiosity to examine with a microscope [illegible] of charcoal, which he found to be filled with pores [illegible] arranged, and passing through its whole length [illegible] it appears that [illegible] is no charcoal into which [illegible] does not introduce itself. This observer, in the [illegible] part of an inch, counted 150 of these pores; from which it follows, that in a piece of charcoal, an inch in diameter, there are about [illegible]7,2000.

On this subject we have been obliged, agreeably to our plan, to [illegible]ngly brief; but, to supply this deficiency, we shall here point out the principal works which contain micrographic observations, and the authors who

have particularly applied to this kind of study. The first we shall mention is Father Bonnani, a Jesuit, author of a book entitled *Ricreazione dell'ochio è della mente*, part of which is entirely devoted to this subject. The celebrated Lewenhoek spent almost the whole of his life in the same occupation, and published the observations he made in his *Arcana Naturæ*. A great many observations of this kind may be found scattered here and there throughout all the Journals and Memoirs of learned Societies. But few have made so many researches on this subject as M. Joblot, author of a quarto volume, entitled *Description et usages de plusieurs nouveaux Microscopes, &c, avec de nouvelles observations sur un multitude innombrable d'insectes, &c, qui naissent dans les liqueurs*, &c: Paris 1716. He infused in water a great number of different substances, and caused the small animals produced in these infusions to be engraved: to the greater part of them he has even given names, derived from their resemblance to known bodies, or from other circumstances. But we must refer the reader to the work itself, which was republished in 1754, considerably enlarged, under this title: *Observations d'Histoire Naturelle, faites avec le Microscope sur un grand nombre d'Insectes, et sur les Animalcules qui se trouvent dans les liqueurs preparées et non preparées, &c*, 4to, with a great number of plates. Needham, in the year 1750, published his work, called New Microscopical observations. Buffon's observations on spermatic molecule may be seen in his work on Natural History. We have also Baker's works, entitled the Microscope made easy, and Employment for the Microscope. The first part contains a description of the apparatus and the method of using different kinds of microscopes, and the second a very long detail of microscopical observations made on various natural objects. This work was attended with great success, and is exceedingly instructive. The abbé Spallanzani caused his microscopical observations, in which he several

times contradicts Needham, to be printed in Italian; a French translation, entitled *Nouvelles Observations Microscopiques*, was published in octavo at Paris, in 1769, with notes by the above philosopher. If to these be added various Memoirs, by Fontana, Roffredi, Spallanzani, &c, published in the *Journal de Physique*, we shall have enumerated all the writings, or at least the principal ones, which have hitherto appeared on this subject.

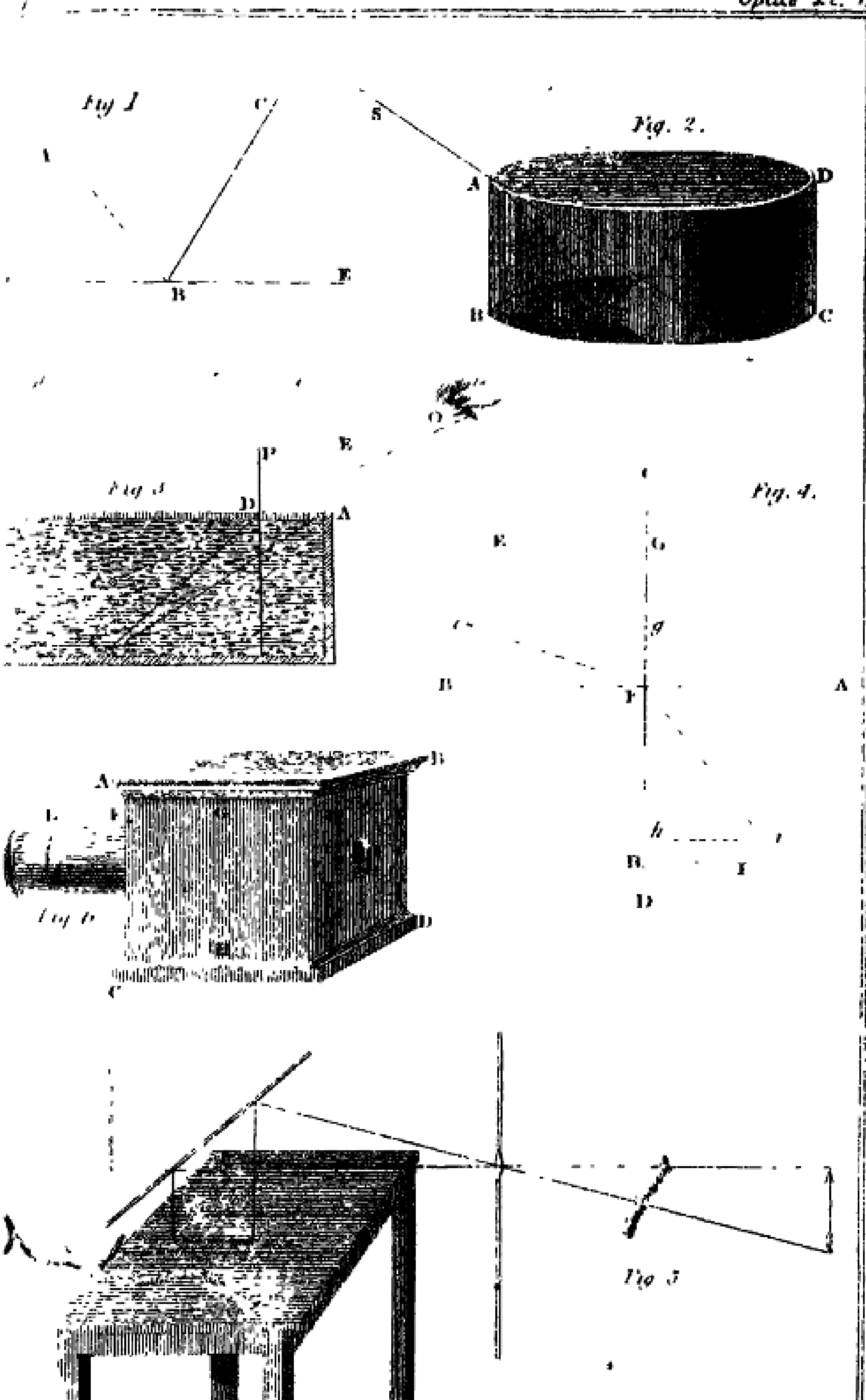


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Fig. 8.

Fig. 7.

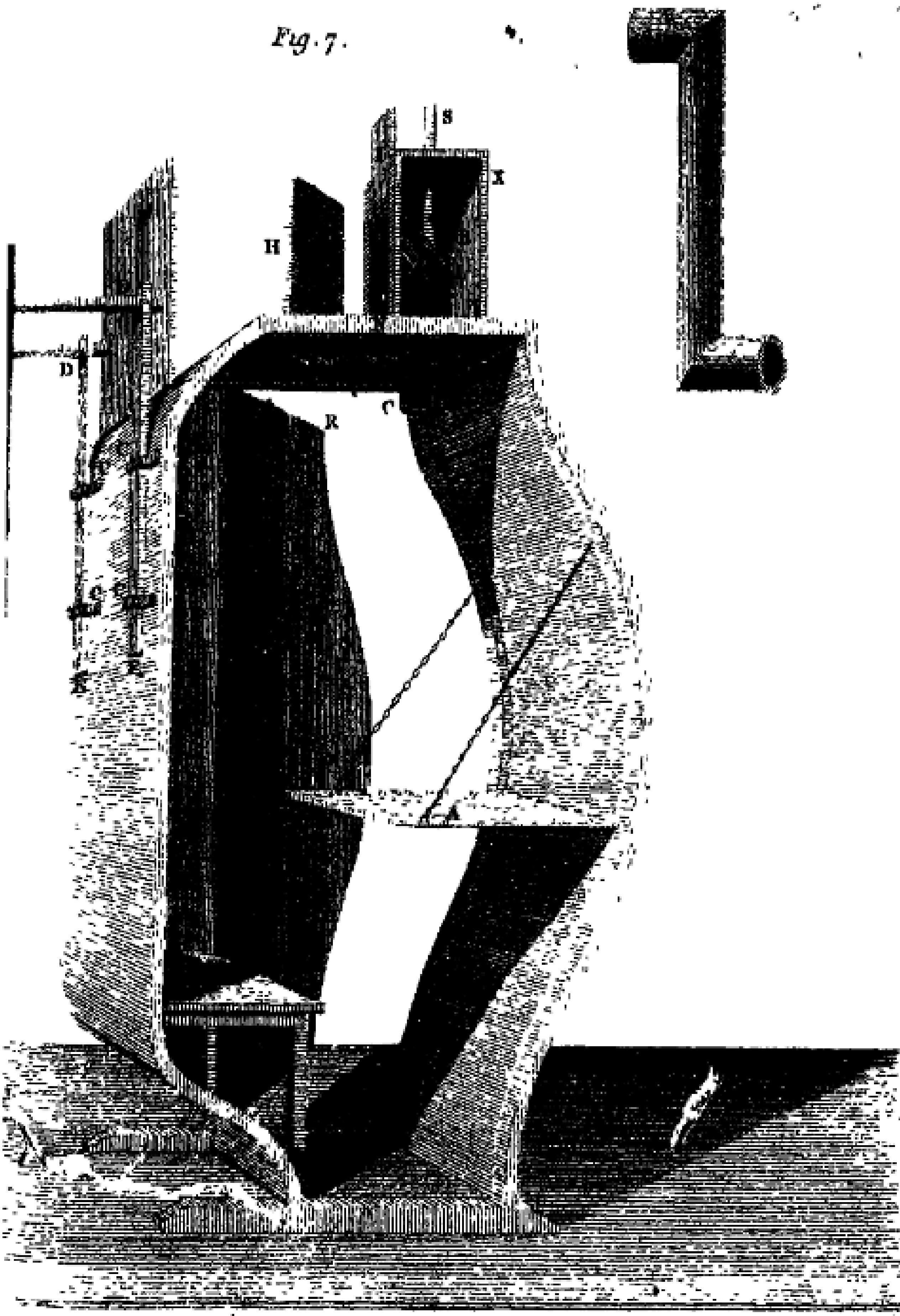
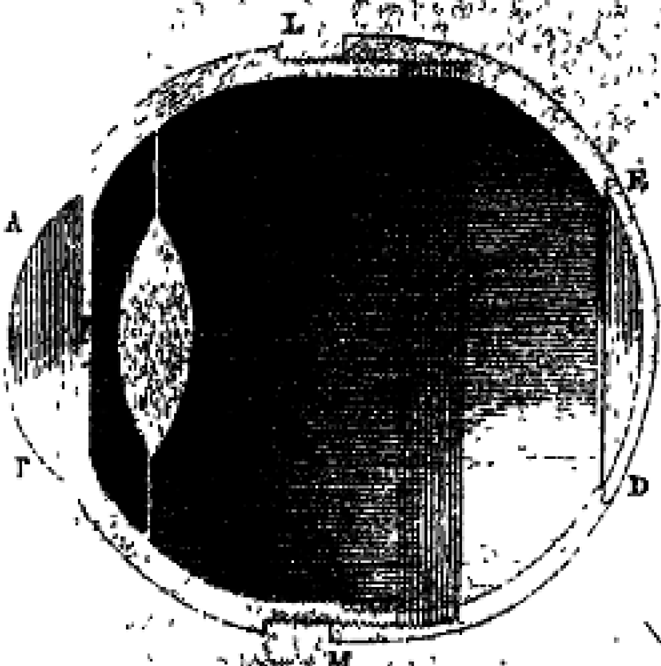


Fig. 9.

Fig. 10.



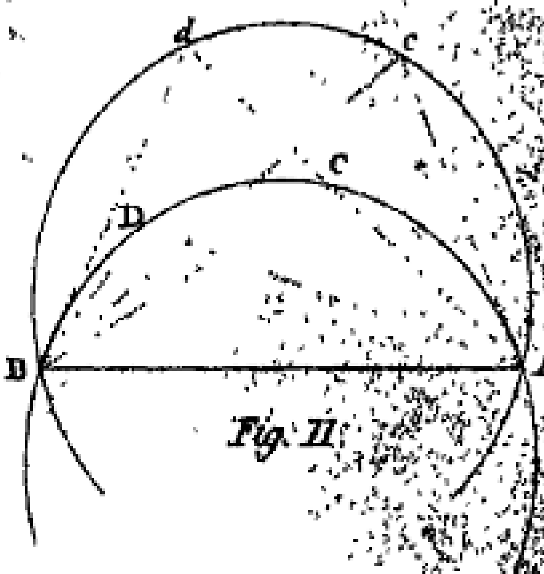
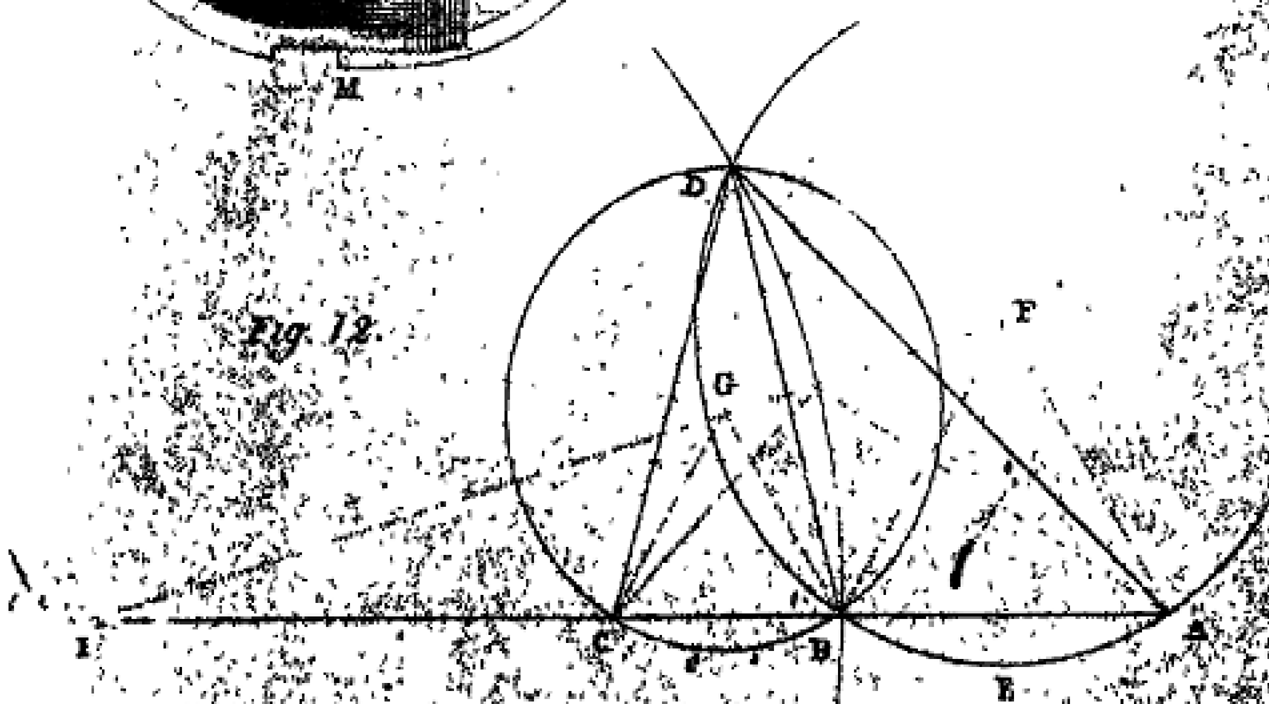


Fig. 11.

Fig. 12.



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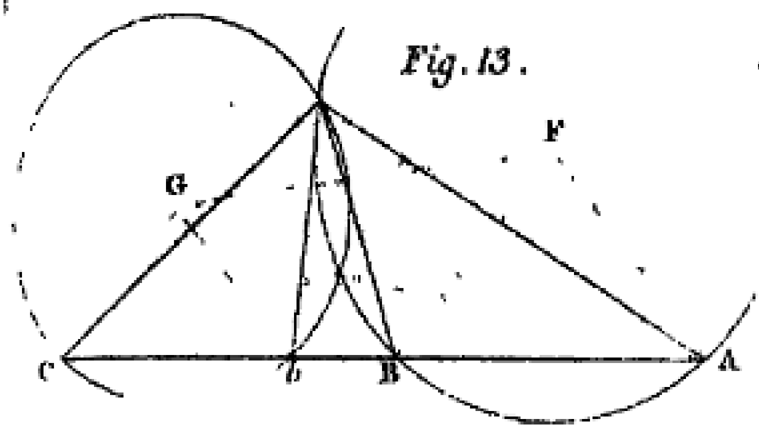

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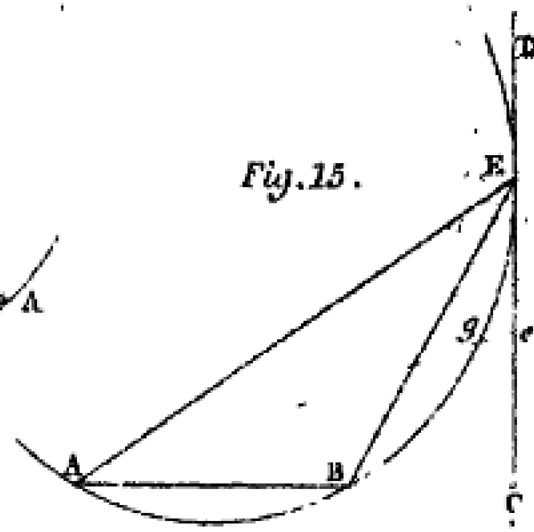


Fig 14

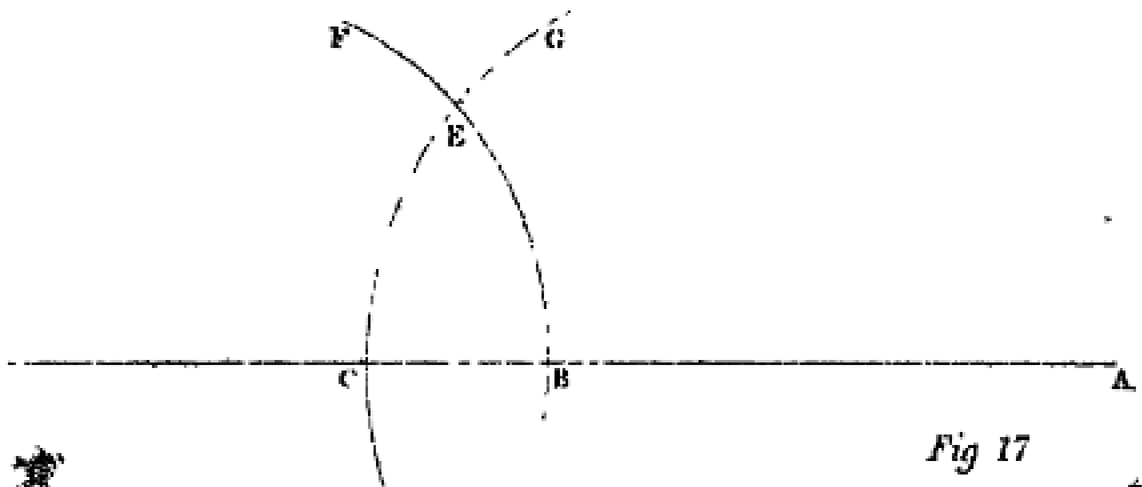
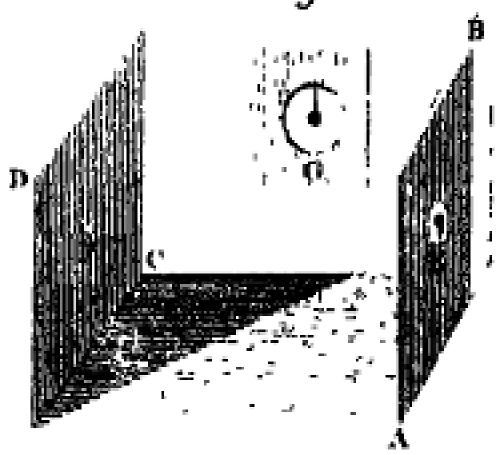


Fig 17



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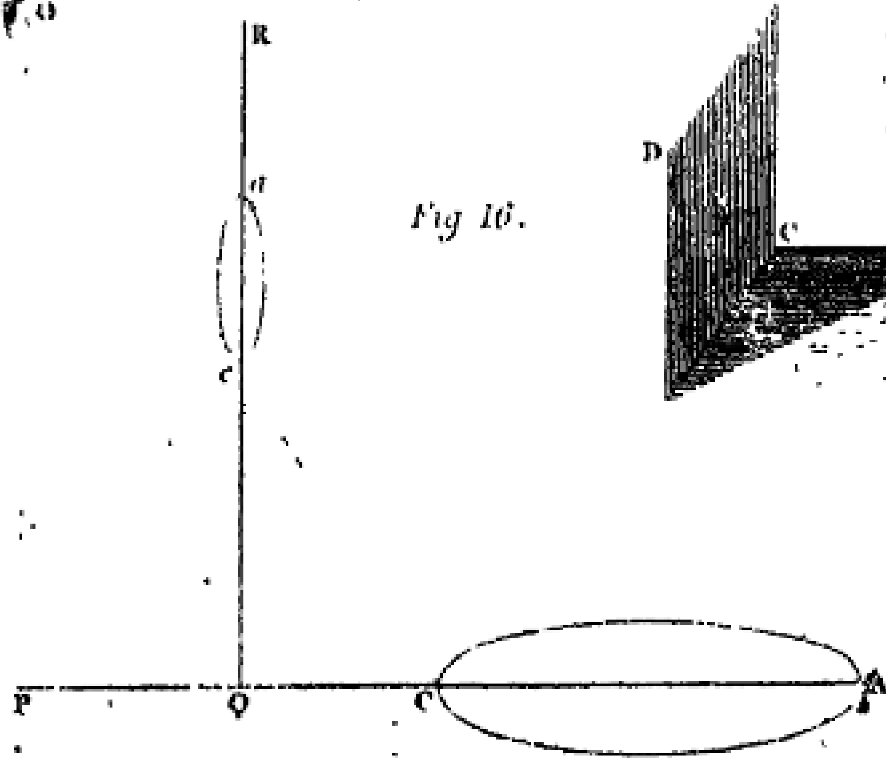

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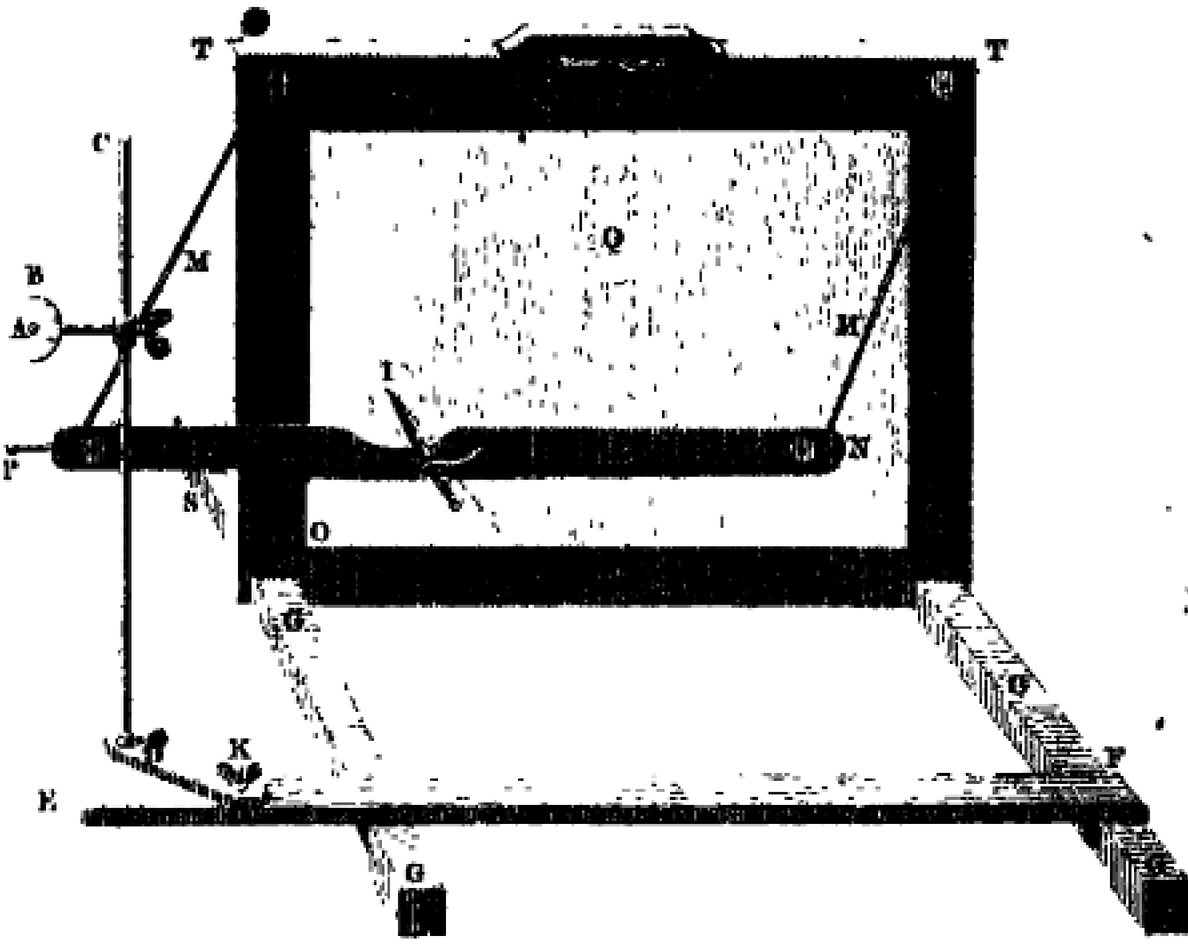

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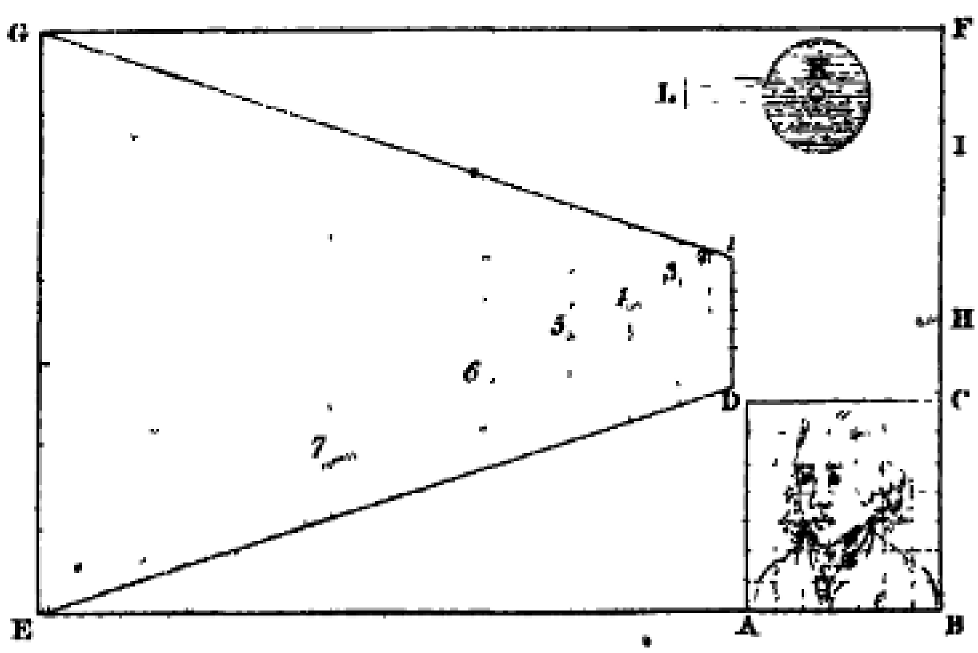

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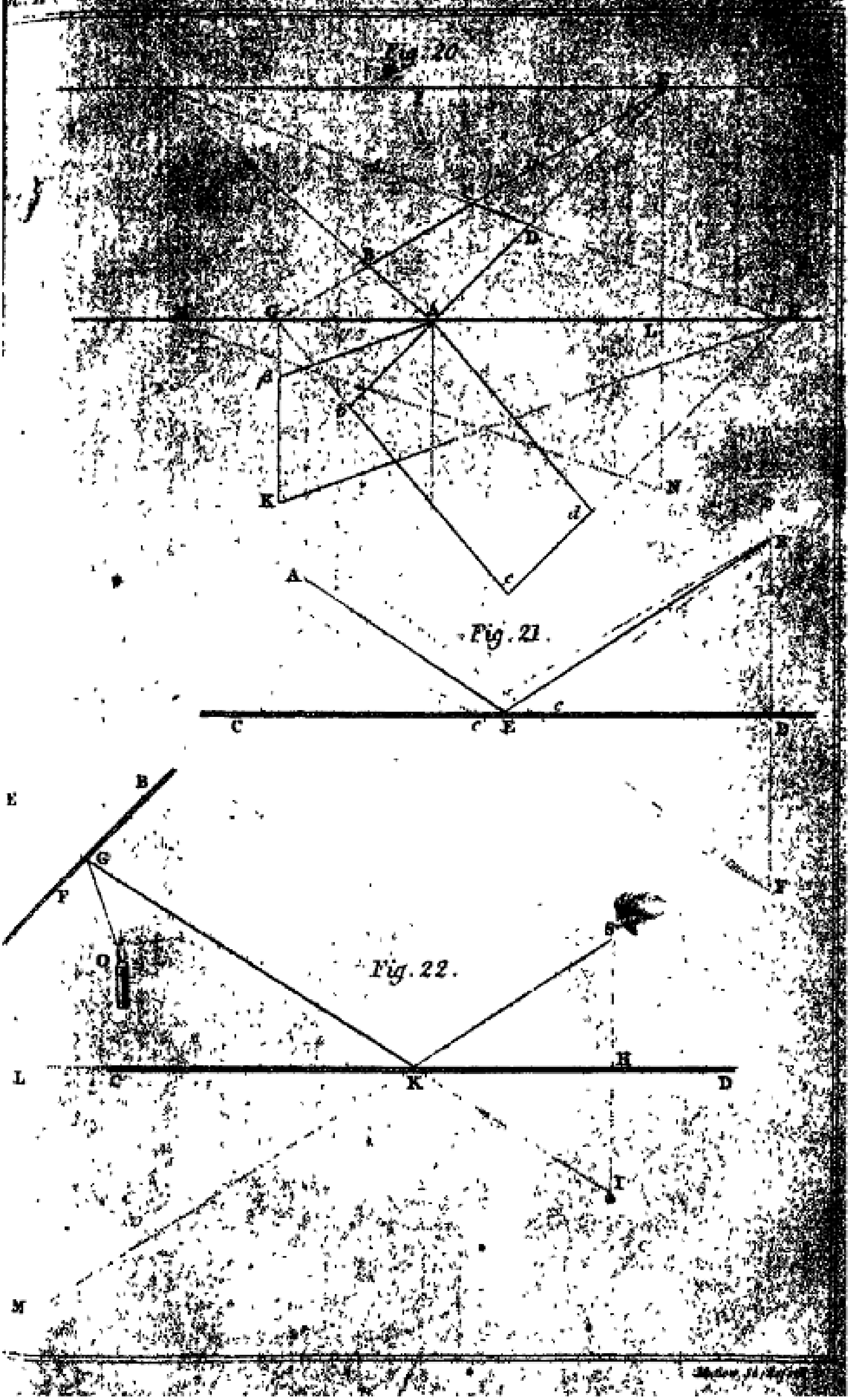

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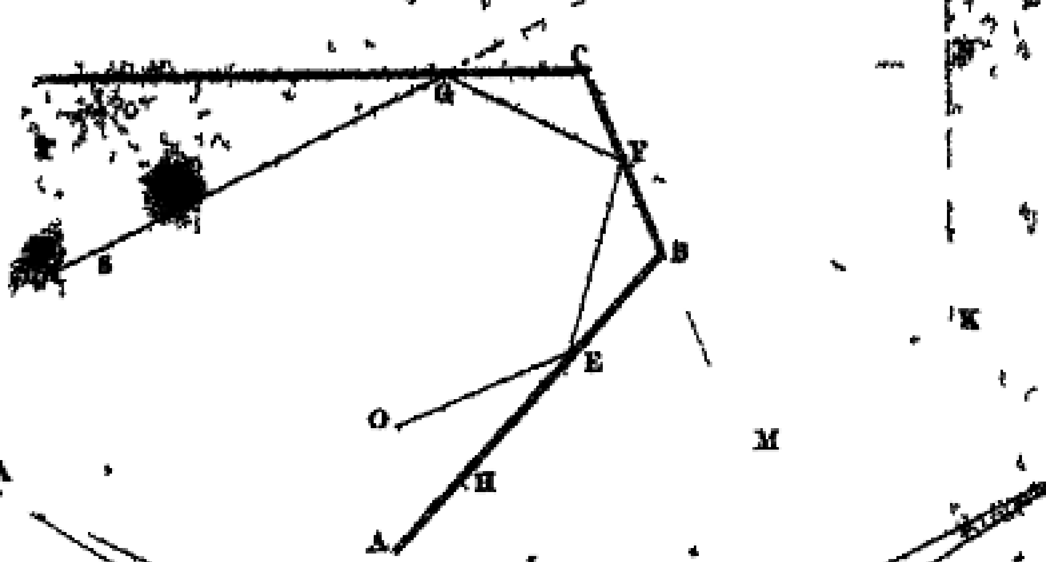


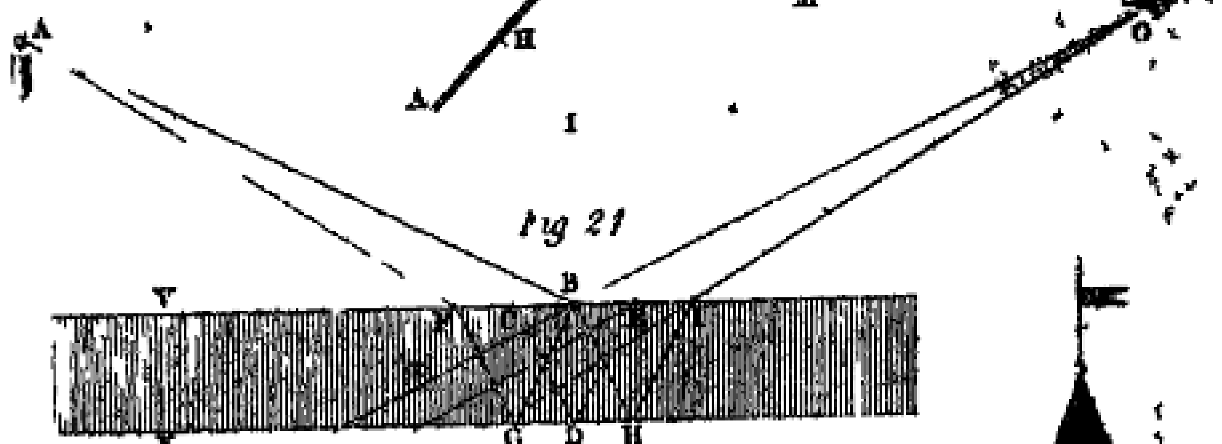
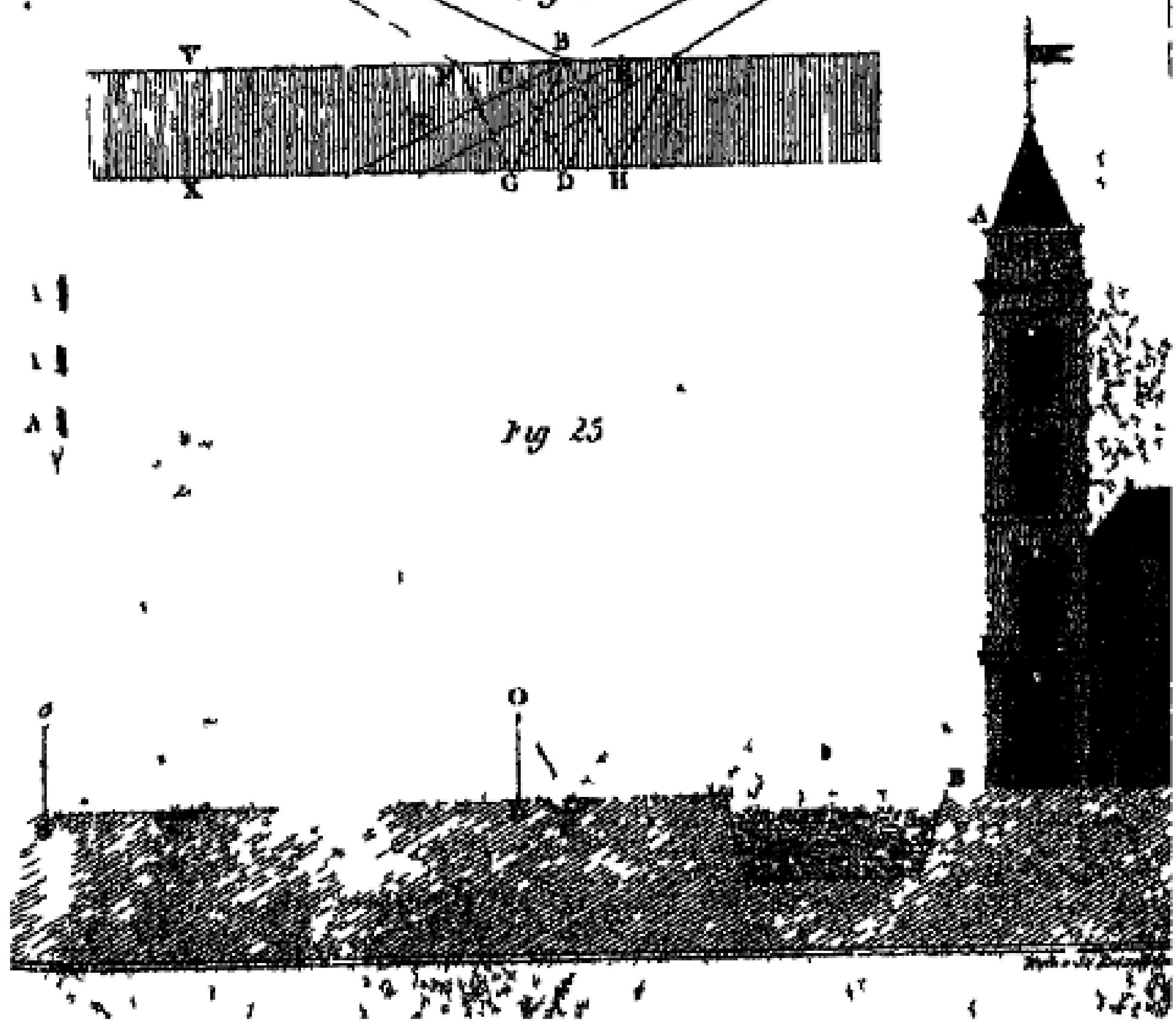


Fig 21

Fig 25



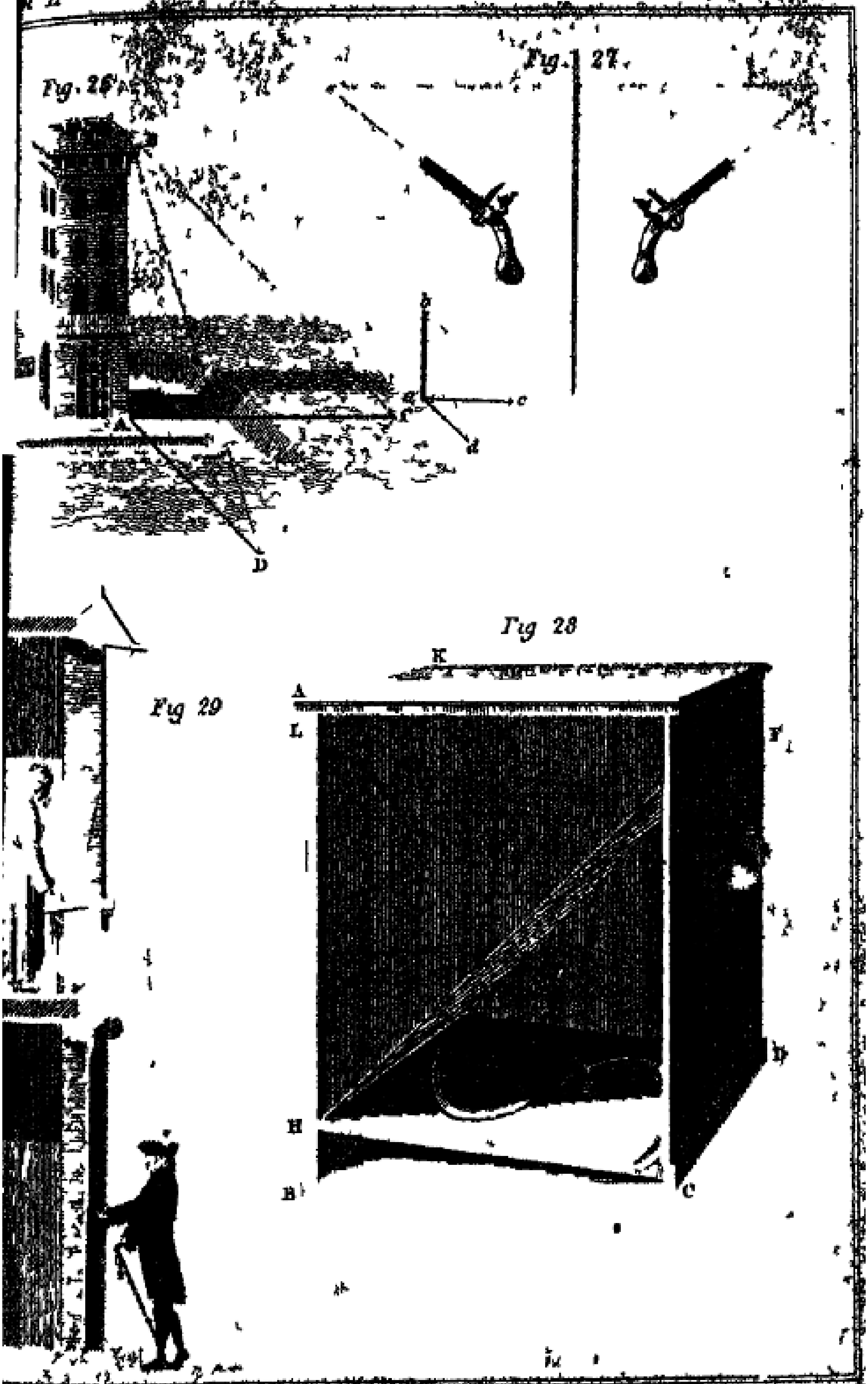


Fig. 26

Fig. 27

Fig. 28

Fig. 29

Fig 31

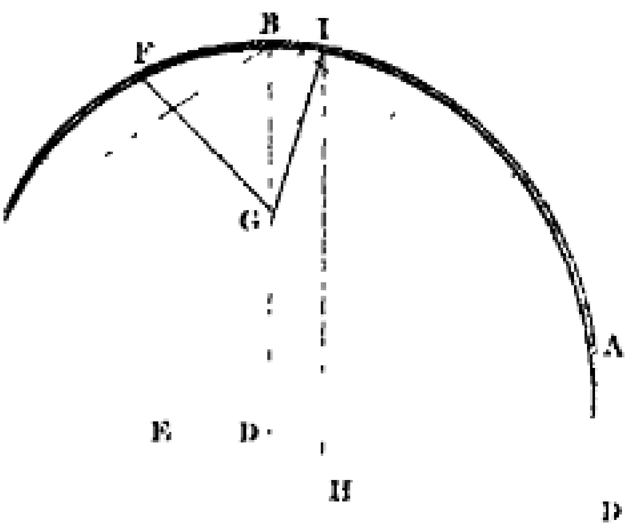


Fig 30

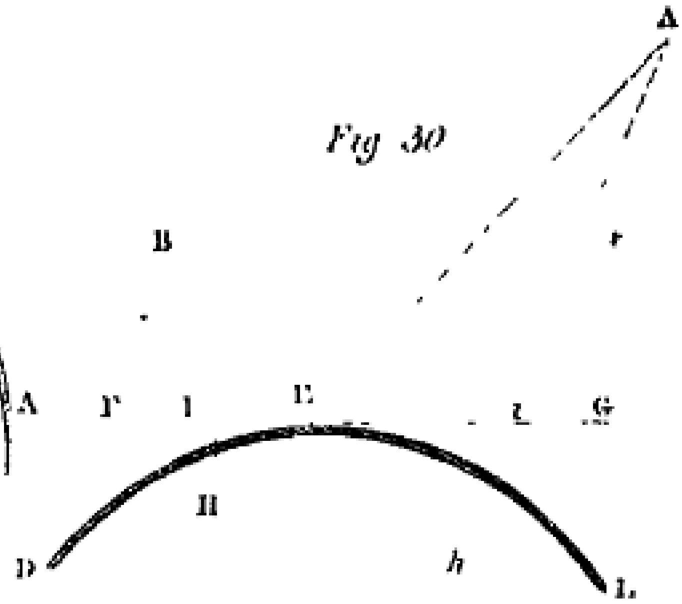
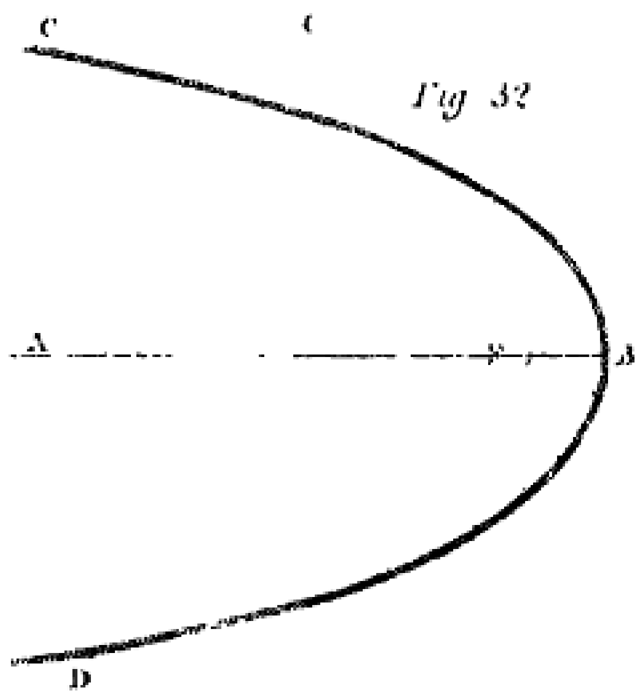


Fig 33

Fig 32



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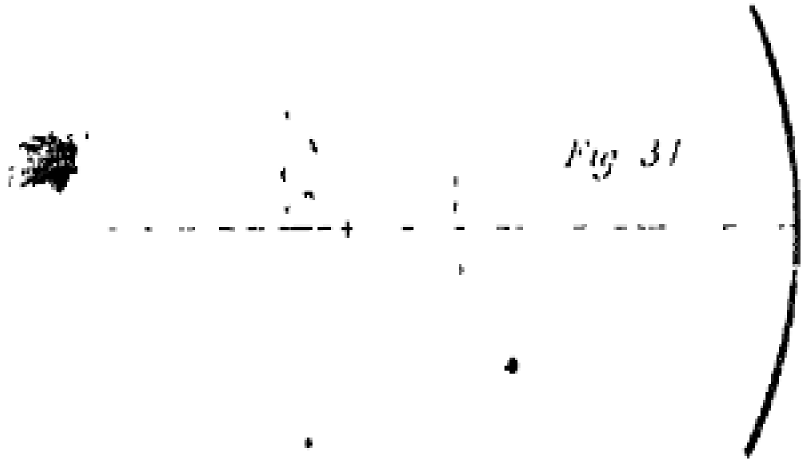

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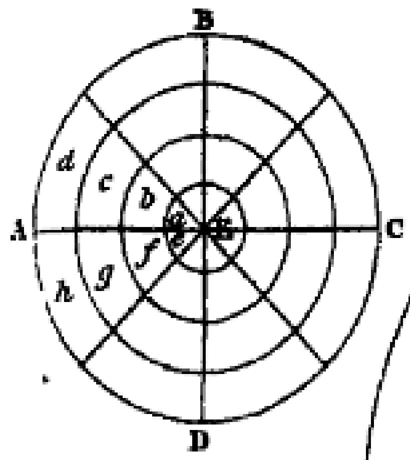

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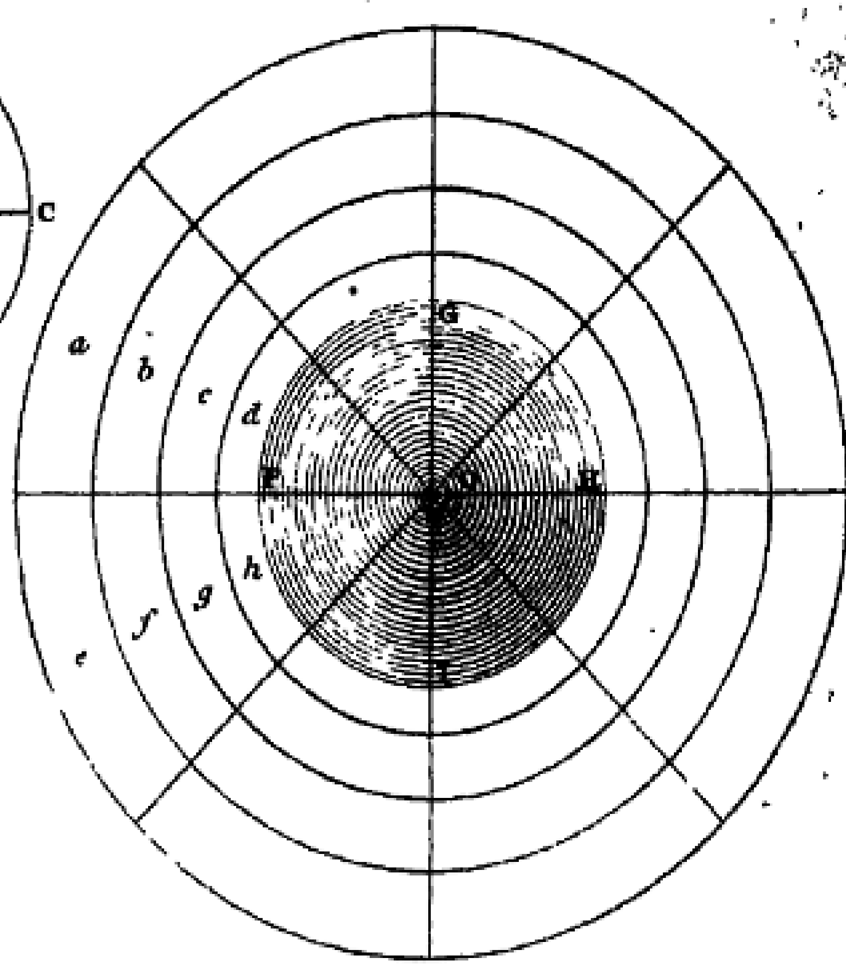

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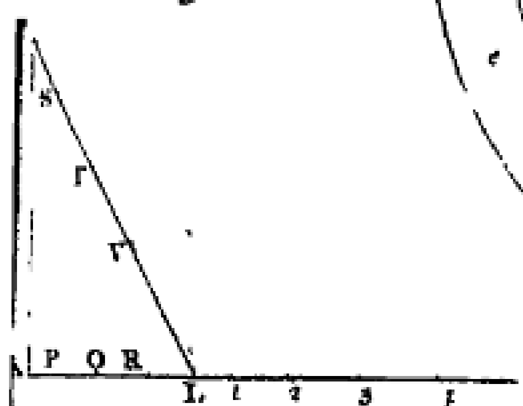

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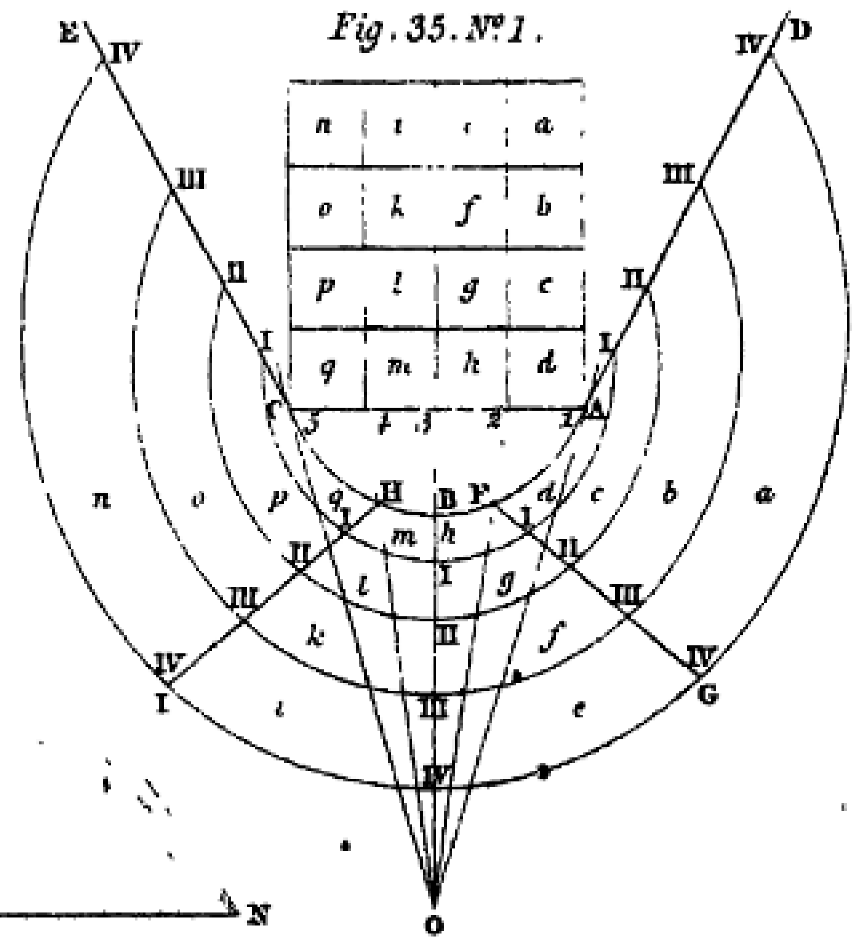

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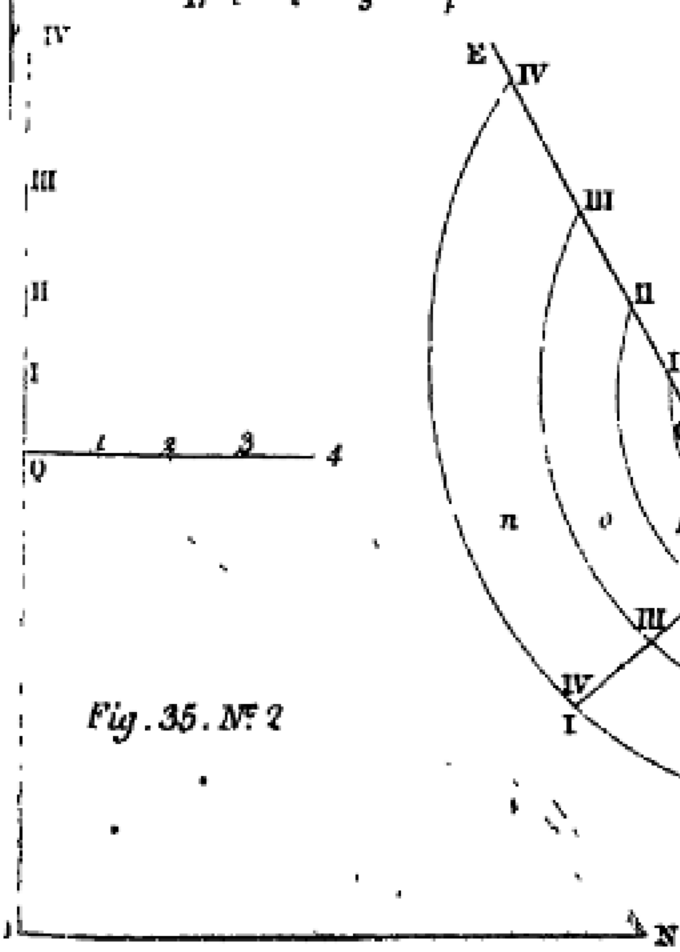

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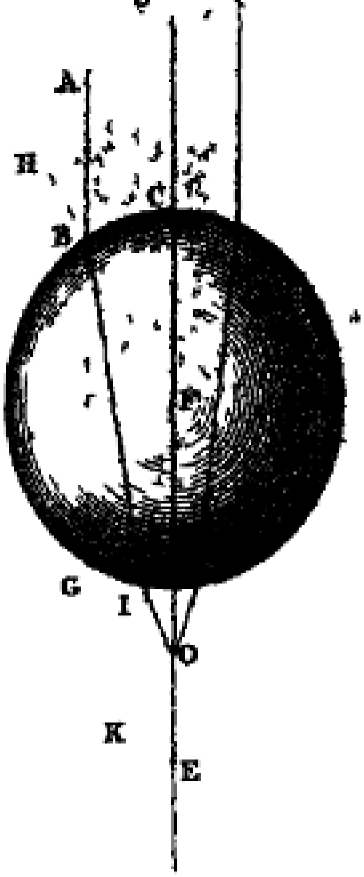
Fig. 36.



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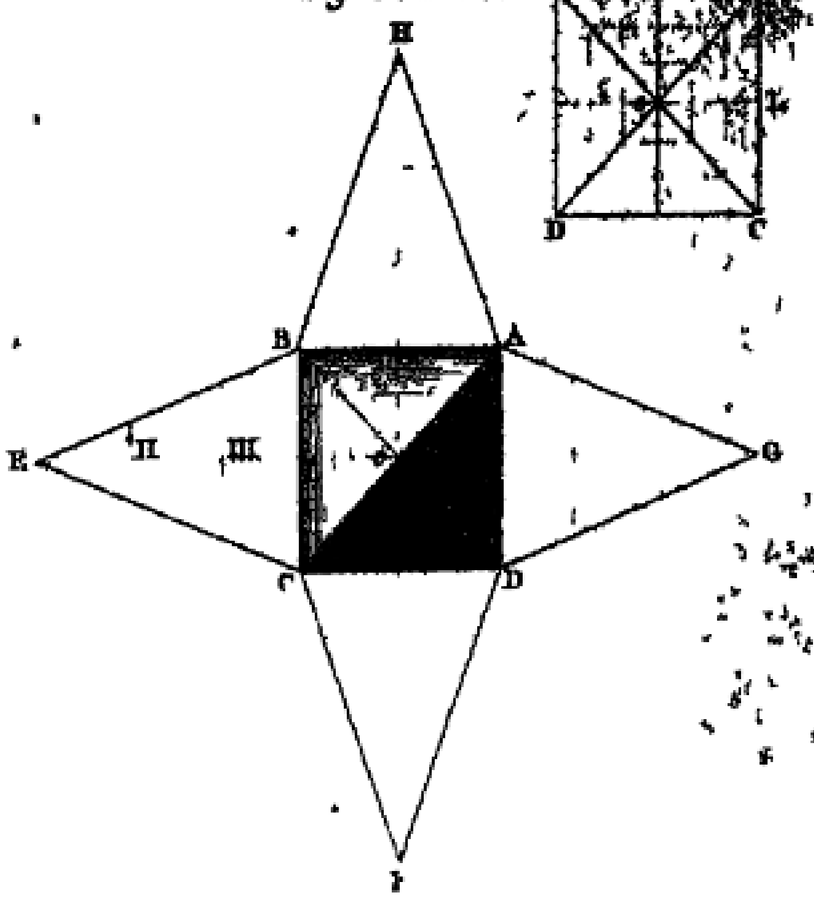
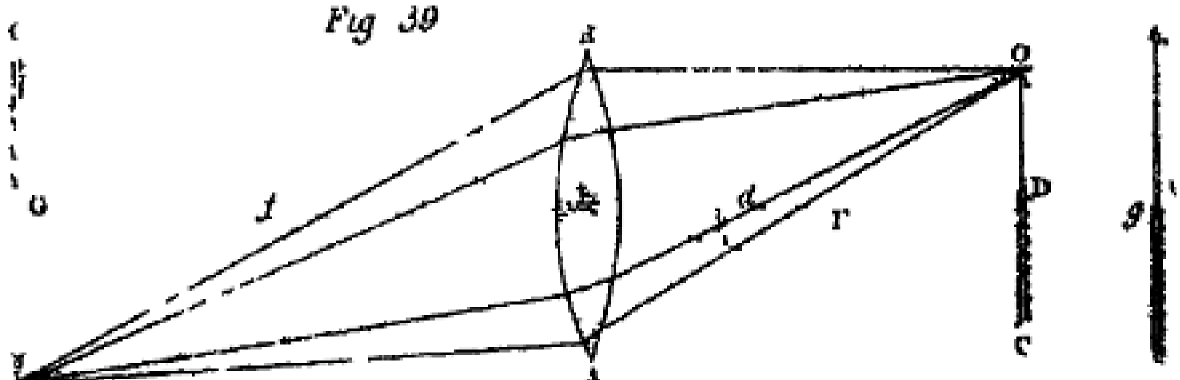


Fig 39



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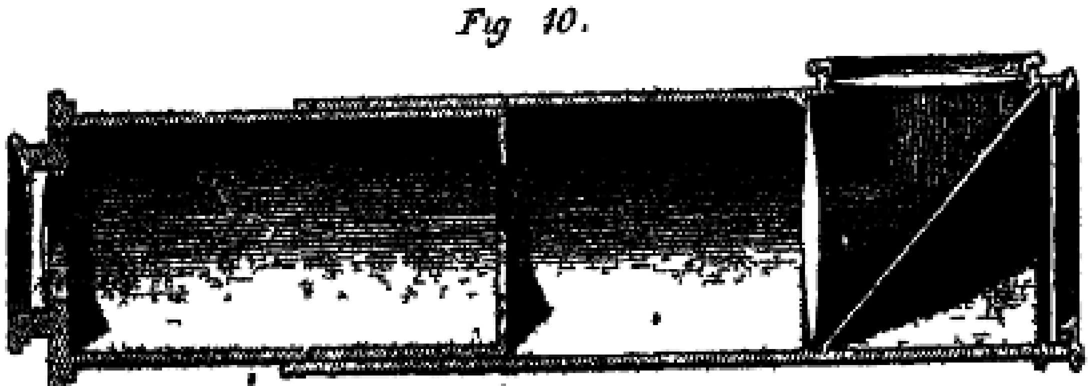

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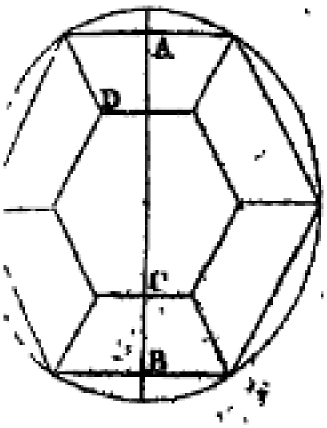

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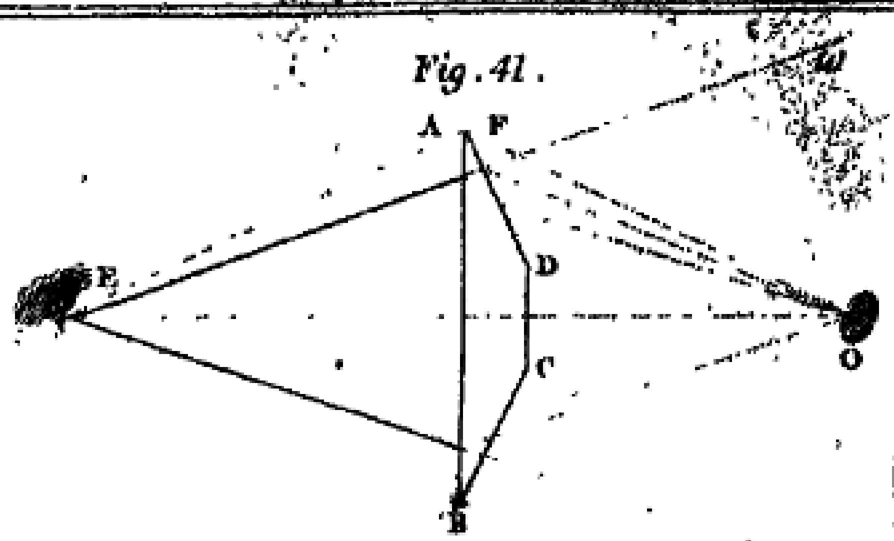

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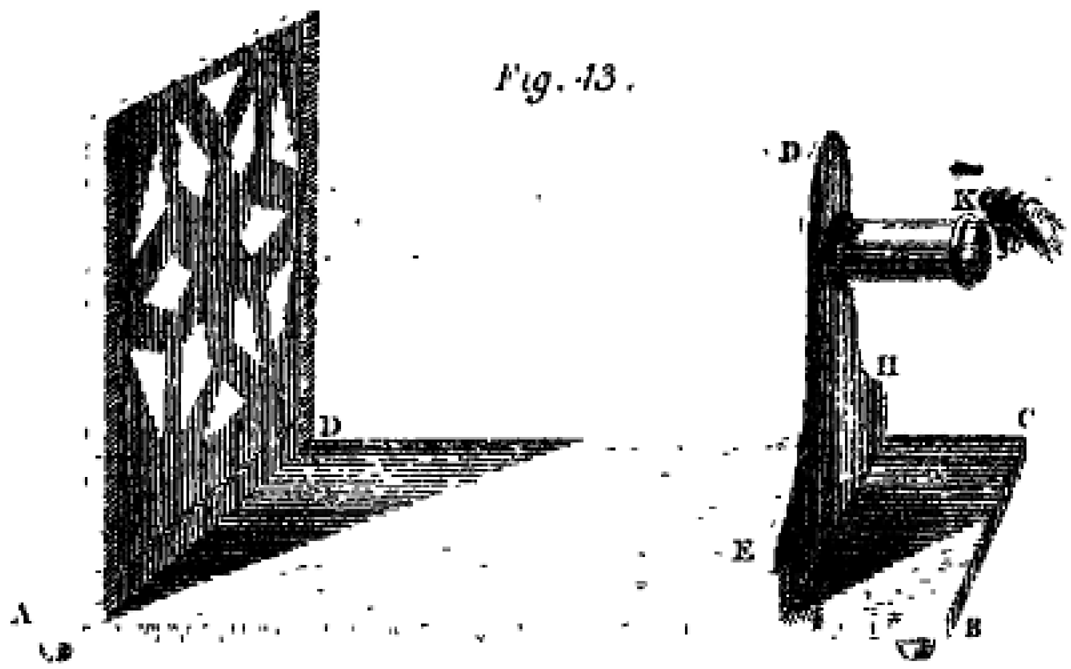

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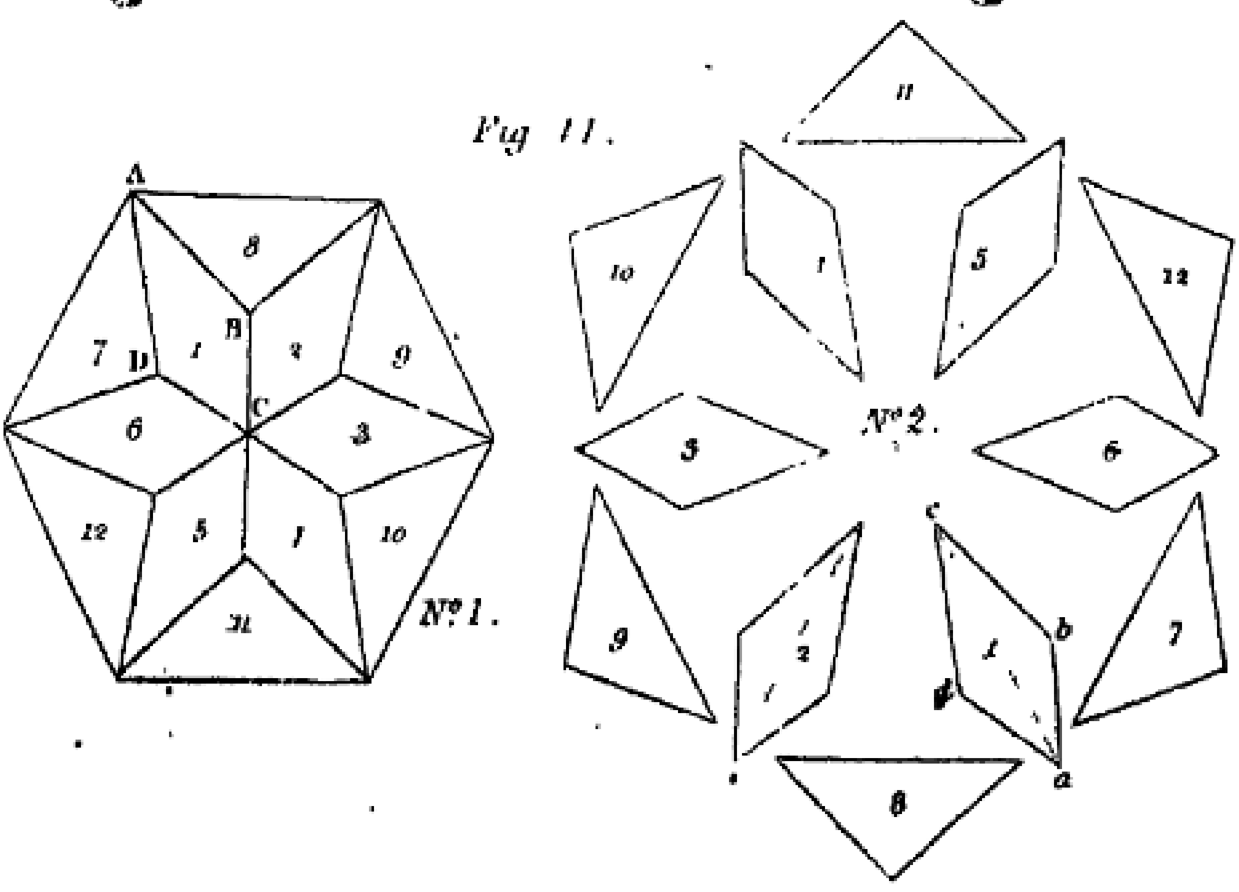


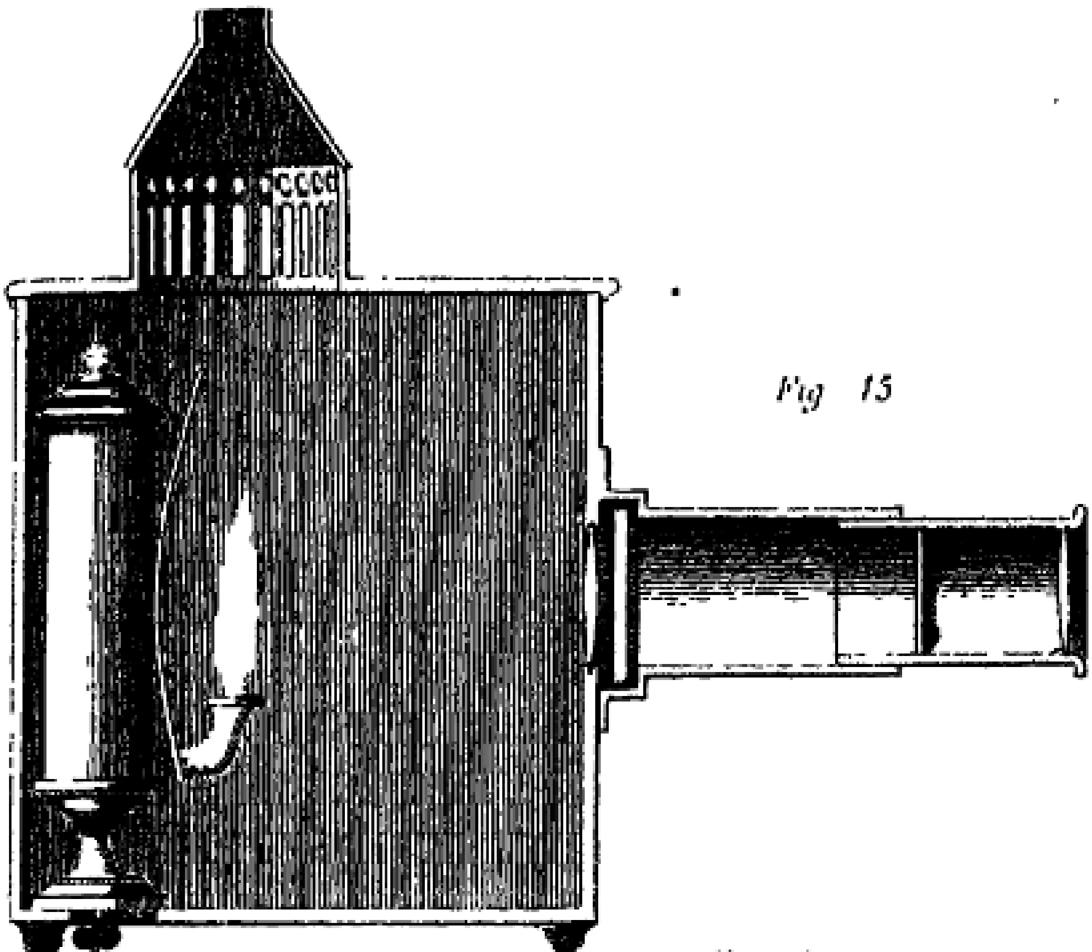


Fig 15

Fig 16

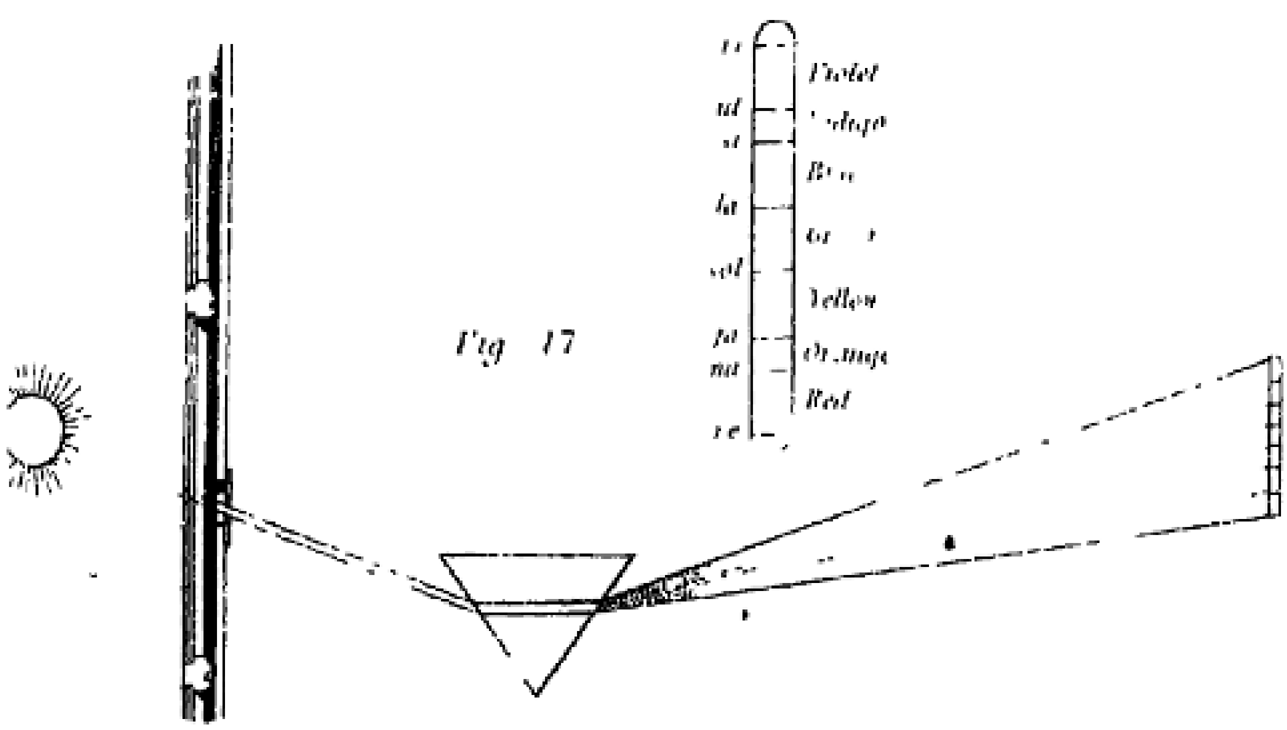
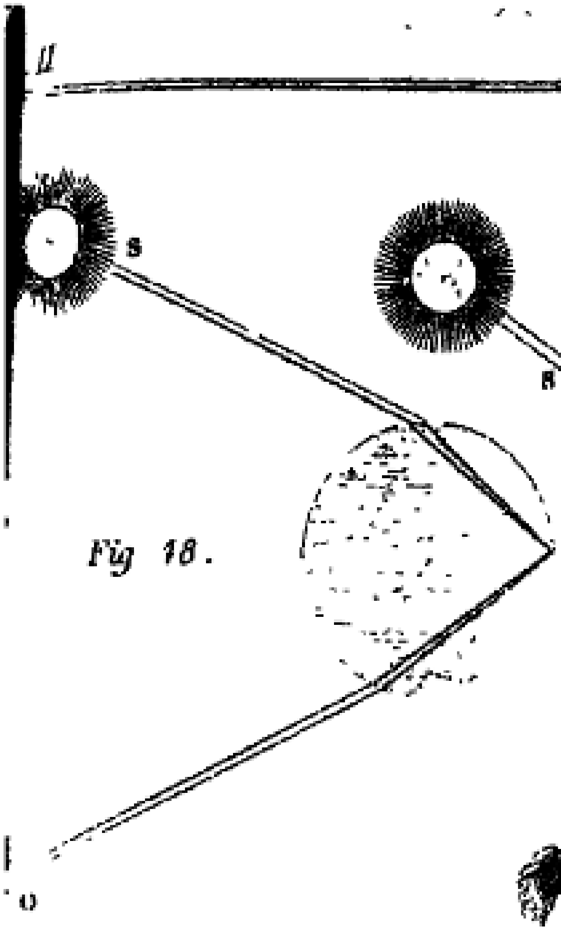


Fig 17



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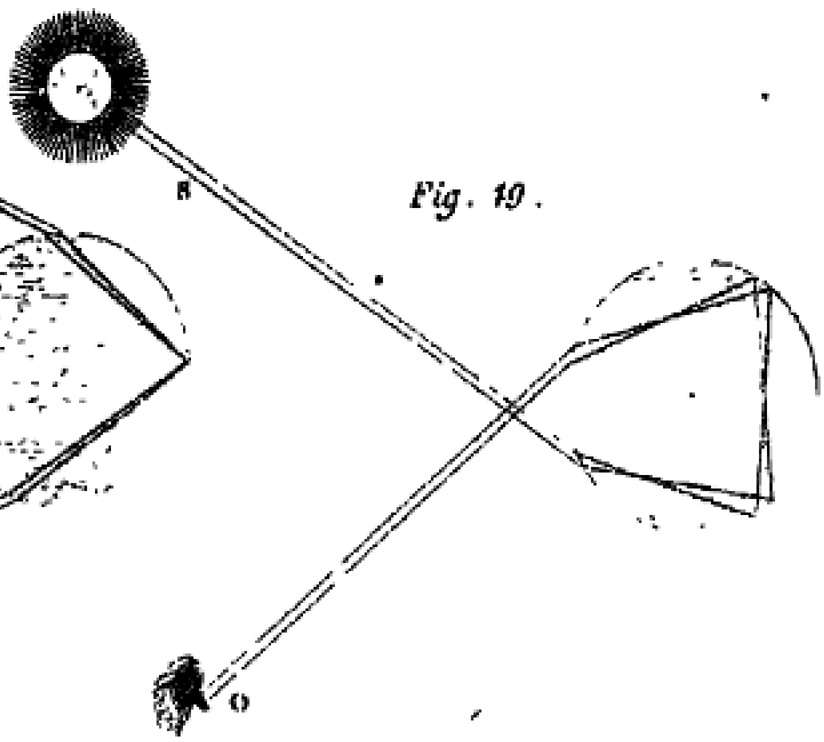


Fig. 49.

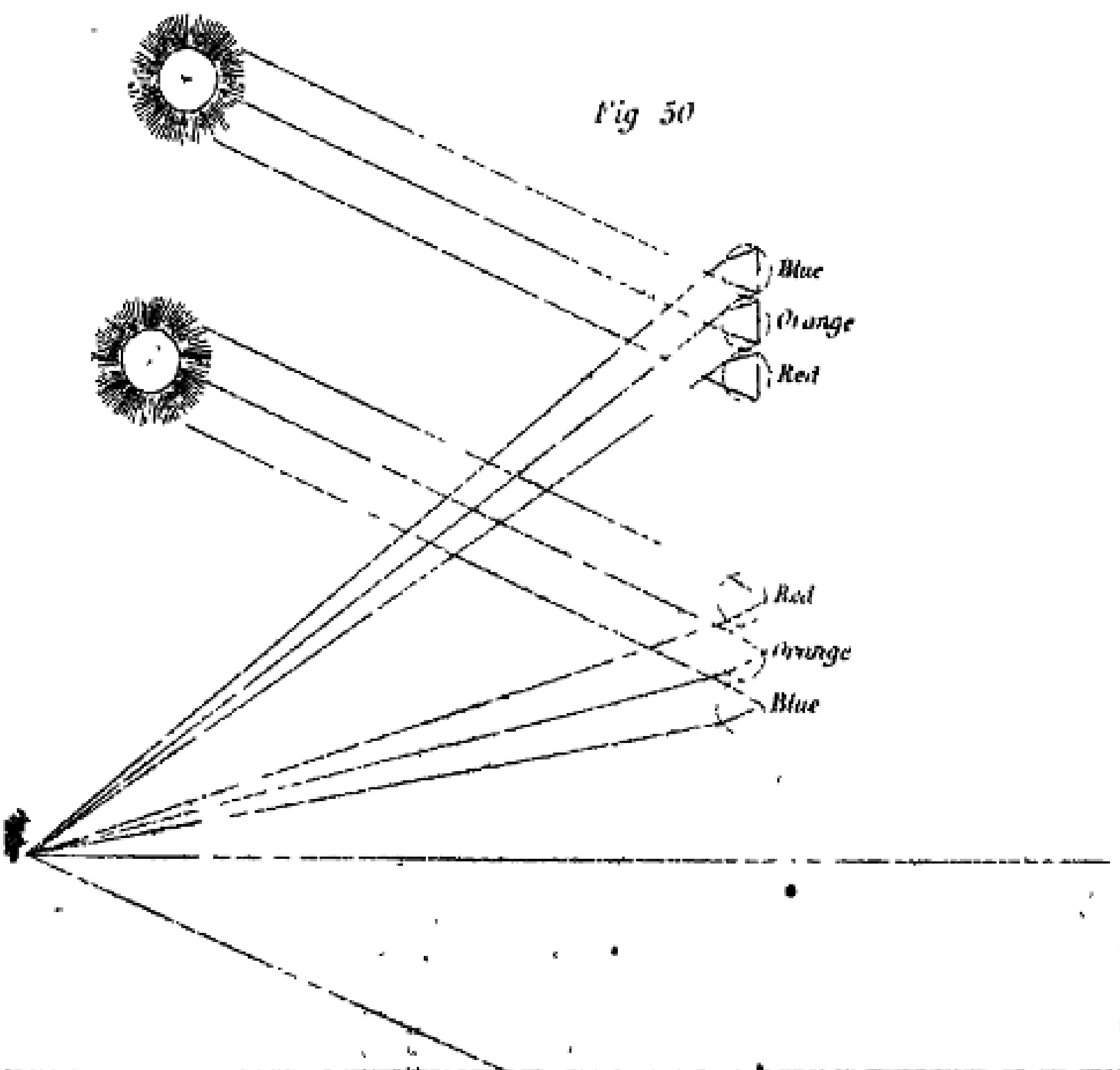
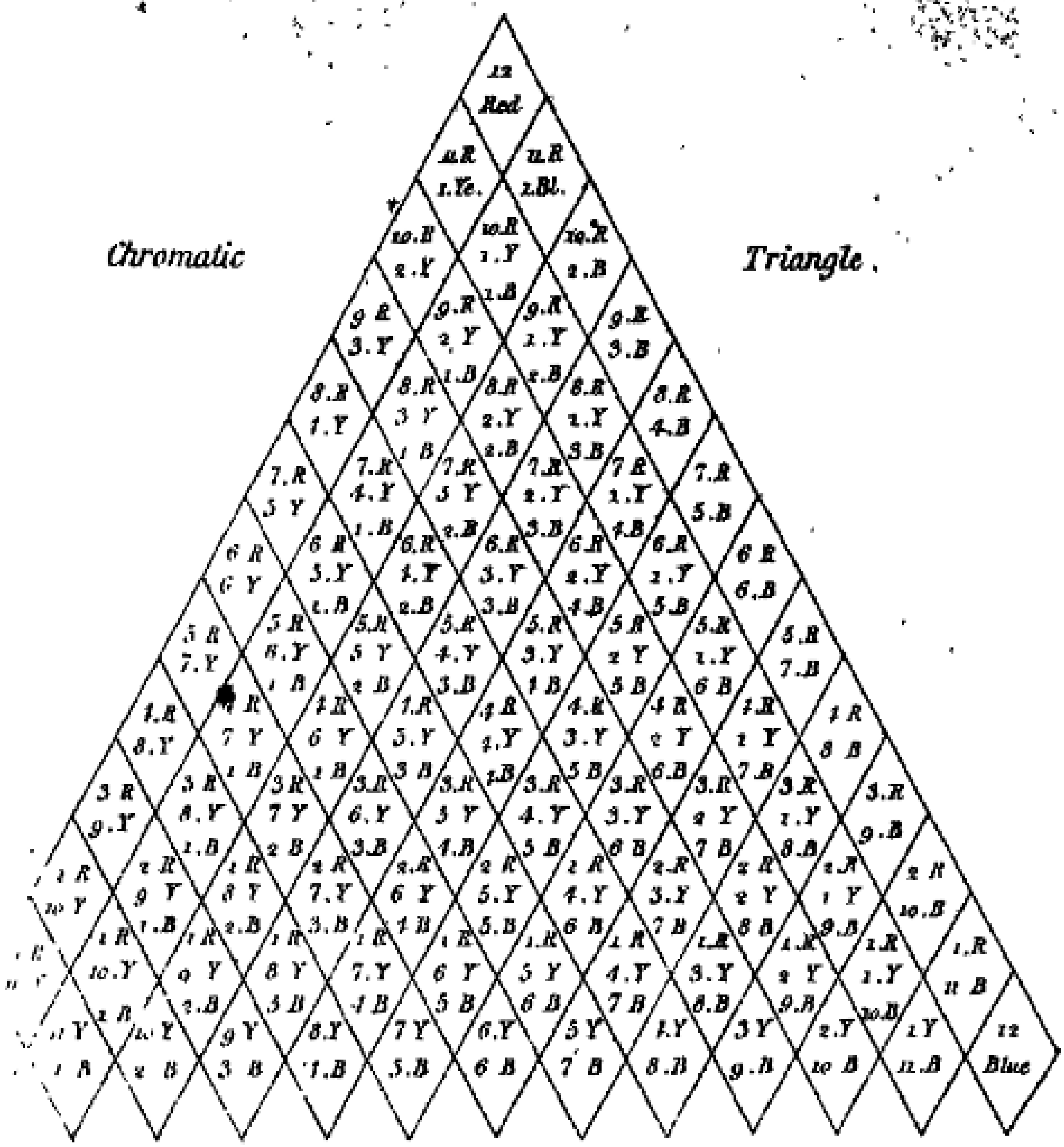


Fig 50

Fig. 51.

Chromatic Triangle.



Acoustics or Music.

Fig 1.

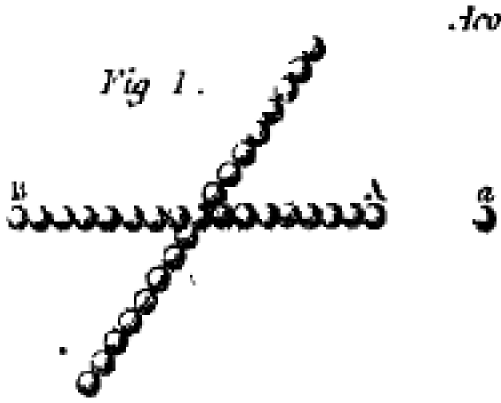


Fig. 2.

Fig. 3.

Fig. 52

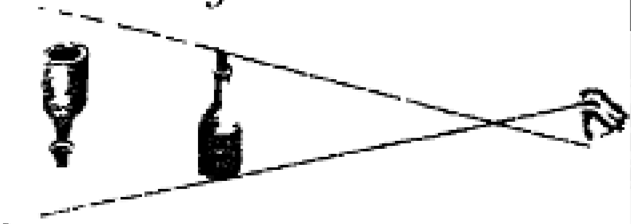


Fig. 54

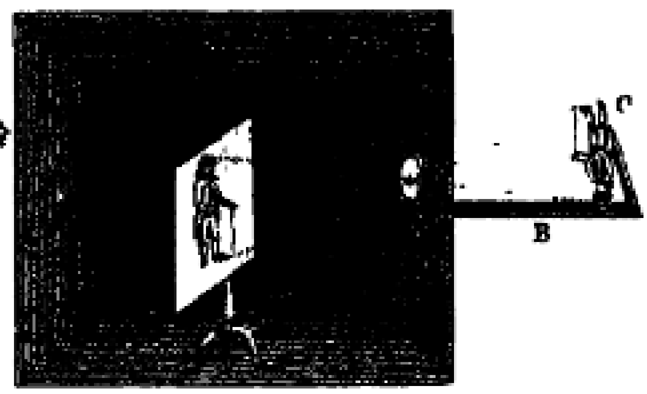


Fig. 53

Fig. 58.

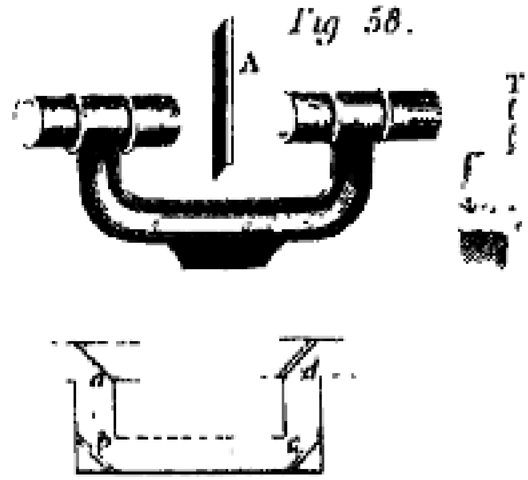


Fig. 55

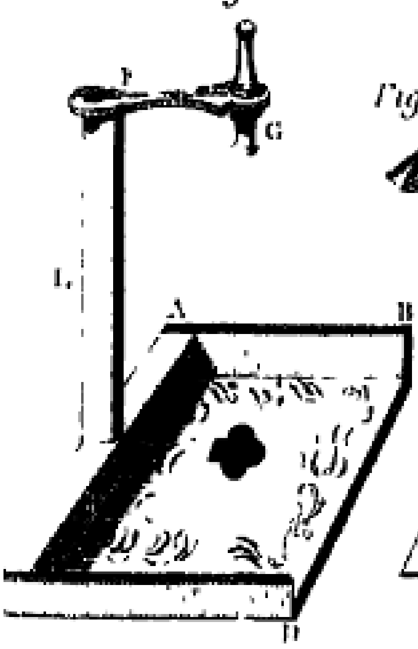


Fig. 56

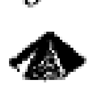


Fig. 57

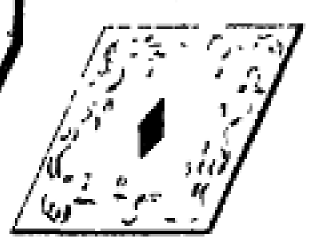


Fig. 60

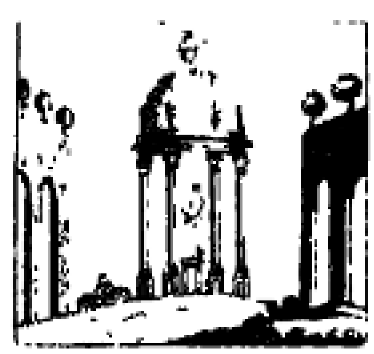


Fig. 59

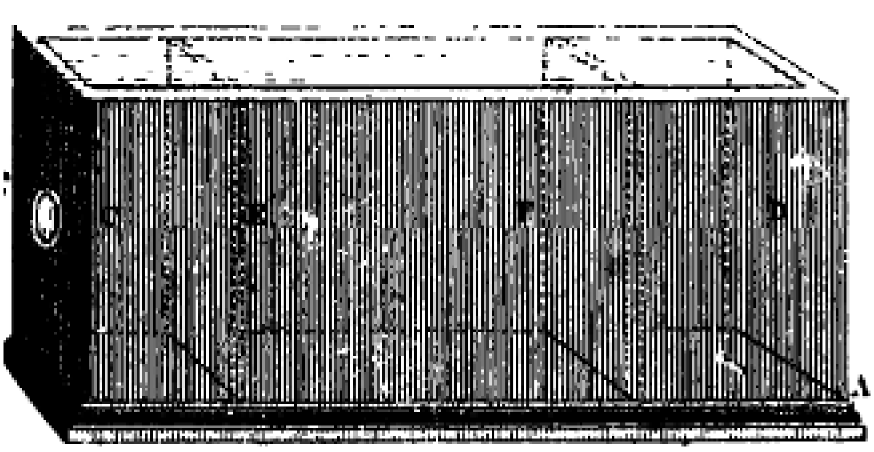


Fig. 61

Fig 62

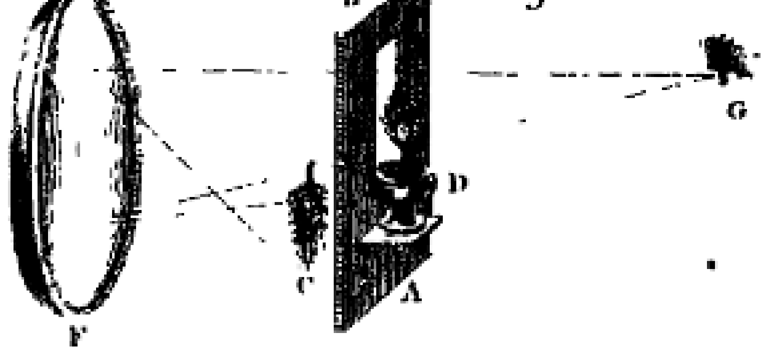
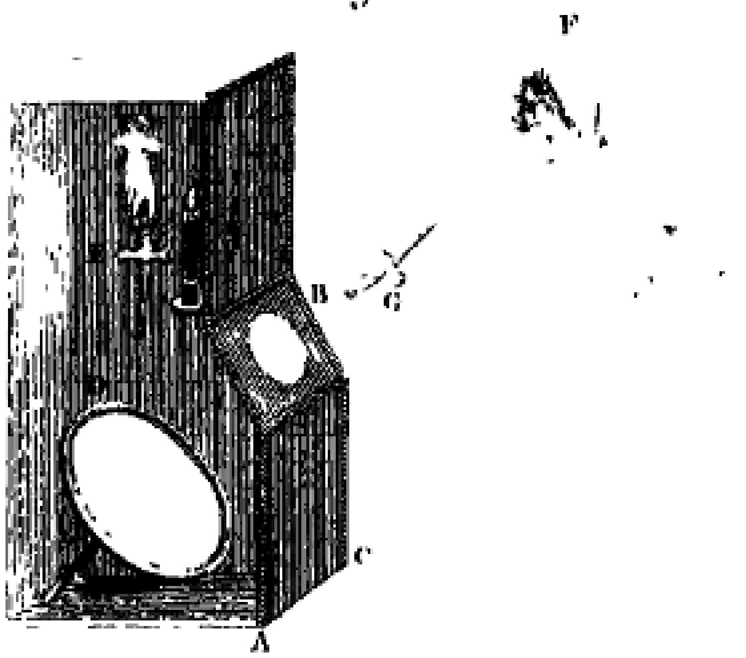


Fig 63



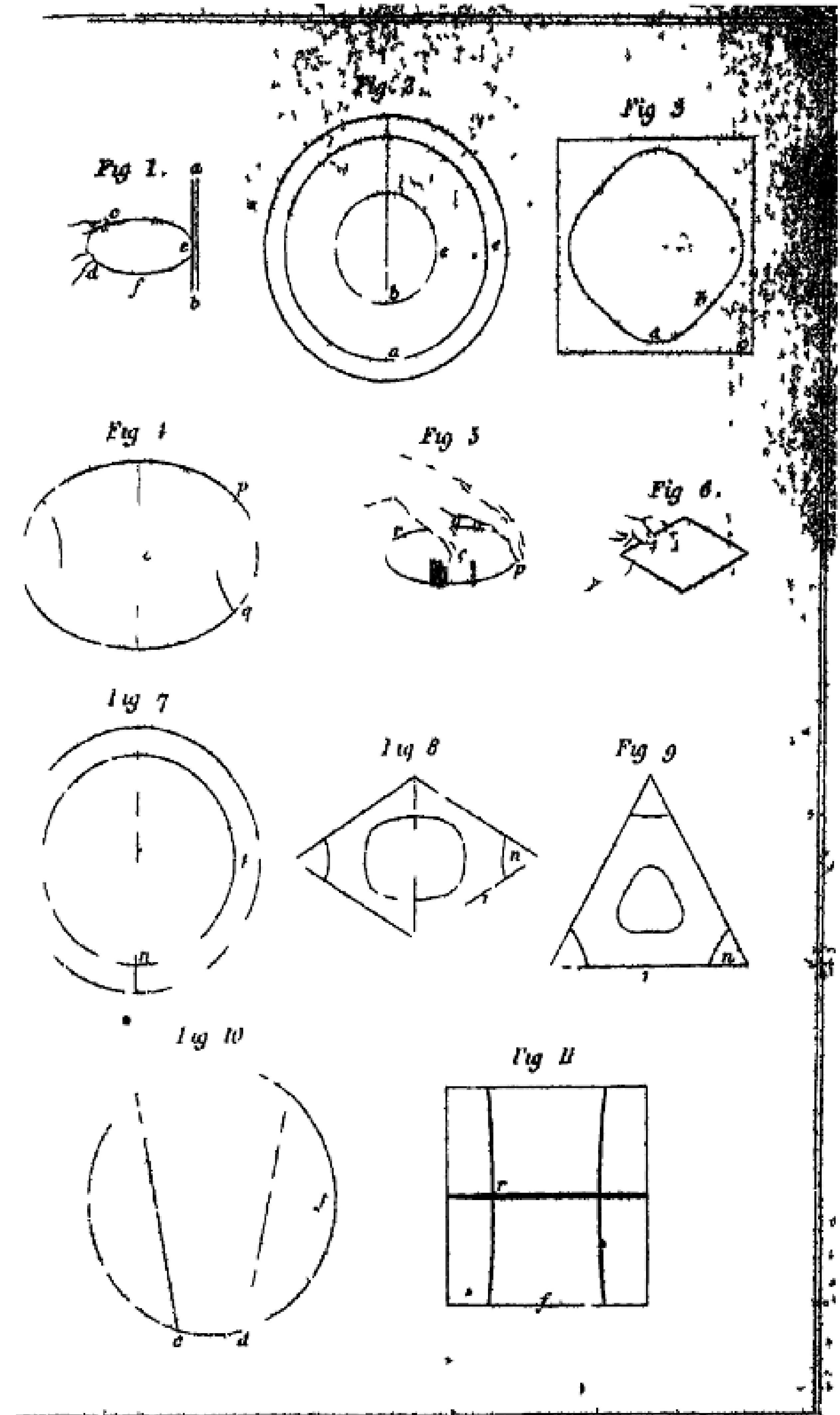


Fig 1.
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Fig 9
Fig 10
Fig 11

MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART FIFTH.

Containing every thing most curious in regard to Acoustics and Music.

THE ancients seem to have considered sounds under no other point of view than that of music; that is to say, as affecting the ear in an agreeable manner. It is even very doubtful whether they were acquainted with any thing more than melody, and whether they had any art similar to that which we call composition. The moderns, however, by studying the philosophy of sounds, have made many discoveries in this department, so much neglected by the ancients; and hence has arisen a new science, distinguished by the name of acoustics. Acoustics have for their object the nature of sounds, considered in general, both in a mathematical and a philosophical view. This science therefore comprehends music, which considers the ratios of sounds, so far as they are agreeable to the ear, either by their succession, which constitutes melody, or by their simultaneity, which forms harmony. We shall

here give an account of every thing most curious and interesting in regard to this science.

ARTICLE I.

Definition of sound, how diffused and transmitted to our organs of hearing; experiments on this subject, different ways of producing sound.

Sound is nothing else but the vibration of the particles of the air, occasioned either by some sudden agitation of a certain mass of the atmosphere violently compressed or expanded; or by the communication of the vibration of the minute parts of a hard and elastic body.

These are the two best known ways of producing sound. The explosion of a pistol, or any other kind of fire arms, produces a report or sound, because the air or elastic fluid contained in the gunpowder, being suddenly dilated, compresses the external air with great violence: the latter, in consequence of its elasticity, re-acts on the surrounding atmosphere, and produces in its molecule an oscillatory motion, which occasions the sound, and which extends to a greater or less distance according to the intensity of the cause that gave rise to it.

To form a proper idea of this phenomenon, let us conceive a series of springs, all maintaining each other in equilibrium, and that the first is suddenly compressed in a violent manner by some shock, or other cause. By making an effort to recover its former situation, it will compress the one next to it, the latter will compress the third, and the same thing will take place to the last, or at least to a very great distance, for the second will be somewhat less compressed than the first, the third a little less than the second, and so on; so that, at a certain distance the compression will be almost insensible, and at length it will totally cease. But each of these springs, in recovering itself, will pass a little beyond the point of equilibrium, and thus will occasion throughout the whole series put in mo-

tion, a vibration, which will continue for a longer or shorter time, and at length cease. Hence it happens that no sound is instantaneous, but always continues more or less, according to circumstances.

The other method of producing sound, is to excite, in an elastic body, vibrations sufficiently rapid to occasion, in the surrounding parts of the air, a similar motion. Thus, an extended string, when struck, emits a sound, and its oscillations, that is to say its motion backward and forward, may be distinctly seen. The elastic parts of the air, struck by the string during the time it is vibrating, are themselves put into a state of vibration, and communicate this motion to the neighbouring ones. Such is the mechanism by which a bell produces its sound; when struck, its vibrations are sensible to the hand which touches it.

Should these facts be doubted, the following experiments will exhibit the truth of them in the clearest point of view.

EXPERIMENT I.

Half fill a vessel, such as a drinking glass, with water: and having made it wet, moisten your finger a little, and move it round the edge. By these means a sound will be produced, and at the same time you will see the water tremble, and form undulations so as to throw up small drops. What but the vibration of the particles of the water can produce in it such a motion

EXPERIMENT II.

If a bell be suspended in the receiver of an air pump, so as not to touch any part of the machine; it will be found, on the bell being made to sound, that as the air is evacuated and becomes rarer, the sound grows weaker and weaker, and that it ceases entirely when as complete a vacuum as possible has been effected. If the air be gradually re-admitted, the sound will be revived, as we may say, and will

increase in proportion as the air contained in the machine approaches towards the same state as that of the atmosphere.

From these two experiments it results, that sound considered in the sonorous bodies, is nothing else than rapid vibrations of their minute parts; that air is the vehicle of it; and that it is transmitted so much the better when the air by its density is itself susceptible of a similar motion.

In regard to the manner in which sound affects the mind, we must first observe that at the interior entrance of the ear, which contains the different parts of the organ of hearing, there is a membrane extended like that of a drum, and which on that account is called the *tympanum*. It is very probable that the vibrations of the air, produced by the sonorous body, excite vibrations in this membrane; that these produce similar ones in the air, with which the internal cavity of the ear is filled; and that the sound is increased by the peculiar construction of the parts, and the circumvolutions both of the semicircular canals and of the helix. Hence there is occasioned in the nerves that cover the helix, a motion which is transmitted to the brain, and by which the mind receives the perception of sound. Here however we must stop, for it is not possible to ascertain how the motion of the nerves can affect the mind. But it is sufficient for us to know by experience that the nerves are as it were the mediators between our spiritual part, and the external and sensible objects.

Sound always ceases when the vibrations of the sonorous body cease, or become too weak. This is proved also by experiment, for when the vibrations of a sonorous body are damped by any soft body, the sound seems suddenly to cease. In a piano-forte therefore, the quills are furnished with bits of cloth, that by touching the strings when they fall down, they may damp their vibrations. On the other hand, when the sonorous body by its nature is capable of

continuing its vibrations for a considerable time, as is the case with a large bell, the sound may be heard for a long time after.

ARTICLE II.

On the velocity of sound; experiments for determining it; method of measuring distances by it.

Light is transmitted from one place to another with incredible velocity; but this is not the case with sound: the velocity of sound is very moderate, and may be measured in the following manner.

Let a cannon be placed at the distance of several thousand yards, and let an observer with a pendulum that vibrates seconds, or rather half seconds, put the pendulum in motion as soon as he sees the flash, and then count the number of seconds or half seconds which elapse between that period and the moment when he hears the explosion. It is evident that, if the moment when the flash is seen be considered as the signal of the explosion, nothing will be necessary, to obtain the number of yards which the sound has passed over in a second, but to divide the number of the yards between the place of observation and the cannon, by the number of seconds or half seconds which have been counted.

Now the moment when the flash is perceived, whatever be the distance, may be considered as the real moment of the explosion, for so great is the velocity of light, that it employs scarcely a second to traverse 60,000 leagues*.

* The velocity of the particles and rays of light is truly astonishing, as it amounts to nearly 2 hundred thousand miles in a second of time, which is nearly a million times greater than the velocity of a cannon-ball. It has been found by repeated experiments, that when the earth is exactly between Jupiter and the sun, his satellites are seen eclipsed 8¼ minutes sooner than they could be according to the tables, but when the earth is nearly in the opposite point of its orbit, these eclipses happen about 8¼ minutes later than the tables predict them: hence it is certain that the motion of light is not instantaneous, but that it takes up about 16½ minutes of time in passing over a space equal to the diameter of the earth's orbit, which is at least 190 mil-

By similar experi[illegible] of the Royal Academy of Sciences[illegible]ved at the rate of 1172 Parisian feet in [illegible]dus makes its velocity to be 1473 feet in [illegible]Mersenne 1474; Duhamel, in the History of th[illegible]demy of Sciences, 1338; Newton 968; and Derham, in whose measure Flamsteed and Halley concurred, 1142. Though it is difficult to determine among so many authorities, the last estimate, viz 1142 per second, has been generally adopted in this country.

It is to be observed that, according to Derham's experiments, the temperature of the air, whether dry or moist, cold or hot, causes no variation in the velocity of sound. This philosopher had often an opportunity of seeing the flash and hearing the report of cannon fired at Blackheath, 9 or 10 miles distant, from Upminster, the place of his residence; but whatever might be the state of the weather, he always counted the same number of half seconds, between the moment of seeing the flash and that of hearing the report, unless any wind blew from either of these places, in which case the number of the seconds varied from 111 to 112. It may be readily conceived, that if the wind impelled the fluid put into a state of vibration, towards the place of the observer, the vibrations would reach him sooner than if the fluid had been at rest, or had been impelled in a contrary direction.

But notwithstanding what Derham has said, we can hardly be persuaded that the velocity of sound is not affected by the temperature of the air; for when the air is heated, and consequently more rarefied or elastic, the vibrations must be more rapid: observations on this subject ought to be carefully repeated.

An inaccessible distance then may be measured by means

lions of miles in length, or moves at the rate of nearly 200000 miles per second. Hence therefore light takes about 8¼ minutes in passing from the sun to the earth.

of sound. For [redacted]e a pendulum that swings half se[redacted] done by suspending from a thread [redacted]an inch in diameter, in such a manner, tha[redacted]e exactly $9\frac{3}{4}\frac{1}{2}$ inches, or $9\frac{1}{4}$, between the centre [redacted]ll and the point of suspension; then the moment you perceive the flash of a cannon, or musket, let go the pendulum, and count how many vibrations it makes till the instant when you hear the report: if you then multiply this number by 571 feet, you will have the distance of the place where the musket or cannon was fired.

We here suppose the weather to be calm, or that the wind blows only in a transversal direction; for if the wind blows towards the observer from the place where the cannon or gun is fired, and if it be violent, as many times 12 feet, as there have been counted half seconds, must be added to the distance found; and in the contrary case, that is, if the wind blows from the observer, towards the quarter where the explosion is made, they must be subtracted. It is well known that a violent wind makes the air move at the rate of about 24 feet per second, which is nearly the 48th part of the velocity of sound. If the wind be moderate, a 96th may be added or subtracted, and if it be weak, but sensible, a 192d: but this correction, especially in the latter case, seems to be superfluous; for can we ever flatter ourselves that we have not erred a 192d part in the measuring of time?

This method may be employed to determine the distance of ships at sea, or in a harbour, when they fire guns, provided the flash can be seen, and the explosion heard. During a storm also, the distance of a thunder-cloud may be determined in the same manner. But as a pendulum is not always to be obtained, its place may be supplied by observing the beats of the pulse, for when in its usual state, each interval between the pulsations is almost equal to a

second; but when q[illegible]ach pulsation is equal to only two thir[illegible]

AR[illegible] III.

How sounds may be propag[illegible] in every direction without confusion.

This is a very singular phenomenon in the propagation of sounds; for if several persons speak at the same time, or play on instruments, their different sounds are heard simultaneously, or all together, either by one person, or by several persons, without being confounded in passing through the same place in different directions, or without damping each other. Let us endeavour to account for this phenomenon.

The cause no doubt is to be found in the property of elastic bodies. For let us conceive a series of globules equally elastic, and all contiguous, and let us suppose that a globule is impelled with any velocity whatever against the first of the series: it is well known that in a very short time the motion will be transmitted to the other extremity, so that the last globule will have the same motion communicated to it as if it had been itself immediately impelled. Now if two globules with unequal velocities impel at the same time the two extremities of the series, the globule *a*, for example, the extremity A, and the globule *b* the extremity B (fig. 1 pl. 15), it is certain, from the well known properties of elastic bodies, that the globules *a* and *b*, after being a moment at rest, will be repelled, making an exchange of velocity, as if they had been immediately impelled against each other.

If we suppose a second series of globules, intersecting the former in a transversal direction, the motion of this second series will be transmitted by means of the common globule, from one end to the other of this series, in the same manner as if it had been alone. The case will be the

same if two, three, four or more series cross the first one, either in the same point or in different points. The particular motion communicated to the beginning of each series, will be transmitted to the other end, as if that series were alone.

This comparison may serve to show how several sounds may be transmitted in all directions, by the help of the same medium; but it must be allowed that there are some small differences. For, in the first place, we must not conceive the air, which is the vehicle of sound, to be composed of elastic globules, disposed in such regular series as those here supposed: each particle of air is no doubt in contact with several others at the same time, and its motion is thereby communicated in every direction. Hence it happens that the sound, which would reach to a very great distance almost without diminution, if communicated as here supposed, experiences a considerable decrease, in proportion as it recedes from the body which produced it. Though the movement by which sound is transmitted be more complex, there is reason to believe that it is reduced, in the last instance, to something similar to what has been here described.

The second difference arises from the particles of air, by which the organ of hearing is immediately affected, not having a movement of translation, like the last globule of the series, which proceeds with a greater or less velocity; in consequence of the shock that impels the other extremity of the series. But the movement in the air consists merely of an undulation or vibration, which, in consequence of the elasticity of its aerial particles, is transmitted to the extremity of the series, such as it was received at the other. It must be observed that the sonorous body communicates to the air, which it touches, vibrations isochronous with those which it experiences itself; and that the same vibrations are transmitted from the one end to the other of the series, and always with the same velocity: for we are

taught by experience that a grave sound, *cæteris paribus*, does not employ more time than an acute one, to pass through a determinate space.

ARTICLE IV.

Of Echoes; how produced; account of the most remarkable echoes, and of some phenomena respecting them.

Echoes are well known; but however common this phenomenon may be, it must be allowed that the manner in which it is produced is still involved in considerable obscurity, and that the explanation given of it does not sufficiently account for all the circumstances attending it.

All philosophers almost have ascribed the formation of echoes to a reflection of sound, similar to that experienced by light, when it falls on a polished body. But, as D'Alembert observes, this explanation is false; for if it were not, a polished surface would be necessary to the production of an echo; and it is well known that this is not the case. Echoes indeed are frequently heard opposite to old walls, which are far from being polished; near huge masses of rock, and in the neighbourhood of forests, and even of clouds. This reflection of sound therefore is not of the same nature as that of light.

It is evident, however, that the formation of an echo can be ascribed only to the repercussion of sound; for echoes are never heard but when sound is intercepted, and made to rebound by one or more obstacles. The most probable manner in which this takes place is as follows.

For the sake of illustration, we shall resume our comparison of the aerian moleculæ, to a series of elastic globules. If a series of elastic globules then be infinite, it may readily be conceived, that the vibrations communicated to one end, will be always propagated in the same direction, and continually recede; but if the end of the series rest against any fixed point, the last globule will react on the whole series, and communicate to it, in the

contrary direc[illegible]on as it would have communicated [illegible]ries, if it had not rested against a fixed point. [illegible]ught indeed to be the case whether the obstacle be in [illegible]ne with the series or oblique to it, provided the last globule be kept back by the neighbouring ones, only with this difference, that the retrograde motion will be stronger in the latter case, according as the obliquity is less. If the aerian and sonorous moleculæ then rest against any point at one end; and if the obstacle be at such a distance from the origin of the motion, that the direct and repercussive motion shall not make themselves sensible at the same instant, the ear will distinguish the one from the other, and there will be an echo.

But we are taught by experience, that the ear does not distinguish the succession of two sounds, unless there be between them the interval of at least one twelfth of a second: for during the most rapid movement of instrumental music, each measure of which cannot be estimated at less than a second*, twelve notes are the utmost that can be comprehended in a measure, to render the succession of the sounds distinguishable, consequently the obstacle which reflects the sound must be at such a distance, that the reverberated sound shall not succeed the direct sound till after one twelfth of a second; and as sound moves at the rate of about 1142 feet in a second, and consequently about 95 feet in the twelfth of a second, it thence follows that, to render the reverberated sound distinguishable from the direct sound, the obstacle must be at the distance at least of about 48 feet.

There are single and compound echoes. In the former only one repetition of the sound is heard; in the latter there are 2, 3, 4, 5, &c, repetitions. We are even told of echoes that can repeat the same word 40 or 50 times.

* If a piece of music, consisting of 60 measures, were executed in a minute, this, in our opinion, would be a rapidity of which there are few instances in the art.

Single echoes are those where there is only one obstacle: for the sound, being impelled backwards, will continue its course in the same direction without returning; but double, triple, or quadruple echoes may be produced different ways. If we suppose, for example, several walls one behind the other, the remotest being the highest; and if each be so disposed as to produce an echo; as many repetitions of the same sound as there are obstacles will be heard.

Another way in which these numerous repetitions may be produced, is as follows: Let us suppose two obstacles, A and B, (fig. 2 pl. 15), opposite to each other, and the productive cause of the sound to be placed between them, in the point S; the sound propagated in the direction from S to A, after returning from A to S, will be driven back by the obstacle B, and again return to S; having then traversed the space SA, it will experience a new repercussion, which will carry it to S after it has struck the obstacle B; and this would be continued in infinitum if the sound did not always become weaker. On the other hand, since the sound is propagated as easily from S to B as from S to A, it will at first be sent back also from B towards S; having then passed over the space SA, it will be repelled from A towards S; then again from B towards S, after having traversed the distance SB, and so on in succession, till the sound dies entirely away.

The sound therefore produced in S will be heard after times, which may be expressed by 2SA; 2SB; 2SB + 2SA; 4SA + 2SB; 4SB + 2SA, 4SA + 4SB; 6SA + 4SB; 6SB + 4SA; 6SA + 6SB, &c; which will form a repetition of the sound after equal intervals, when SA is equal to SB, and even when SB is double SA; but when SA is a third, for example, of SB, this remarkable circumstance will take place, that after the first repetition, there will be a kind of double silence; three repetitions will then follow, at equal intervals; there will then be a silence double one of these intervals; then three repetitions after intervals equal

to the former; and so on till the sound is quite extinguished. The different ratios of the distances SA, SB, will also give rise to different irregularities in the succession of these sounds, which we have thought it our duty to notice, as being possible, though we do not know that they have been ever observed.

There are some echoes that repeat several words in succession; but this is not astonishing, and must always be the case when a person is at such a distance from the echo, that there is sufficient time to pronounce several words before the repetition of the first has reached the ear.

There are some echoes which have been much celebrated on account of their singularity, or of the number of times that they repeat the same word. Misson, in his description of Italy, speaks of an echo at the villa Simonetta, which repeated the same word 40 times. At Woodstock in Oxfordshire there is an echo which repeats the same sound 50 times*.

The description of an echo still more singular near Roseneath, some miles distant from Glasgow, may be found in the Philosophical Transactions for the year 1698. If a person, placed at the proper distance, plays 8 or 10 notes of an air with a trumpet, the echo faithfully repeats them, but a third lower; after a short silence another repetition is heard in a tone still lower; and another short silence is followed by a third repetition, in a tone a third lower.

A similar phenomenon is perceived in certain halls; where, if a person stands in a certain position, and pronounces a few words with a low voice, they are heard only by another person standing in a determinate place. Muschenbroeck speaks of a hall of this kind in the castle of

* This seems to be a mistake: the echo at Woodstock, according to Dr. Plat, repeats in the day time very distinctly 17 syllables, and in the night time 20. *Nat. Hist. of Oxf.* chap. 1, p. 7.

Cleves; and most of those who have visited the Observatory at Paris have experienced a similar phenomenon in the hall on the first story.

Philosophers unanimously agree in ascribing this phenomenon to the reflection of the sonorous rays; which, after diverging from the mouth of the speaker, are reflected in such a manner as to unite in another point. But it may be readily conceived, say they, that as the sound by this union is concentrated in that point, a person whose ear is placed very near will hear it, though it cannot be heard by those who are at a distance.

We do not know whether the hall in the castle of Cleves, of which Muschenbroeck speaks, is elliptical, and whether the two points where the speaker and the person who listens ought to be placed are the two foci; but in regard to the hall in the Observatory of Paris, this explanation is entirely void of foundation. For,

1st. The echoing hall, or as it is called the *Hall of Secrets,* is not at all elliptical; it is an octagon, the walls of which at a certain height are arched with what are called in architecture *cloister arches;* that is to say, by portions of a cylinder which, in meeting, form re-entering angles, that continue those formed by the sides of the octagonal plan.

2d. The person who speaks does not stand at a moderate distance from the wall, as ought to be the case in order to make the voice proceed from one of the foci of the supposed ellipsis: he applies his mouth to one of the re-entering angles, very near the wall, and the person whose ear is nearly at the same distance from the wall, on the side diametrically opposite, hears the one who speaks on the other side, even when he does so with a very low voice.

It is therefore evident, that, in this case, there is no reflection of the voice according to the laws of catoptrics; but the re-entering angle continued along the arch, from

one side of the hall to the other, forms a sort of canal, which contains the voice, and transmits it to the other side. This phenomenon is entirely similar to that of a very long tube, to the end of which if a person applies his mouth and speaks, even with a low voice, he will be heard by a person at the other end.

The case is the same with the whispering gallery, or dome of St. Paul's church, London; or even with the arched recesses on Westminster bridge; the sound is not heard by a person in the intermediate spaces, but only at the opposite point of the dome, or of the opposite recess of the bridge.

The Memoirs of the Academy of Sciences, for the year 1692, speak of a very remarkable echo in the court of a gentleman's seat called le Genetay, in the neighbourhood of Rouen. It is attended with this singular phenomenon, that a person who sings or speaks in a low tone, does not hear the repetition of the echo, but only his own voice; while those who listen hear only the repetition of the echo, but with surprising variations; for the echo seems sometimes to approach and sometimes to recede, and at length ceases when the person who speaks removes to some distance, in a certain direction. Sometimes only one voice is heard, sometimes several, and sometimes one is heard on the right, and another on the left. An explanation of all these phenomena, deduced from the semicircular form of the court, may be seen in the above collection.

ARTICLE V.

Experiments respecting the vibrations of musical strings, which form the basis of the theory of music.

If a string of metal or catgut, such as is used for musical instruments, made fast at one of its extremities, be extended in a horizontal direction over a fixed bridge; and if a weight be suspended from the other extremity, so as to stretch it; this string, when struck, will emit a sound

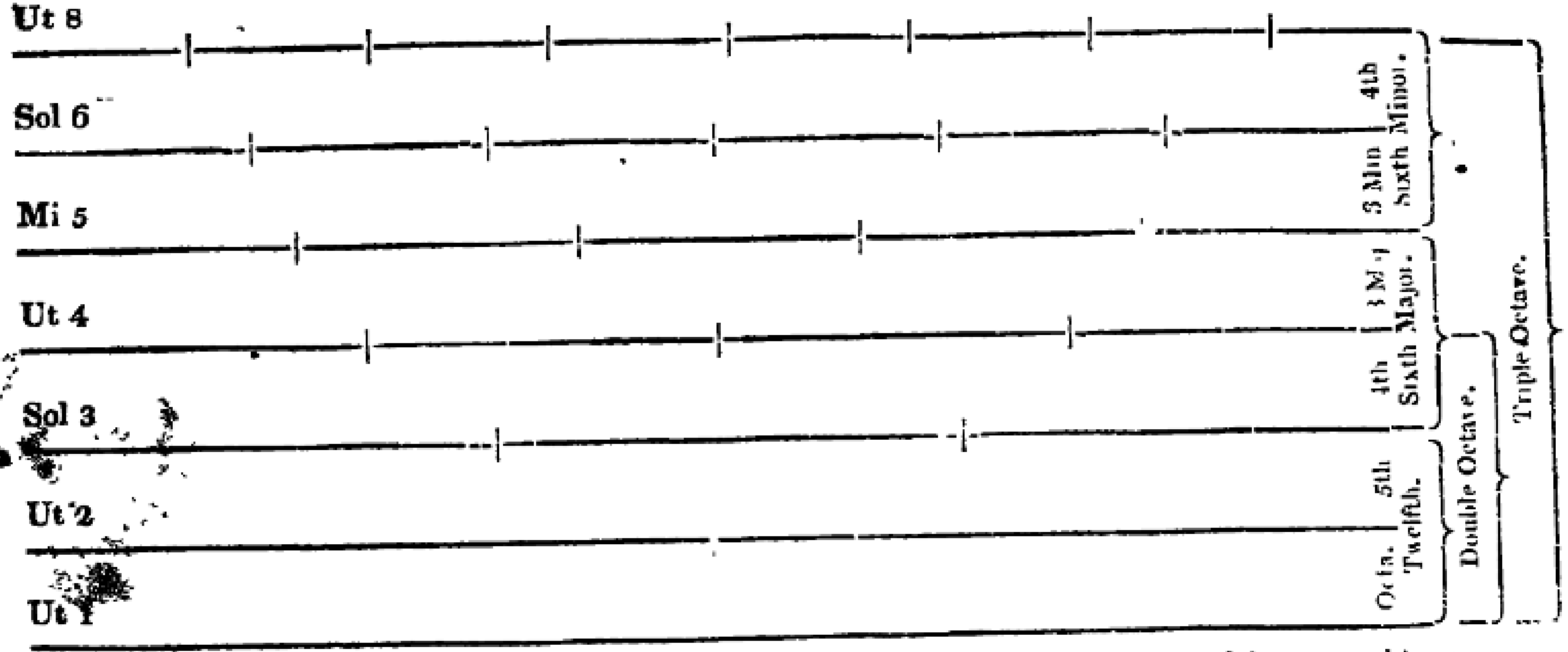
produced by reciprocal vibrations, which are sensible to the sight.

If the part of the string made to vibrate be shortened, and reduced to one half of its length, any person who has a musical ear will perceive, that the new sound is the octave of the former, that is to say twice as sharp.—If the vibrating part of the string be reduced to two thirds of its original length, the sound it emits will be the fifth of the first.—If the length be reduced to three fourths, it will give the fourth of the first.—If it be reduced to $\frac{4}{5}$, it will give the third major; if to $\frac{5}{6}$, the third minor. If reduced to $\frac{8}{9}$, it will give what is called the tone major; if to $\frac{9}{10}$, the tone minor; and if to $\frac{15}{16}$, the semi-tone, or that which in the gamut is between *mi* and *fa*, or *si* and *sol*.

The same results will be obtained if a string be fastened at both ends, and $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of it, be successively intercepted by means of a moveable bridge.

As this subject will be better understood if the reader has a clear idea of the relation of the sounds in the diatonic progression, we shall here insert the following table.

Ingenious Manner in which Rameau expresses the relation of the Sounds in the Diatonic Progression.



It may here be seen that if these seven lines represent seven strings of equal length, the order of the principal harmonic concords will be determined by the following numbers

Thus			4 to 5	denotes	The third major	3 to 5	denotes	The sixth major
1 to 2	denotes	The octave	5 to 6		The third minor	5 to 8		The sixth minor
2 to 3		The fifth	6 to 8		The fourth	1 to 4		The double octave
3 to 4		The fourth	1 to 3		The twelfth	1 to 8		The triple octave.

Such is the result of a determinate degree of tension given to a string, when the length of it has been made to vary. Let us now suppose that the length of the string is constantly the same, but that its degree of tension is varied. The following is what we are taught by experiment on this subject.

If a weight be suspended at one end of a string of a determinate length, made fast by the other, and if the tone it emits be fixed; when another weight quadruple of the first has been applied, the tone will be the octave of the former; if the weight be nine times as heavy, the tone will be the octave of the fifth; and if it be only a fourth part of the first, the tone will be the octave below. Nothing more is necessary to prove that the effect produced, by successively reducing a string to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c, will be produced also by suspending from it in succession weights in the ratio of 4, $\frac{9}{4}$, $\frac{16}{9}$, &c, that is to say, the squares of the weights, or the degrees of tension, must be reciprocally as the squares of the lengths proper for emitting the same tones.

Pythagoras, we are told, was led to this discovery by the following circumstance. Harmonious sounds proceeding from the hammers striking on an anvil in a smith's shop happening one day to reach his ear, while walking in the street, he entered the shop, and found, by weighing the hammers which had occasioned these sounds, that the one which gave the octave was exactly the half of that which produced the lowest tone; that the one which produced the fifth, was two thirds of it, and that the one which produced the third major, was four fifths. When he returned home, meditating on this phenomenon, he extended a string, and after successively shortening it to one half, two thirds, and four fifths, perceived that it emitted sounds which were the octave, the fifth, and the third major, of the tone emitted by the whole string. He then suspended weights from it, and found that those which gave the

octave, the fifth, and the third major, ought to be respectively as 4, $\frac{9}{4}$, $\frac{25}{16}$, to that which emitted the principal tone; that is in the inverse ratio of the squares of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$.

Whatever may be the degree of credit due to this anecdote, which is appreciated as it deserves in the History of the Mathematics, such were the first facts that enabled mathematicians to subject the musical intervals to calculation. The sum of what the moderns have added to them, is as follows: It can be demonstrated at present by the principles of mechanics:

1st. That if a string of a uniform diameter, extended by the same weights, be lengthened or shortened, the velocity of its vibrations, in these two states, will be in the inverse ratio of the lengths. If this string then be reduced to one half of its length, its vibrations will have a double velocity, that is to say, it will make two vibrations for one which it made before; if it be reduced to two thirds, it will make three vibrations, for two which it made before. When a string therefore performs two vibrations, while another performs one, the tones emitted by these strings will be octaves to each other; when one vibrates three times while another vibrates twice, the one will be the fifth to the other, and so on.

2d. The velocity of the vibrations performed by a string, of a determinate length, and distended by different weights, is as the square roots of the stretching weights: quadruple weights therefore will produce double velocity, and consequently double the number of vibrations in the same time; a nonecuple weight will produce vibrations of triple velocity, or a triple number in the same time.

3d. If two strings, differing both in length and in weight, be stretched by different weights, the velocities of their vibrations will be as the square roots of the distending weights, divided by the lengths and the weights of the strings: thus. if the string A, stretched by a weight of 6 pounds, weigh six grains, and be a foot in length; while

the string B, stretched by a weight of 10 pounds, weighs five grains, and is half a foot in length; the velocity of the vibrations of the former, will be to that of the vibrations of the latter, as the square root of $6 \times 6 \times 1$ to that of $5 \times 10 \times \frac{1}{2}$, that is, as the square root of 36, which is 6, to that of 25, or 5: the first therefore will perform 6 vibrations while the second performs 5.

From these discoveries it follows, that the acuteness or gravity of sounds, is merely the effect of the greater or less frequency of the vibrations of the string which produces them; for since we know by experience, on the one hand, that a string when shortened, if subjected to the same degree of tension, emits a more elevated tone; and on the other, both by theory and experience, that its vibrations are more frequent the shorter it is, it is evident that it is only the greater frequency of the vibrations that can produce the effect of elevating the tone.

It thence results also, that a double number of vibrations produces the octave of the tone produced by the single number; that a triple number produces the octave of the fifth; a quadruple number, the double octave; a quintuple the third major, above the double octave, &c; and if we descend to ratios less simple, three vibrations for two will produce the concord of fifth; four for three, that of the fourth, &c.

The ratios of tones therefore may be expressed, either by the lengths of the equally stretched strings which produce them, or by the ratio of the number of the vibrations performed by these strings; thus, if the principal tone be denoted by 1, the octave above is expressed mathematically by $\frac{1}{2}$, or by 2, the fifth by $\frac{2}{3}$ or $\frac{3}{2}$; the third major by $\frac{4}{5}$ or $\frac{5}{4}$, &c. In the first case, the respective lengths of the strings are denoted; in the second the respective numbers of vibrations. In calculation, the results will be the same, whichever method of denomination be adopted.

PROBLEM.

To determine the number of the vibrations made by a string, of a given length and size, and stretched by a given weight; or, in other words, the number of the vibrations which form any tone assigned.

Hitherto we have considered only the ratios of the number of the vibrations, performed by strings which give the different concords; but a more curious, and far more difficult problem, is, to find the real number of the vibrations performed by a string which gives a certain determinate tone; for it may be readily conceived that their velocity will not admit of their being counted. Geometry however, with the help of mechanics, has found means to resolve this question, and the rule is as follows:

Divide the stretching weight by that of the string; multiply the quotient by the length of the pendulum that vibrates seconds, which at London is $39\frac{1}{8}$ inches, or $469\frac{1}{2}$ lines, and divide the product by the length of the string from the fixed point to the bridge; extract the square root of this new quotient, and multiply it by the ratio of the circumference of the circle to the diameter, viz, $3\frac{1}{7}$ nearly, or the fraction $\frac{355}{113}$, in decimals 3·1416 nearly; the product will be the number of the vibrations performed by the string in the course of a second.

Let a string of a foot and a half in length, for example, and weighing 8 grains, be stretched by a weight of 4 pounds Troy weight, or 23040 grains: the quotient of 23040 divided by 8 is 2880; and as the length of the pendulum which swings seconds is $469\frac{1}{2}$ lines, the product of 2880 by this number will be 1352160; if this product be divided by 216, the lines in a foot and a half, we shall have 6260, the square root of which will be 79·1201: this number multiplied by $\frac{355}{113}$ or 3·1416, gives 248·563, which is the number of the vibrations made by the above string in the course of a second.

A very ingenious method, invented by M. Sauveur, for finding the number of these vibrations, may be seen in the Memoirs of the Academy of Sciences for 1700. Having remarked when two organ pipes, very low, and having tones very near to each other, were sounded at the same time, that a series of pulsations or beats was heard in the sounds; by reflecting on the cause of this phenomenon he found, that these beats arose from the periodical meeting of the coincident vibrations of the two pipes. Hence he concluded, that if the number of these pulsations, which took place in a second, could be ascertained by a stop watch, and if it were possible also to determine, by the nature of the consonance of the two pipes, the ratio of the vibrations which they made in the same time, he should be able to ascertain the real number of the vibrations made by each.

We should here suppose, for example, that two organ pipes are exactly tuned, the one to *mi* flat, and the other to *mi*; as it is well known that the interval between these two tones is a semi-tone minor, expressed by the ratio of 24 to 25, the higher pipe will perform 25 vibrations while the lower performs only 24; so that at each 25th vibration of the former or 24th of the latter, there will be a pulsation; if 6 pulsations therefore are observed in the course of 1 second, we ought to conclude that 24 vibrations of the one and 25 of the other are performed in the 10th part of a second: and consequently that the one performs 240 vibrations, and the other 250, in the course of a second.

M. Sauveur made experiments according to this idea, and found that, an open organ-pipe, 5 feet in length, makes 100 vibrations per second; consequently one of 40 feet, which gives the triple octave below, and the lowest sound perceptible to the ear, would make only $12\frac{1}{2}$: on the other hand, a pipe of one inch less $\frac{1}{16}$, being the shortest the sound of which can be distinguished, will give in a second 6400 vibrations. The limits therefore of the

slowest and quickest vibrations, appreciable by the ear, are according to M. Sauveur $12\frac{1}{2}$ and 6400.

We shall not enlarge farther on these details, but proceed to a very curious phenomenon respecting strings in a state of vibration.—Make fast a string at both its extremities, and by means of a bridge divide it into aliquot parts, for example 3 on the one side, and 1 on the other; and put the larger part, that is to say the $\frac{3}{4}$, in a state of vibration; if the bridge absolutely intercepts all communication from the one part to the other, these $\frac{3}{4}$ of the string, as is well known, will give the tone of the fourth of the whole string; if $\frac{4}{5}$ be intercepted, the tone will be the third major.

But if the bridge only prevents the whole of the string from vibrating, without intercepting the communication of motion from the one part to the other, the greater part will then emit only the same sound as the less; and the $\frac{3}{4}$ of the string, which in the former case gave the fourth of the whole string, will give only the double octave, which is the tone proper to the fourth of the string. The case is the same if this fourth be struck: its vibrations, by being communicated to the other three fourths, will make them sound, but in such a manner as to give only this double octave.

The following reason, which may be rendered plain by an experiment, is assigned for this phenomenon: when the bridge absolutely intercepts all communication between the two parts of the string, the whole of the largest part vibrates together, and if it be $\frac{3}{4}$ of the whole string, it makes, agreeably to the general law, 4 vibrations in the time that the whole string would make 3: its sound therefore is the fourth of the whole string.

But in the second case, the larger part of the string divides itself into as many portions as the number of times it contains the less, which in the present example is 3, and each of these portions, as well as the fourth, performs its

particular vibrations: at the points of division, as B, C, D, (fig. 3 pl. 15,) there are established fixed points, between which the portions of the string AB, BC, CD, DE, each vibrate separately, forming alternate bendings in a contrary direction, as if these parts were alone and invariably fixed at their extremities.

This explanation is founded on a fact which M. Sauveur rendered sensible to the eyes in the presence of the Royal Academy of Sciences (*Hist. de l'Acad. année* 1700). On the points C and D, pl. 15 fig. 3, he placed small bits of paper; and having put the small part of the string AB in a state of vibration, the vibrations being communicated to the remaining part BE, the spectators saw, with astonishment, the small bits of paper placed on the points C and D remain motionless, while those placed on the other parts of the string were thrown down.

If the part AB of the string, instead of being exactly an aliquot part of the remainder BE, be for example $\frac{2}{5}$ of it, the whole string AE will divide itself into 7 portions, of which AB will contain two, and each of these portions will vibrate separately, and emit only that sound which belongs to the $\frac{1}{7}$ of the string.

If the parts AB and BE be incommensurable, they will emit a sound absolutely discordant, and which almost immediately ceases on account of the impossibility of bendings and invariable points of rest being established.

ARTICLE VI.

Method of adding, subtracting, multiplying, and dividing concords.

It is necessary for those who wish to understand the theory of music, to know what concords result from two or more concords, either when added or subtracted, &c, by each other. For this reason we shall give the following rules.

PROBLEM I.

To add one concord to another.

Express the two concords by the fractions which represent them, and then multiply these two fractions together; that is to say, first the numerators, and then the denominators; the number thence produced, will express the concord resulting from the sum of the two concords given.

EXAM. I. *Let it be required to add the fourth and fifth together.*

The expression for the fifth is $\frac{2}{3}$, and that for the fourth $\frac{3}{4}$; the product of these two is $\frac{6}{12} = \frac{1}{2}$, being the expression for the octave. It is indeed well known that the octave is composed of a fifth and a fourth.

EXAM. II. *What is the concord arising from the addition of the third major and the third minor?*

The expression for the third major is $\frac{4}{5}$, and that of the third minor is $\frac{5}{6}$, the product of which is $\frac{20}{30}$ or $\frac{2}{3}$, which expresses the fifth; and this concord indeed is composed of a third major and a third minor.

EXAM. III. *What is the concord produced by the addition of two tones major?*

A tone major is expressed by $\frac{8}{9}$, consequently, to add two tones major, $\frac{8}{9}$ must be multiplied by $\frac{8}{9}$. The product $\frac{64}{81}$ is a fraction less than $\frac{64}{80}$ or $\frac{4}{5}$, which expresses the third major; hence it follows, that the concord expressed by $\frac{64}{81}$ is greater than the third major; and consequently two tones major are more than a third major, or form a third major false by excess.

On the other hand, by adding two tones minor, which are each expressed by $\frac{9}{10}$, it will be found that their sum $\frac{81}{100}$ is greater than $\frac{80}{100}$ or $\frac{4}{5}$, which denotes the third major: two tones minor therefore, added together, make more than a third major. This third indeed is composed

of a tone major and a tone minor, as may be proved by adding together the concords $\frac{8}{9}$ and $\frac{9}{10}$, which make $\frac{72}{90}$ = $\frac{8}{10}$ or $\frac{4}{5}$.

It might be proved, in like manner, that two semi-tones major make more than a tone major, and two semi-tones minor less even than a tone minor; and, in the last place, that a semi-tone major and a semi-tone minor, make exactly a tone minor.

PROBLEM II.

To subtract one concord from another.

Instead of multiplying together the fractions which express the given concords, they must here be divided; or invert that which expresses the concord to be subtracted from the other, and then multiply them together as before: the product will give a fraction expressing the quotient, or concord required.

EXAM. I. *What is the concord which results from the fifth subtracted from the octave?*

The expression of the octave is $\frac{1}{2}$, that of the fifth $\frac{2}{3}$, which inverted gives $\frac{3}{2}$; and if $\frac{1}{2}$ be multiplied by $\frac{3}{2}$, we shall have $\frac{3}{4}$, which expresses the fourth.

EXAM. II. *What is the difference between the tone major and the tone minor?*

The tone major is expressed by $\frac{8}{9}$, and the tone minor by $\frac{9}{10}$, which when inverted gives $\frac{10}{9}$; the product of $\frac{8}{9}$ by $\frac{10}{9}$ is $\frac{80}{81}$, which expresses the difference between the tone major and the tone minor: this is what is called the *great comma.*

PROBLEM III.

To double a concord, or to multiply it any number of times at pleasure.

In this case, nothing is necessary but to raise the terms of the fraction, which expresses the given concord, to the

power denoted by the number of times it is to be multiplied; that is, to the square if it is to be doubled, to the cube if to be tripled, and so on.

Thus, the concord arising from the tone major tripled is $\frac{512}{729}$, for as the expression of the tone major is $\frac{8}{9}$, we shall have $8 \times 8 \times 8 = 512$, and $9 \times 9 \times 9 = 729$. This concord $\frac{512}{729}$ corresponds to the interval between *ut* and a *fa* higher than *fa* sharp of the gammut.

PROBLEM IV.

To divide one concord by any number at pleasure, or to find a concord which shall be the half, third, &c, of a given concord.

To answer this problem, take the fraction which expresses the given concord, and extract that root of it which is denoted by the determinate divisor: that is to say, the square root if the concord is to be divided into two; the cube root, if it is to be divided into three, &c; and this root will express the concord required.

EXAMPLE.—As the octave is expressed by $\frac{1}{2}$, if the square root of it be extracted, it will give $\frac{7}{10}$ nearly; but $\frac{7}{10}$ is less than $\frac{3}{4}$, and greater than $\frac{2}{3}$; consequently the middle of the octave is between the fourth and the fifth, or very near *fa* sharp.

ARTICLE VII.

Of the resonance of sonorous bodies; the fundamental principle of harmony and melody; with some other harmonical phenomena.

EXPERIMENT I.

If you listen to the sound of a bell, especially when very grave, however indifferent your ear may be, you will easily distinguish, besides the principal sound, several others more acute; but if you have an ear accustomed to appreciate the musical intervals, you will perceive that one of these sounds is the twelfth or fifth above the octave,

and another the seventeenth major, or third major above the double octave. If your ear be exceedingly delicate, you will distinguish also its octave, its double, and even its triple octave: the latter indeed are somewhat more difficult to be heard, because the octaves are almost confounded with the fundamental sound, in consequence of that natural sensation which makes us confound the octave with unison.

The same effect will be perceived if the bow of a violoncello be strongly rubbed against one of its large strings, or the string of a trumpet-marine.

In short, if you have an experienced ear, you will be able to distinguish these different sounds, either in the resonance of a string, or in that of any other sonorous body, and even in the voice.

Another method of making this experiment.

Suspend a pair of tongs by a woollen or cotton cord, or any other kind of small string, and twisting the extremities of it around the fore finger of each hand, put these two fingers into your ears. If the lower part of the tongs be then struck, you will first hear a loud and grave sound, like that of a large bell at a distance; and this tone will be accompanied by several others more acute; among which, when they begin to die away, you will distinguish the twelfth and the seventeenth of the lowest tone.

The truth of this phenomenon, in regard to the multiplicity of sounds, is confirmed by another experiment, mentioned by Rameau, in his *Harmonical generation.* If you take, says he, those stops of the organ called *bourdon*, *prestant* or *flute*, *nazard* and tierce, which form the octave, the twelfth and seventeenth major of the *bourdon*, and if you draw out in succession each of the other stops, while the *bourdon* alone is sounding, you will hear their sounds successively mixed with each other; you may even distinguish them while they are all sounding together; but if

you prelude for a moment, by way of amusement, on the same set of keys, and then return to the single key first touched, you will think you hear only one tone, that of the bourdon, the gravest of all which corresponds to the sound of the whole system.

REMARK.—This experiment, respecting the resonance of sonorous bodies, is not new. It was known to Dr. Wallis, and to Mersenne, who speak of it in their works; but it appeared to them a simple phenomenon, with the consequences of which they were entirely unacquainted. Rameau first discovered its use in deducing from it all the rules of musical composition, which before had been founded on mere sentiment, and on experience, incapable of serving as a guide in all cases, and of accounting for every effect. It forms the basis of his theory of fundamental bass, a system which has been opposed with much declamation, but which however most musicians seem at present to have adopted.

All his harmony then is multiple, and composed of sounds which would be produced by the aliquot parts of the sonorous body $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and we might add $\frac{1}{7}$, $\frac{1}{8}$, &c. But the weakness of these sounds, which go on always decreasing in strength, renders it difficult to distinguish them. Rameau however says that he could distinguish very plainly the sound expressed by $\frac{1}{7}$, which is the double octave of a sound divided nearly into two equal parts, being the interval between *la* and *si* flat below the first octave: he calls it a lost sound, and totally excludes it from harmony. It would indeed be singularly discordant with all the other sounds given by the fundamental tone.

We must however observe that the celebrated Tartini, in regard to this sound, was not of the same opinion as Rameau. Instead of calling it a lost sound, he maintains that it may be employed in melody as well as harmony; he distinguishes it by the name of the *seventh consonant*. But we shall leave it to musicians to appreciate this idea of

Tartini, whose celebrity in composition, as well as execution, required a refutation of a different kind, from that to be found at the end of a work printed in 1767, entitled *Histoire de la Musique*.

EXPERIMENT II.

If you tune several strings to the octave, to the twelfth, and to the seventeenth major, of the determinate sound emitted by another string, both ascending and descending; as often as you make that which gives the determinate sound to resound strongly, and with continuance, you will immediately see all the rest put themselves in a state of vibration: you will even hear those sound which are tuned lower, if care be taken to damp suddenly, by means of a soft body, the sound of the former.

Most persons have heard the glasses on a table sound, when a person near them has been singing with a strong and a loud voice. The strings of an instrument, though not touched, are often heard to sound, in consequence of the same cause, especially after swelling notes long continued.

This phenomenon arises, no doubt, from the vibrations of the air being communicated to the string, or to the sonorous body, elevated to the above tones: for it may be easily conceived that the vibrations of strings, tuned to unison or to the octave, or to the twelfth, &c, of that put in motion, are disposed to recommence regularly, and at the same time as those of that string, one vibration corresponding to another, in the case of unison; two to one, in the case of the octave; or three to one in that of the twelfth: the small impulsions therefore of the vibrating air, produced by the string put in motion, will always concur to increase those movements, at first insensible, which they have occasioned in the other strings; because they will take place in the same direction, and will at length render them sensible. Thus a gentle breath of air, continued

always in the same direction, is at length able to elevate the waters of the ocean. But when the strings in question are stretched in such a manner, that their vibrations can have no correspondence with those of the string which is struck, they will in this case be sometimes assisted and sometimes opposed, and the small movement which can be communicated to them, will be annihilated as soon as produced, consequently they will remain at rest.

QUESTION.

Do the sounds heard with the principal sound derive their source immediately from the sonorous body, or do they reside only in the air or the organ?

It is very probable that the principal sound is the only one that derives its origin immediately from the vibrations of the sonorous body. Philosophers of eminence have endeavoured to discover whether, independently of the total vibrations made by the body, there are not also partial vibrations; but hitherto they have been able to observe only simple vibrations. Besides, how can it be conceived that the whole of a string should be in vibration, and that during its motion it should divide itself into two or three parts that perform also their distinct vibrations?

It must then be said that these harmonical sounds of octave, twelfth, seventeenth, &c, are in the air or the organ: both suppositions are probable; for since a determinate sound has the property of putting into a state of vibration bodies disposed to give its octave, its twelfth, &c, we must allow that this sound may put in motion the particles of the air susceptible of vibrations of double, triple, quadruple, and quintuple velocity. What however appears most probable in this respect is, that these vibrations exist only in the ear: it seems indeed to be proved, by the anatomy of this organ, that sound is transmitted to the soul only by the vibrations of those nervous fibres which cover the interior part of the ear; and as they are of dif-

ferent lengths, there are always some of them which perform their vibrations isochronous to those of the given sound. But, at the same time, and in consequence of the property above mentioned, this sound must put in motion those fibres which are susceptible of isochronous vibrations; and even those which can make vibrations of double, triple, quadruple, &c, velocity. Such, in our opinion, is the most probable explanation that can be given of this singular phenomenon.

EXPERIMENT III.

For this experiment we are indebted to the celebrated Tartini of Padua. If you draw from two instruments, at the same time, any two sounds whatever, you will hear in the air a third, which will be the more perceptible the nearer your ear is placed to the middle of the distance between the two instruments. Let us suppose then, for example, two sounds which succeed each other in the order of consonances, as the octave and the twelfth, the double octave and the seventeenth major, &c; the sound resulting, says Tartini, will be the octave of the principal sound.

This experiment was repeated in France with the same success, as we are assured by M. Serres, in his *Principes de l'Harmonie*, printed in 1753; but with this exception, that M. Serres found the latter sound to be lower by an octave. As the octaves are easily confounded, this difference needs excite no surprize. We must however here observe, that the celebrated musician of Padua, established on this phenomenon a system of harmony and composition; but it does not seem to have met with so favourable a reception as that of Rameau.

ARTICLE VIII.

Of the different systems of Music; the Grecian and the Modern, together with their peculiarities.

§ I.

Of the Grecian Music.

During the infancy of music among the Greeks, their lyre had four strings, the sounds of which would have corresponded to *si, ut, re, mi*; but they afterwards added other three *fa, sol, la*. The first diatonic scale therefore of the Greeks, translated into our musical language, was *si, ut, re, mi, fa, sol, la*, and was composed of two tetrachords, or systems of four sounds, *si, ut, re, mi*; *mi, fa, sol, la*; in which the last of the one and the first of the other were common, and on this account they were called *conjunct tetrachords*.

We must here observe that, however singular this disposition of sounds may appear to those who are acquainted only with the modern diatonic order, it is no less natural and agreeable to the rules of harmony; for Rameau has shown that it is nothing else than a chant, the fundamental bass of which would be, *sol, ut, sol, ut, fa, ut, fa*. It possesses also the advantage of having only one altered interval, viz the third minor from *re* to *fa*, which, instead of being in the ratio of 5 to 6, is in that of 27 to 32; which is somewhat less, and consequently too low by a *comma* of from 80 to 81.

But this perfection in the Grecian gammut was counterbalanced by two great imperfections, viz 1st. that it did not complete the octave; 2d. that it did not terminate by a rest, which leaves to the ear that kind of uneasiness resulting from a song begun and not finished. It could neither ascend to *si*, nor descend to *la*; and therefore the musicians who, to complete the octave, added the latter note below, considered it to be foreign, as we may say, and give it the name of *proslambanomenos*.

For this reason they endeavoured to discover another remedy for this defect, and Pythagoras, as is said, proposed the succession of sounds *mi, fa, sol, la*; *si, ut, re*,

mi, composed as it appears of two *disjunct tetrachords*. This diatonic scale is almost the same as ours, with this difference, that ours begins and ends with the tonic note, while the former begins and ends with the mediante, or third major. This termination, almost reprobated at present, was very common among the Greeks, and is still so in the chants or vocal music of our churches.

But here, in consequence of the harmonic generation, the values of the sounds and intervals are not the same as in the first scale. In the first, the interval from *sol* to *la* was a tone minor, in the second it is a tone major. In the last place, according to this second arrangement there are three intervals altered or false, viz the tierce major, from *fa* to *la*, too high; the tierce minor, from *la* to *ut*, too low; and the fifth, from *la* to *mi*, too high. These are the same faults as those of our diatonic scale; but the temperament corrects them.

To these sounds the Greeks afterwards added a conjunct tetrachord descending, *si*, *ut*, *re*, *mi*, and another ascending, *mi*, *fa*, *sol*, *la*; by which they nearly supplied all the wants of melody, so far as it was confined to one tone. Ptolemy speaks of a combination, by means of which they joined the second primitive tetrachord to the first, lowering the *si* a semitone, which made *si* flat, *ut*, *re*, *mi*. This, no doubt, answered the purpose when they passed from the tone of *ut* to that of its lower fifth *fa*; a transition common in the Grecian music, as well as in our church music; for in that case a *si* flat is required. Plutarch also speaks of a combination where the two last tetrachords were disjointed, by raising the *fa* a semi-tone, and that no doubt of its lower octave. Who does not here perceive our *fa**, which is necessary when we pass from the tone of *ut* to that of its upper fifth *sol*? The strings which corresponded to *si* flat and *fa* sharp, were no doubt merely added, and not substituted in the room of *si* and *fa*.

It is well known that in the Grecian music there were

three genera, viz. the diatonic, chromatic, and enharmonic. What has been hitherto said relates only to the diatonic.

What characterises the enharmonic is, that it employs, either ascending or descending, several semitones in succession. The chromatic gammut of the Greeks was *si*, *ut*, *ut* sharp, *mi*, *fa*, *fa* sharp, *la*. This disposition, by which they passed immediately from *ut* sharp to *mi*, omitting the *re* must no doubt appear very strange; but it is certain that this was the gammut employed by the Greeks in the chromatic genus. It is however not known whether the Greeks had considerable pieces of music of this kind, or whether, like us, they employed it only in very short passages of cantatas; for we also have a chromatic kind, though in a different acceptation. This transition from semi-tones to semi-tones, is less natural than the diatonic succession; but it has more energy to express certain peculiar sensations: the Italians therefore, who are great colorists in music, make frequent use of it in their airs.

In regard to the enharmonic music of the Greeks, though considered by the ancients as the most perfect kind, it is to us still an enigma. To give some idea of it, let us assume the sign *, as that of the enharmonic diesis or sharp, which raises the note a quarter of a tone: the enharmonic scale then was *si*, *si**, *ut*, *mi*, *mi**, *fa*, *la*, where it appears that, after two fourths of a tone, from *si* to *ut*, or from *mi* to *fa*, they proceed to *mi* or *la*. It can hardly be conceived how there could be ears so well exercised as to appreciate fourths of a tone, and even if we suppose that there were, what modulation could they make with these sounds? It is however very certain that this kind of music was long held in high estimation in Greece; but on account of its difficulty it was at length abandoned, so that not even a fragment of Grecian music in the enharmonic kind has been handed down to us; nor any in the chromatic, though we have some in the diatonic.

We must however here observe, that this enharmonic

music of the Greeks is not perhaps so remote from nature as has been hitherto supposed; for does not Tartini, in proposing the use of his consonant seventh, which is nearly a mean sound between *la* and *si* flat, pretend that this intonation, *la*, si^{bb}, si^{b}, *re*, *re*, si^{b}, si^{bb}, *la*, is not only supportable, but highly agreeable. Tartini does more; for he assigns to this succession the sounds of its bass, *fa*, *ut*, *sol*, *sol*, *ut*, *fa*, marking *ut* with this sign b^7, which signifies consonant seventh. If this pretension of Tartini should find partizans, may we not say that the enharmonic music of the Greeks has been revived?

It now remains that we should say a few words respecting the modes of the Grecian music. However obscure this matter may be, if we can believe the author of *Histoire des Mathematiques*, who founds his ideas on certain tables of Ptolemy, these modes are nothing else than the tones of our music, and he gives the following comparison.

The Dorian being taken hypothetically for the mode of *ut*, these modes, some lower than the Dorian and others higher, were:

The Hypodorian . . .	corresponding to *sol*	
The Hypophrygian	*la* flat	
The Hypophrygian acutior	*la*	
The Hypolydian or Hypo-æolian . . .	*si* flat	
The Hopolydian acutior	*si*	
The Dorian	*ut*	
The Iastian or Ionian	*ut* sharp	
The Phrygian	*re*	
The Æolian	*re* sharp	
The Lydian	*mi*	
The Hyperdorian	*fa*	
The Hyperiastian or Mixolydian . .	*fa* sharp	
The Hypermixolydian	*sol*	The Replicate of the first.

But this question might be asked: If the difference of the

Grecian modes consisted in the greater or less height of the tone of the modulation, how can we explain, what is told us of the characters of these different modes, some of which excited fury, others appeased it, &c? There is reason therefore to think that they depended on something more; and it is not improbable, that besides difference of tone, there was a character of modulation peculiar to each. The Phrygian, for example, which originated among a hardy and warlike people of that name, had a masculine and warlike character, while the Lydian, which was derived from a soft and effeminate people, had an analogous character, and consequently was proper for calming the transports excited by the former.

As we have here said enough respecting the Grecian music, we shall now proceed to the modern.

§ II.

Of the Modern Music.

Every person acquainted with music knows, that the gammut, or diatonic scale of the moderns, is represented by these sounds, *ut, re, mi, fa, sol, la, si, ut*, which complete the whole extent of the octave*; and, we shall add, that from its generation, as explained by Rameau, it follows that between *ut* and *re* there is a tone major; from *re* to *mi*, a tone minor; from *mi* to *fa*, a semitone major; from *fa* to *sol* a tone major, as well as from *sol* to *la*; from *la* to *si* a tone minor, and from *si* to *ut* a semitone major.

Hence it is concluded, that in this scale there are three intervals, which are not entirely just, viz, the third minor from *re* to *fa*, and indeed, being composed of a tone minor and a semitone major, it is only in the ratio of 27 to 32,

* Of the seven notes in the French scale *Ut, re, mi, fa, sol, la, si*, four only are generally used among us, as *mi, fa, sol, la*, which are applied to the scale in this order, *fa, sol, la, fa, sol, la, mi, fa*, and express the natural series from C. It is of little consequence however which method be used; the principles still remain the same.

which is somewhat less, viz, an 80th, than that of 5 to 6, the just ratio of the sounds which compose the third minor.

In like manner, the third major, from *fa* to *la*, is too high, being composed of two tones major, whereas it ought to be composed of a tone major and a tone minor, to be exactly in the ratio of 4 to 5. In the last place, the third minor, from *la* to *ut*, is also altered, and for the same reason as that from *re* to *fa*.

If this disposition of tones major and minor were arbitrary, they might no doubt be arranged in such a manner that fewer intervals should be altered; it would be sufficient for this purpose to make the tone from *ut* to *re* minor, and that from *re* to *mi* major; the tone from *sol* to *la* might also be made minor, and that from *la* to *si* major. For it will be found, that by this method there would be no more than a single third altered; whereas, according to the other disposition, there are three. This circumstance has given rise to disputes among the musicians respecting the distribution of the tones minor and major; some being desirous, for example, that there should be a tone major between *ut* and *re*, and others a tone minor. The harmonic generation of the diatonic scale, as explained by Rameau, will not however allow this disposition, but only the former, which is that indicated by nature; and notwithstanding its imperfections, which the temperament corrects, in the execution, it is preferable to the first of the Grecian scales, a scale very deficient, as it did not comprehend the whole extent of the octave; it is superior also to the second, *mi, fa, sol*, &c, ascribed to Pythagoras, because its desinence is more perfect, and conveys to the ear a rest, which is not in that of Pythagoras, on account of its fall on the tonic note, announced and preceded by the note *si*, the third of the fifth *sol*, the effect of which is so striking to our musical ears, that it has been distinguished by the name of the *sensible note*.

Two modes, properly so called, are known in music

the characters of which are exceedingly striking to ears possessed of any musical sensibility: these are the *major mode* and the *minor mode*. The major mode is, when in the diatonic scale the third of the tonic note is major; such is the third from *ut* to *mi*. The above gammut, or diatonic scale, therefore is in the major mode.

But if the third of the tonic note be minor, it indicates the minor mode. This mode has its scale as well as the major. Thus, for example, if we assume *la* as the tonic note, the scale of the minor mode ascending will be *la*, *si*, *ut*, *re*, *mi*, *fa*, *sol**, *la*. We here make use of the term ascending, because it is a singularity of the minor mode, that its scale descending, is different from what it is ascending; and indeed in descending we ought to say *la*, *sol*, *fa*, *mi*, *re*, *ut*, *si*, *la*. If the tone were in *ut*, the ascending scale would be *ut*, *re*, *mi*^{b}, *fa*, *sol*, *la*^{b}, *si*, *ut*, and descending *ut*, *si*^{b}, *la*^{b}, *sol*, *fa*, *mi*^{b}, *re*, *ut*. Hence the reason why, in airs in the minor mode, we so often find, without the tone being changed, accidental *flats* or *sharps*, or *naturals*, which soon destroy their effect, or that of those which are in the clef. This is one of those singularities, of the necessity of which the ear made musicians sensible: the cause of it however, which depends on the progress of the fundamental bass, was first explained by Rameau.

To these two modes shall we add a third, proposed by M. de Blainville, under the name of the *mixed mode*, the generation and properties of which he explains in his History of Music? His scale is *mi*, *fa*, *sol*, *la*, *si*, *ut*, *re*, *mi*. We shall here only observe, that musicians do not seem to have given a very favourable reception to this new mode, and we confess that we are not sufficiently versed in these matters to be able to decide whether they are right or wrong. But however this may be, the character of the major mode is sprightliness and gaiety; while in the minor

mode there is something gloomy and sad, which renders it peculiarly fitted for expressions of that kind.

The modern music has its genera as well as the ancient. The diatonic is the most common; and is that most agreeable to what is pointed out by nature; but the moderns have their chromatic also, and even in certain respects their enharmonic, though in a sense somewhat different from that assigned to these words by the ancients.

The modulation is chromatic when several semitones are passed over in succession, as if we should say *fa*, *mi*, *mi*$^{\flat}$, *re*, or *sol*, *fa**, *fa mi*. It is very rare to have more than three or four semitones following each other in this manner; yet in an air of the second act of *la Zingara*, or the Gypsy, an Italian *intermede*, there is a whole lower octave almost from *ut* to *re* in consecutive semitones. It is the longest chromatic passage with which we are acquainted.

Rameau finds the origin of this progression in the nature of the fundamental bass, which, instead of proceeding from fifth to fifth, which is its natural movement, proceeds from third to third. But it must here be remarked, that in the first passage from *mi* to *mi*$^{\flat}$, there ought strictly to be only a semitone minor, and from *mi*$^{\flat}$, to *re* a semitone major; but the temperament and constitution of most instruments, by confounding the *re** with *mi*$^{\flat}$, divide into equal parts the interval from *re* to *mi*, and the ear is affected by them exactly in the same manner, especially by means of the accompaniment.

There are two enharmonic genera, the one called the *diatonic enharmonic*, and the other the *chromatic enharmonic*, but they are very rarely employed by musicians. These genera are not so called because quarters of a tone are employed in them, as in the ancient enharmonic; but because, from the progress of the fundamental bass, there result sounds, which, though taken one for the other,

really differ a quarter of a tone, called by the ancients enharmonic, or are in the ratio of 125 to 128. In the diatonic enharmonic, the fundamental bass goes on alternately by fifths and thirds, and in the chromatic enharmonic it goes on alternately by third major and minor. This progression introduces, both into the melody and the harmony, sounds which, belonging neither to the principal tone nor its relatives, convey astonishment to the ear, and affect it in a harsh and extraordinary manner, but which are proper for certain terrible and violent expressions. It was for this reason that Rameau employed the diatonic enharmonic in the trio of the Fates, in his opera of *Hippolitus and Aricia*; and though he was not able to get it executed, he was firmly persuaded that it would have produced a powerful effect had he found performers disposed to fall into his ideas, so that he suffered it to remain in the partition which was printed. He mentions, as a piece of the enharmonic kind, a scene of the Italian opera of *Coriolano*, beginning with these words, *O iniqui Marmi!* which he says is admirable. Specimens of this genus are to be found also in two of his own pieces for the harpsichord, the *Triumphante* and the *Enharmonique*, and he did not despair of being able to employ the chromatic enharmonic at least in symphonies. And why indeed might he not have done so, since Locatelli, in his first concertos, employed this genus, leaving the flats and sharps to exist, and distinguishing for example the *re* from *mi* ♭. This, says a modern historian of music, M. de Blainville, is a piece truly infernal, which throws the soul into a violent state of apprehension and terror.

We cannot terminate this article better than by giving a few specimens of the music of different nations. For this purpose we have caused to be engraved some Grecian, Persian, Chinese, Armenian and Tartar airs, which will serve to give an idea of the modulation that characterises the music of these people (plate 16).

ARTICLE IX.

Musical Paradoxes.

§ I.

It is impossible to intonate justly the following intervals, sol, ut, la, re, sol; *that is to say, the interval between* sol *and* ut *ascending, that from* ut *to* la *re-descending from third minor, then ascending from fourth to* re, *and that between* re *and* sol *descending from fifth, and to make the second* sol *in unison with the first.*

It will be found indeed by calculation, that if the first *sol* be represented by 1, the *ut*, ascending from fourth, will be $\frac{3}{4}$; consequently the *la*, descending from third minor, will be $\frac{9}{10}$; the *re* above then will be $\frac{27}{40}$, and in the last place the *sol*, descending from fifth, will be $\frac{81}{80}$. But the sound represented by $\frac{81}{80}$, is lower than that represented by 1, therefore the last *sol* is lower than the first.

But how comes it that experience is contrary to this calculation? In answer to this question we shall observe, that the difference arises merely from the remembrance of the first tone *sol*. If the ear however were not affected by this tone, and if the performer's whole attention were directed to the just intonation of the above intervals, it is evident that he would end with a lower *sol*. It therefore often happens that a voice, without an accompaniment, after having chanted a long air, in which several tones are passed through, remains, in ending, higher or lower than the tone by which it began.

This arises from the necessary alteration of some intervals in the diatonic scale. In the preceding example, from *la* to *ut*, there is only a third minor in the ratio of 27 to 32, and not of 5 to 6; but it is the latter which is intonated if the voice be true and well exercised; consequently the person who chants, lowers by a comma more than is necessary, and therefore it is not astonishing that

the last *sol* should always be lower, by a comma, than the first.

§ II.

In instruments constructed with keys, such as the harpsichord, it is impossible that the thirds and the fifths should be both just.

This may be easily demonstrated in the following manner.—Let there be a series of tones, fifths to each other ascending, as *ut*, *sol*, *re*, *la*, *mi*; if *ut* be denoted by 1, *sol* will be $\frac{2}{3}$, *re* $\frac{4}{9}$, *la* $\frac{8}{27}$, *mi* $\frac{16}{81}$: this *mi* ought to form the third major with the double octave of *ut* or $\frac{1}{4}$, that is to say they ought to be in the ratio of 1 to $\frac{4}{5}$, or of 5 to 4, or of 80 to 64; but this is not the case, for $\frac{1}{4}$ and $\frac{16}{81}$ are to each other as 81 to 64: this *mi* therefore does not form the third major with the double octave of *ut*; or if both are lowered from the double octave, *ut* and *mi* are not thirds to each other, if *mi* is a just fifth to *la*.

In instruments with keys then, such as the harpsichord, however well tuned, all the intervals, the octaves excepted, are either false or altered. This necessarily follows from the manner in which that instrument is tuned; for when all the *ut's* are made octaves to each other, as they ought to be, the *sol* is made the fifth to *ut*, *re* the fifth to *sol*, and the octave is lowered, because it is too high; *la* is then made the fifth to *re*, thus lowered, and *mi* the fifth to *la*, and this *mi* is lowered from octave. By continuing in this manner to ascend twice from fifth, and then to descend from octave, the series of sounds *si*, *fa* , *ut* , *sol**, *re**, *la**, *mi**, *si** are obtained. But the latter *si* , which ought at most to be in unison with the *ut*, the octave of the first, is found to be higher; for calculation shows that it is expressed by $\frac{262144}{531441}$, which is less than $\frac{1}{2}$ the value of the octave of *ut*; this renders necessary what is called temperament, which consists in lowering gently and equally all the fifths, so that the latter *si** is found to be exactly the octave of

the first *ut*. Such at least is the method taught by Rameau, and it is no doubt the most rational. But whatever may be the method employed, it always consists in rejecting in a more or less equal manner from the notes of the octave, this excess of *si** above *ut*, which cannot be done without altering, in some measure, the fifths, thirds, &c.

We have just seen that the *si**, given by the progression of fifths, is higher than *ut*; but if the following progression of thirds be employed, *ut*, *mi*, *sol**, *si**, this *si** will be very different from the former; for it will be found that it is expressed by $\frac{64}{125}$, while the octave of *ut* is $\frac{1}{2}$. But $\frac{1}{2}$ is less than $\frac{64}{125}$, consequently this *si** is below *ut* expressed by $\frac{1}{2}$, and the interval of these two sounds is expressed by the ratio of 128 to 125, which is the fourth of the enharmonic tone.

§ III.

A lower note, for example re, *affected by a sharp, is not the same thing as the higher note*, mi, *affected by a flat; and the case is the same with other notes which are a whole tone distant from each other.*

The sharps are generally given by the major mode, and even by the minor, provided the subtonic note is not distant from the tonic more than a semitone major, as the *si* is from *ut*, in the tone of *ut*; then, as from *re* to *mi* there is a tone minor, which is composed of a semitone major and a semitone minor, if we take away a semitone major, by which *re* ought to be lower than *mi*, the remainder will be a semitone minor, by which the same *re** ought to be higher than *re*. If the distance between the notes were a tone major, the sharp would raise the lower note by an interval equal to a semitone minor, plus a comma of 80 to 81, which is a mean semitone between the major and the minor. The note therefore is raised by the sharp only a mean semitone, or a semitone minor.

Flats are generally introduced in modulation by the

minor mode, when it is necessary to lower the note a third, so that it shall form with the tonic a third minor: *mi* flat therefore ought to form with *ut* a third minor: consequently if from the third major *ut mi*, which is $\frac{4}{5}$, we take the third minor, which is $\frac{5}{6}$, the remainder $\frac{24}{25}$ is the quantity which expresses how much the flat lowers the *mi* below the natural tone: *mi* flat then is higher than *re* sharp.

In practice however the one is taken for the other, especially in instruments constructed with keys: the flat in these is lowered, and the sharp gradually raised, till they coincide with each other; and we do not know whether practice would gain much by making a distinction between them.

ARTICLE X.

On the cause of the pleasure arising from music—The effects of it on man and on animals.

It has often been asked, why two sounds, which form to each other the fifth and the third, excite pleasure, while the ear experiences a disagreeable sensation by hearing sounds which are no more than a tone or a semitone distant from each other. Though it is difficult to answer this question, the following observations may tend to throw some light on it.

Pleasure, we are told, arises from the perception of relations, as may be proved by various examples taken from the arts. The pleasure therefore derived from music, consists in the perception of the relations of sounds. But are these relations sufficiently simple for the soul to perceive and distinguish their order? Sounds will please when heard together in a certain order; but, on the other hand, they will displease if their relations are too complex, or if they are absolutely destitute of order.

This reasoning will be sufficiently proved by an enumeration of the known concords. In unison, the vibrations of two sounds continually coincide throughout the whole time of their duration; this is the simplest kind of relation.

Unison also is the first concord. In the octave, the two sounds, of which it is composed, perform their vibrations in such a manner, that two of the one are completed in the same time as two of the other. Thus unison is succeeded by the octave. It is so natural to man, that he who, through some defect in his voice, cannot reach a sound too grave or too acute, falls into the higher or lower octave.

When the vibrations of two sounds are performed in such a manner, that three of the one correspond to one of the other, these give the simplest relation, next to those above mentioned. Who does not know, that the concord most agreeable to the ear is the twelfth, or the octave of the fifth? In this respect it even surpasses the fifth, the ratio of which, a little more compounded, is that of 2 to 3.

Next to the fifth is the double octave of the third, or the seventeenth major, which is expressed by the ratio of 1 to 3. This concord therefore, next to the twelfth, is the most agreeable; and if it be lowered from the double octave, to obtain the third, it will still be in consonance; the ratio of 4 to 5, by which it is then expressed, being very simple.

In the last place, the fourth, expressed by $\frac{3}{4}$, the third minor, expressed by $\frac{5}{6}$, and the sixths, both major and minor, expressed by $\frac{5}{8}$ and $\frac{3}{5}$, are concords, and for the same reason.

But it appears that all the other sounds, after these relations, are too complex for the soul to perceive their order: of this kind are the intervals called the tone major and the tone minor, expressed by $\frac{8}{9}$ and $\frac{9}{10}$, and much more so the semitones major and minor, expressed by $\frac{15}{16}$ and $\frac{24}{25}$. Such also are the concords of third and fifth, however little they may be altered; for the third major, raised a comma, is expressed by $\frac{27}{32}$; and the fifth diminished by the same quantity, has for its expression $\frac{27}{40}$: in the last place, the tritone, as from *ut* to *fa**, is one of the most disagreeable discords, and is expressed by $\frac{32}{45}$.

The following very strong objection however may be made to this reasoning. How can the pleasure arising from concords consist in the perception of relations, since the soul often does not know whether such relations exist between the sounds? The most ignorant person is no less pleased with a harmonious concert than he who has calculated the relation of all its parts: what has hitherto been said may therefore be more ingenious than solid.

We cannot help acknowledging that we are rather inclined to think so; and it appears to us that the celebrated experiment on the resonance of sonorous bodies, may serve to account, in a still more plausible manner, for the pleasure arising from concords; because, as every sound degenerates into mere noise, when not accompanied by its twelfth and its seventeenth major, besides its octaves, is it not evident that, when we combine any sound with its twelfth or its seventeenth major, or with both at the same time, we only imitate the process of nature, by giving to that sound, in a fuller and more sensible manner, the accompaniment which nature itself gives it, and which cannot fail to please the ear on account of the habit it has acquired of hearing them together? This is so agreeable to truth, that there are only two primitive concords, the twelfth and the seventeenth major; and that the rest, as the fifth, the third major, the fourth, and the sixth, are derived from them. We know also that these two primitive concords are the most perfect of all, and that they form the most agreeable accompaniment that can be given to any sound; though on the harpsichord, for example, to facilitate execution, the third major and the fifth itself, which with the octave form what is called perfect harmony, are substituted in their stead. But this harmony is perfect only by representation, and the most perfect of all would be that in which the twelfth and the seventeenth were combined with the fundamental sound and its octaves. Rameau therefore adopted it as often as he could in his

choruses, and particularly in his Pygmalion. We might enlarge farther on this idea, but what has been already said will be sufficient for every intelligent reader.

Some very extraordinary things are related in regard to the effects produced by the music of the ancients, which on account of their singularity we shall here mention. We shall then examine them more minutely, and show that, in this respect, the modern music is not inferior to the ancient.

Agamemnon, it is said, when he set out on the expedition against Troy, being desirous to secure the fidelity of his wife, left her under the care of a Dorian musician, who by the effect of his airs rendered fruitless, for a long time, the attempts of Ægisthus to obtain her affection; but that Prince having discovered the cause of her resistance, got the musician put to death, after which he triumphed without difficulty over the virtue of Clytemnestra.

We are told also that, at a later period, Pythagoras composed songs or airs capable of curing the most violent passions, and of recalling men to the paths of virtue and moderation: while the physician prescribes draughts for curing bodily diseases, an able musician might therefore prescribe an air for rooting out a vicious passion.

The story of Timotheus, the director of the music of Alexander the Great, is well known.—One day, while the prince was at table, Timotheus performed an air in the Phrygian mode, which made such an impression on him that, being already heated with wine, he flew to his arms, and was going to attack his guests, had not Timotheus immediately changed the style of his performance to the Sub-Phrygian. This mode calmed the impetuous fury of the monarch, who resumed his place at table. This was the same Timotheus who, at Sparta, experienced the humiliation of seeing publicly suppressed four strings which he had added to his lyre. The severe Spartans thought that this innovation would tend to effeminate the manners,

by introducing a more extensive and more variegated kind of music. This at any rate proves that the Greeks were convinced that music had a peculiar influence on manners; and that it was the duty of government to keep a watchful eye over that art.

Who indeed can doubt that music is capable of producing such an effect? Let us only interrogate ourselves, and examine what have been our sensations on hearing a majestic or warlike piece of music, or a tender and pathetic air sung or played with expression. Who does not feel that the latter tends as much to melt the soul, and dispose it to pleasure, as the former to rouse and exalt it? Several facts in regard to the modern music place it on a level in this respect with the ancient.

The modern music indeed has also had its Timotheus, who could excite or calm, at his pleasure, the most impetuous emotions. Henry III. king of France, says *le Journal de Sancy*, having given a concert on occasion of the marriage of the Duke de Joyeuse, Claudin le Jeune, a celebrated musician of that period, executed certain airs, which had such an effect on a young nobleman, that he drew his sword and challenged every one near him to combat; but Claudin, equally prudent as Timotheus, instantly changed to an air, apparently Sub-Phrygian, which appeased the furious youth.

But, what shall we say of Stradella, the celebrated composer, whose music made the daggers drop from the hands of his assassins? Stradella having carried off the mistress of a Venetian musician, and retired with her to Rome, the Venetian hired three desperadoes to assassinate him; but fortunately for Stradella they had an ear sensible to harmony. These assassins, while waiting for a favourable opportunity to execute their purpose, entered the church of St. John de Latran, during the performance of an Oratorio composed by the person whom they intended to destroy, and were so affected by the music, that they

abandoned their design, and even waited on the musician to forewarn him of his danger. Stradella however was not always so fortunate; other assassins, who apparently had no ear for music, stabbed him some time after at Genoa: this event took place about the year 1670.

Every person almost has heard that music is a cure for the bite of the tarantula. This cure, which was formerly considered as certain, has by some been contested: but, however this may be, Father Schott, in his *Musurgia Curiosa*, gives the tarantula air, which appears to be very dull, as well as that employed by the Sicilian fishermen to entice the thunny fish into their nets.

Various anecdotes are related respecting persons whose lives have been preserved, by music effecting a sort of revolution in their constitutions. A woman being attacked for several months with the vapours, and confined to her apartment, had resolved to starve herself to death: she was however prevailed on, but not without difficulty, to see a representation of the *Serva Padrona*, at the conclusion of which she found herself almost cured, and, renouncing her melancholy resolution, was entirely restored to health by a few more representations of the like kind.

There is a celebrated air in Swisserland, called *Ranz des Vaches*, which had such an extraordinary effect on the Swiss troops in the French service, that they always fell into a deep melancholy when they heard it: Louis XIV. therefore forbade it ever to be played in France, under the pain of a severe penalty. We are told also of a Scotch air (*Lochaber no more*) which has a similar effect on the natives of Scotland.

Most animals, and even insects, are not insensible to the pleasure of music. There are few musicians perhaps who have not seen spiders suspend themselves by their threads in order to be near the instruments. We have several times had that satisfaction. We have seen a dog who, at an adagio of a sonata by Sennaliez, never failed

to show signs of attention, and some peculiar sensation by howling.

The most singular fact however is that mentioned by Bonnet, in his History of Music. This author relates that an officer, being shut up in the Bastille, had permission to carry with him a lute, on which he was an excellent performer; but he had scarcely made use of it for three or four days, when the mice issuing from their holes, and the spiders suspending themselves from the ceiling by their threads, assembled around him to participate in his melody. His aversion to these animals made their visit at first disagreeable, and induced him to lay aside his recreation; but he was soon so accustomed to them, that they became a source of amusement. We are informed by the same author, that he saw, in 1688, at the country seat of Lord Portland, the English ambassador in Holland, a gallery in a stable, employed, as he was told, for giving a concert once a week to the horses, which seemed to be much affected by the music. This, it must be allowed, was carrying attention to horses to a very great length. But it is not improbable that this anecdote was told to Bonnet by some person, in order to make game of him.

ARTICLE XI.

Of the properties of certain instruments, and particularly wind instruments.

I. We are perfectly well acquainted with the manner in which stringed instruments emit their sounds; but erroneous ideas were long entertained in regard to wind instruments, such as the flute; for the sound was ascribed to the interior surface of the tube. The celebrated Euler first rectified this error, and it results from his researches: —1st. That the sound produced by a flute, is nothing else than that of the cylinder of air contained in it.—2d. That the weight of the atmosphere which compresses it, acts the part of a stretching weight.—3d. That the sound of

this cylinder of air, is exactly the same as that which would be produced by a string of the same mass and length, extended by a weight equal to that which compresses the base of the cylinder.

This fact is confirmed by experiment and calculation: for Euler found that a cylinder of air, of 7½ Rhinlandish feet, at a time when the barometer is at a mean height, must give C—*sol*—*ut*; and such is nearly the length of the open pipe of an organ which emits that sound. The reason of its being made generally 8 feet is, because that length is required at those times when the weight of the atmosphere is greater.

Since the weight of the atmosphere produces, in regard to the sounding cylinder of air, the same effect as that produced by the weight which stretches a string, the more that weight is increased, the more will the sound be elevated; it is therefore observed that during serene warm weather, the tone of wind instruments is raised; and that during cold and stormy weather it is lowered. These instruments also emit a higher sound, in proportion as they are heated; because the mass of the cylinder of heated air becoming less, while the weight of the atmosphere remains unchanged, the case is exactly the same as if a string should become less and be still stretched by the same weight: every body knows that such a string would emit a higher tone.

But as stringed instruments must become lower, because the elasticity of the strings insensibly decreases, it thence follows that wind and stringed instruments, however well tuned they may be to each other, soon become discordant: for this reason the Italians never admit the former into their Orchestras.

II. A very singular phenomenon is observed in regard to wind instruments, such as the flute and huntsman's horn: with a flute, for example, when all the holes are stopped, if you blow faintly into the mouth aperture, a certain tone

will be produced; if you blow a little stronger, the tone instantly rises to the octave; and by blowing successively with more force, you will produce the twelfth, or fifth above the octave; then the double octave or seventeenth major.

The cause of this effect is the division of the cylinder of air contained in the instrument: when you breathe into the flute gently, the whole column resounds, and it emits the lowest tone; but if you endeavour, by a stronger inspiration, to make it perform quicker vibrations, it divides itself into two parts, which perform their vibrations separately, and which consequently must give the octave: a still stronger inspiration makes the column divide itself into three portions, which give the twelfth, &c.

III. It remains for us to speak of the trumpet marine. This instrument is only a monochord of a singular construction, being composed of three boards that form a triangular body. It has a very long neck, and one thick string, mounted on a bridge, which is firm on the one side, and tremulous on the other. It is struck by a bow with one hand, and with the other the string is stopped or pressed on the neck by means of the thumb, applied to the divisions indicated for the different tones.—The trembling of the bridge, when the string is struck, makes it imitate the sound of the trumpet; and this it does to such perfection, that it is scarcely possible to distinguish the one from the other. Hence it had its name: but whereas in common stringed instruments the tone becomes lower as the part of the string struck is longer, the case here is the contrary; for if the half of the string, for example, gives *ut*, the two thirds give the *sol* above, and the three fourths give the octave.

M. Sauveur first assigned the reason of this singularity, and proved it in a sensible manner, by showing that when the string, by the gentle application of the finger, is divided into two parts which are to each other as 1 to 2,

whatever part be touched, the greater immediately divides itself into two equal portions, which consequently perform their vibrations in the same time, and give the same sound as the less. But the less being the third of the whole, and the two thirds of the half, it must give the fifth or *sol* when the half gives *ut*. In like manner, the three fourths of the string divide themselves into three portions, each equal to the remaining fourth, and as they perform their vibrations separately, they must emit the same sound, which can be only the octave of the half. The case is the same with the other sounds of the trumpet marine, which may be easily explained on the same principle.

ARTICLE XII.

Of a fixed sound; method of preserving and transmitting it.

Before the effects of the temperature of the air on sound, and on the instruments by which it is produced, were known, this would not have formed the subject of a question, but to the few possessed of an ear exceedingly fine and delicate, and in which the remembrance of a tone is perfect: to others no doubt would remain that a flute, not altered, would always give the same tone. Such an opinion however would be erroneous, and if the means of transmitting to St. Domingo, for example, or to Quito, or only to posterity, the exact pitch of our opera were required, to solve this problem would be attended with more difficulty than might at first be imagined.

Notwithstanding what may be generally said in this respect, we shall here begin by a sort of paradox. It is every where said that the degree of the tone varies according to the weight of the atmosphere, or the height of the barometer. This we can by no means admit; and we flatter ourselves that we can prove the contrary.

It has been demonstrated by the formulæ of Euler, and no one entertains any doubt in regard to their truth, that if G represents the weight which compresses the column

of air in a flute, L the length of that column, and W its weight, the number of the vibrations it makes will be expressed by $\sqrt{\frac{G}{WL}}$, that is to say, will be in the compound ratio of the square root of G, or the compressing weight taken directly, and the product of the length by the weight taken inversely. Let us suppose then that the length of the column of air put in vibration is invariable, and that the gravity of the atmosphere only, or G, is variable, as well as the weight of the vibrating column. In this case we shall have the number of the vibrations proportional to the expression $\sqrt{\frac{G}{W}}$. But the density of any stratum of air being proportional to the whole weight of that part of the atmosphere immediately above it, it thence follows that W, which in equal lengths is as the density, is as G. The fraction $\frac{G}{W}$ therefore is constantly the same, when difference of heat does not alter the density. The square root of $\frac{G}{W}$ then is always the same; consequently there will be no variation in the number of the vibrations, or in the tone, at whatever height in the atmosphere the instrument may be situated, or whatever be the gravity of the air, provided its temperature has not changed.

This reasoning, in our opinion, is unanswerable; and if the gravity of the air has hitherto been reckoned among those causes which alter the tone of wind instruments, it is because it has been implicitly believed that the weight of the column of air put in vibration is invariable. It is however evident that under the same temperature it must be more or less dense, according to the greater or less density of the atmosphere; since it has a communication with the surrounding stratum of air, the density of which is proportional to that gravity. But the gravity in equal volumes is proportional to the density; therefore, &c.

Nothing then remains to be considered but the temperature of the air, which is the only cause that can produce variations in the tone of a wind instrument. But whatever

may be the degree of heat or of cold, the tone might be fixed in the following manner. For this purpose provide an instrument, such as a German flute, the cylinder of air in which can be lengthened or shortened by moving the joints closer to or farther from each other; and have another so constructed, as to remain invariable, and which ought to be preserved in the same temperature, such as 54 degrees of Fahrenheit's thermometer. The first flute being at the same degree of temperature, bring them both into perfect unison, and then heat the first to 74° of Fahrenheit, which will necessarily communicate to the cylinder of air contained in it the same degree of heat, and lengthen it by the quantity necessary to restore perfect unison: it is evident that if this elongation were divided into 20 parts, each of them would represent the quantity by which the flute ought to be lengthened for each degree of Fahrenheit's thermometer.

But it may be readily conceived that the quantity of this elongation, which at most would be but a few lines, could not be divided into so many parts; and therefore it ought to be executed by the motion of a screw, that is to say one of the joints of the instrument should be screwed into the other; for it would then be easy to make this elongation correspond to a whole revolution, and hence it might be divided into a great number of equal parts.

By these means the opera at Lima, if required, where the heat frequently rises to 110 of Fahrenheit's thermometer, might be made to have exactly the same pitch or tone as at Paris. But this is sufficient on a subject the utility of which would not be worth the trouble necessary for attaining to such a degree of precision.

ARTICLE XIII.

Singular application of music to a question in mechanics.

This question was formerly proposed by Borelli; and though we do not think that it can at present be a subject

of controversy, it has occasioned some difference of opinion among a certain class of mechanicians.

Fasten a string at one end to a fixed point; and having stretched it over a kind of bridge, suspend from it a weight, such as 10 pounds for example.

Now if, instead of the fixed point, which maintains the string in its place in opposition to the action of the weight, a weight equal to the former be substituted, will the string in both cases be equally stretched?

We have no doubt that every well informed mechanician will readily believe that in both cases the tension will be the same; and this necessarily follows from the principle of equality between action and re-action. According to this principle, the immoveable point, which in the first case counteracts the weight suspended from the other end of the string, opposes to it a resistance exactly equal to the action which it exercises: if a weight equal to the former be therefore substituted instead of the fixed point, every thing remains equal in regard to the tension experienced by the parts of the string, and which tends to separate them.

But music furnishes us with a method of proving this truth to the reason, by means of the sense of hearing; for as the tone is not altered while the tension remains the same, nothing is necessary but to make the following experiment. Take two strings of the same metal, and the same size, and having fastened one of them by one end to a fixed point, stretch it over a bridge, so as to intercept between it and the fixed point a determinate length, such as a foot for example; and suspend from the other end of it a given weight, such as 10 pounds. Then extend the second string over two bridges, a foot distant from each other, and suspend from each extremity of it a weight of 10 pounds; if the tone of these two strings be the same, there will be reason to conclude that the tension also is the same. We do not know whether this experiment was ever

made; but we will venture to assert that it will decide in favour of equality of tension.

This ingenious application of music to mechanics, is the invention of Diderot, who proposed it in his *Memoires sur différentes sujets de Mathematique et de Physique*, printed at Paris in octavo in the year 1748.

ARTICLE XIV.

Some singular considerations in regard to the flats and sharps, and to their progression on their different tones.

Those in the least acquainted with music know that, according to the different keys employed in modulation, a certain number of sharps or flats are required; because in the major mode, the diatonic scale, with whatever tone we begin, must be similar to that of *ut*, which is the simplest of all, as it has neither sharp nor flat. These flats or sharps have a singular progress, which deserves to be observed; it is even susceptible of a sort of analysis, and as we may say algebraic calculation.

To give some idea of it, we shall first remark, that a flat may and ought to be considered as a negative sharp, since its effect is to lower the note a semitone, whereas the sharp raises it the same quantity. This consideration alone may serve to determine all the sharps and flats of the different tones.

It may be readily seen that when a melody in *ut* major is raised a fifth, or brought to the tone of *sol*, a sharp is required on the *fa*. It may therefore be thence concluded, that this modulation, lowered a fifth or brought to *fa*, will require a flat; and indeed one is required on the *si*.

It hence follows also, that if the air be raised another fifth, that is to say to *re*, one sharp more will be required; and this is the reason why two are necessary. But to rise two fifths, and then descend an octave to approach the primitive tone, is to rise only one tone; consequently to raise the air one tone, two sharps must be added. The

tone of *re* indeed requires two sharps, and for the same reason the tone of *mi* requires four.

The tone of *fa* requires one flat, and that of *mi* requires four sharps; therefore, when an air is raised a semitone, five flats must be added; for, a flat being a negative sharp, it is evident that such a number of flats must be added to the four sharps of *mi*, as shall efface these four sharps, and leave one flat remaining; which cannot be done but by five flats, for, according to the language of analysis,—$5x$ must be added to $4x$, to leave as remainder—x. For the same reason, if the modulation be lowered a semitone, five sharps must be added: thus, as the tone of *ut* has neither sharps nor flats, five sharps will be found necessary for *si*, which is indeed the case. If the modulation be still lowered a tone, to be in *la*, we must add two flats, in the same manner as two sharps are added when we rise a tone. But five sharps plus two flats, is the same thing as five sharps minus two sharps, or three sharps. We still find therefore, by this method, that the tone of *la* requires three sharps.

But, before we proceed farther, it will be necessary to observe, that all the chromatic tones, that is to say all those inserted between the tones of the natural diatonic scale, may be considered as sharps or flats; for it is evident that *ut*♯ or *re*♭ are the same thing. It is very singular however, that according as this note is considered an inferior one affected by a sharp, or a superior one affected by a flat, the number of sharps required by the tone of the first, *ut*♯, for example, and that of the flats required by the tone of the second, *re*♭, always make 12; which evidently arises from the division of the octave into 12 semitones: therefore, since *re*♭, as above shown, requires five flats, if, instead of this tone, we consider it as *ut*♯, seven sharps will be required; but for the facility of execution it is much better, in the present case, to consider this tone as *re*♭, than *ut*♯.

This change therefore ought always to be made when the number of the sharps exceeds six; so that, since ten sharps, for example, would be found in the tone of *la**, we must call it *si*♭, and we shall have for that tone two flats, because two flats are the complement of ten sharps. On the other hand, in following the progression of the semitones descending, if we should find a greater number of sharps than 12, we ought to reject 12, and the remainder will be that of the tone proposed: for example, as *ut* has neither sharp nor flat, we have five sharps for the lower tone *si*; ten sharps for the semitone below *la**; fifteen sharps for the still lower semitone *la*: if twelve sharps therefore be rejected, there will remain three, which are indeed the number of sharps necessary in the tone of A—*mi*—*la*.

The tone of *sol** ought to have 8 or 4 flats, if we call it *la*♭.

The tone of *sol* will have 13 sharps, from which if 12 be deducted, one sharp will remain, as is well known.

The tone of *fa** will have 6 sharps, or 6 flats, if we call it *sol*♭.

The tone *fa* ought to have 6 flats plus 5 sharps; that is, 1 flat, as the 5 sharps destroy the same number of flats.

That of *mi* will have 1 flat plus 5 sharps; that is 4 sharps, as the flat destroys one of them.

That of *re** will have 9 sharps or 3 flats, if it be considered as *mi*♭.

That of *re* will have 14 sharps; that is to say 2, by rejecting 12, or 3 flats plus 5 sharps = 2 sharps.

That of *ut* will have 7 sharps, or 5 flats, if we call it *re*♭.

In the last place, the tone *ut* natural will have 12 sharps; that is, none, or 5 flats plus 5 sharps, which destroy each other.

The very same results would be obtained in ascending by semitone after semitone from *ut*, and adding 5 flats for each; taking care to reject 12 when they exceed that number. Our readers, by way of amusement, may make

the calculation. By calculating the number of the semitones, either ascending or descending, we might in like manner find that of the sharps or flats of any tone given.

Let us take, for example, that of *fa**: from *ut* ascending there are six semitones, and six times 5 flats makes 30 flats; from which if we deduct 24, a multiple of 12, the remainder will be 6: *sol*^{b} therefore will have 6 flats.

The same *fa** is 6 tones lower than *ut;* consequently there must be 6 times 5 or 30 sharps; from which if 24 be deducted, 6 sharps will remain, as we have found by another method.

The tone of *sol* is 5 semitones lower than *ut;* consequently there must be 5 times 5, or 25 sharps; from which if 24 be deducted, there will remain only one sharp.

As the same tone is 7 semitones higher than *ut*, there must be 7 times 5 or 35 flats; from which if 24 be deducted, the remainder will be 11 flats, that is, one sharp.

This progression appeared to us so curious as to be worthy of this notice; but in order that it may be exhibited under a clearer and more favourable point of view, we shall form it into a table, which will at any rate be useful to those who are beginning to play on the harpsichord. For this purpose we shall present each chromatic note as flattened or sharpened, and on the left of the former we shall mark the sharps it requires, and the flats on the right of the latter.

0 sharp . . .	*ut** . . .	0 flats
7 sharps .	*ut** or *re*^{b}* .	5 flats
2 sharps . . .	*re** . . .	
9 sharps .	*re** or *mi*^{b} .	3 flats
4 sharps . . .	*mi** . . .	
11 sharps . . .	*fa** . . .	1 flat
6 sharps .	*fa** or *sol*^{b}* .	6 flats
1 sharp . . .	*sol** . . .	
8 sharps .	*sol* or *la*^{b}* .	4 flats

3 sharps	. . .	*la** .		
10 sharps	. *la**	or *si*^{b}*		2 flats
5 sharps	. . .	*si** .		
0 sharp	. . .	*ut** .		0 flat

Of these tones, we have marked those usually employed with a *; for it may be easily conceived that by employing *re** under this form, we should have 9 sharps, which would give two notes with double sharps, viz *fa***, *ut***; so that the gamut would be *re** *mi**, or *fa*, *fa***, or *sol*, *sol**, *la**, *si** or *ut*, *ut*** or *re*, *re**; which it would be exceedingly difficult to execute: but by taking *mi*^{b}, instead of *re**, we have only 3 flats, which renders the gamut much simpler, as it then becomes

mi^b, *fa*, *sol*, *la*^{b}, *si**, *ut*, *re*, *mi*^{b}.

We are almost inclined to ask pardon of our readers for having amused them with this frivolous speculation; but we hope the title of our work will plead our excuse.

ARTICLE XV.

Method of improving barrel-instruments, and of making them fit to execute airs of every kind.

The mechanism of that instrument, called the barrel organ, is well known. It consists of a great number of pipes, graduated according to the tones and semitones of the octave, or at least those semitones which the progress of modulation in general requires. But these pipes never sound except when the wind of a bellows, kept in continual action, is made to penetrate to them by means of a valve. This valve is shut by a spring, and opened when necessary by a small lever, raised by spikes implanted in a wooden cylinder, which is put in motion by a crank. The crank serves also to move the bellows, which must continually furnish the air, destined to produce the different sounds by its introduction into the pipes.

But, in order that the subject of this article may be

properly comprehended, it will be necessary that the reader should have a perfect idea of the manner in which the notes are arranged on the cylinder.

The different small levers, which must be raised to produce the different tones, being placed at a certain distance from each other, that of half an inch for example, circular lines are traced out at that distance on the cylinder. One of these lines is intended for receiving the spikes that produce the sound *ut*, the next for those that sound *ut**, the next for those that give *re*, and so on. There are as many lines of this kind as there are pipes; but it may be easily conceived that the duration of an air or tune cannot exceed one revolution of the cylinder.

Let us suppose then that the air consists of 12 measures. Each of these circumferences is divided into 12 equal parts at least, by 12 lines drawn parallel to the axis of the cylinder; and if we suppose that the shortest note of the air is a quaver, and that the air is in triple time, denoted by $\frac{3}{4}$, each interval must be divided into six equal portions; because, in this case, a measure will contain six quavers. Let us now suppose that the first notes of the air are *la*, *ut*, *si*, *re*, *ut*, *mi*, *re*, &c, all equal notes, and all simple crotchets. At the beginning of the line for receiving the *la*, and of the first measure, a spike must be placed of such a construction, as to keep raised up during the third of a measure the small lever that makes the *la* sound; then, in the line destined for the *ut*, at the end of the second division or beginning of the third, a spike similar to the first must be fixed in the cylinder; and in the line destined for the *si*, another of the same kind must be placed; it is evident that, when the cylinder begins to turn, the first spike will make *la* sound during the third of a measure. The second, as soon as the first third of the measure is elapsed, will catch the lever and make *ut* sound; and the third will in like manner make *si* sound

during the last third. The instrument therefore will say *la*, *ut*, *si*, &c.

If, instead of three crotchets, there were six quavers, which in this measure are the first long, the second short, the third long, and so on alternately, which are called dotted quavers, it may be easily perceived that after the spikes of the first, third, and fifth notes, have been fixed in the respective places of the division where they ought to be, nothing will be necessary but to take care that the spike of the first quaver, which in this time ought to be equal to a quaver and a half, will have its head constructed in such a manner as to raise the lever during one part and a half of the six divisions into which the measure is divided; which may be done by giving it a tail behind of the necessary length. In regard to the short quavers, the spikes representing them ought to be removed back half a division, and to be formed in such a manner, as to keep the lever corresponding to them raised up only during the revolution of a semidivision of the cylinder. By these examples it may be easily seen what must be done in the other cases, that is, when the notes have other values.

Were the cylinder immoveable in the direction of its axis, only one air could be performed; but as the spikes move the small levers merely by touching them beneath in a very narrow space, such as the breadth of a line at most, which is a mechanism that may be easily conceived, it will be readily seen that by giving to the cylinder the small lateral motion of a line, none of the spikes can communicate motion to the levers. Another line therefore, to receive spikes arranged so as to produce a different air, may be drawn close to each of the first set of lines, and the number of the different sets of lines may be six or seven, according to the interval between the first lines, which is the same as that between the middle of one pipe and the middle of the neighbouring one: by these means,

if the cylinder be moved a little in the direction of its axis the air may be changed.

Such is the mechanism of the hand or barrel organ, and other instruments constructed on the same principle; but it may be easily seen that they are attended with this inconvenience, that they can perform only a very small number of airs. But as a series of five, six, eight, or a dozen of tunes, is soon exhausted, it might be a matter of some importance to discover a method by which they might be changed at pleasure.

We agree in opinion with Diderot, who has given some observations on this subject, in the work above quoted, that this purpose might be answered by constructing the cylinder in the following manner. Let it be composed of a piece of solid wood, covered with a very hard cushion, and let the whole be pushed into a hollow cylinder, of about a line in thickness. On this inner cylinder draw the lines destined to receive the spikes, placed at the proper intervals for producing the different tones; and let holes be pierced in these lines at certain distances, six for example in each division of the measure if it be triple time, or eight in the measure if it be common time, denoted by c: we here suppose that no air is to be set that has shorter notes than plane quavers. Twelve holes per measure will be required in the first case, and sixteen in the second, if the air contains semi-quavers.

It may now be readily conceived that on a cylinder of this kind any air whatever might be set; nothing will be necessary for this purpose, but to thrust into the holes of the exterior cylinder spikes of the proper length, taking care to arrange them as above explained; they will be sufficiently firm in their places in consequence of the elasticity of the cushion *, strongly compressed between the inner cylinder and the hollow outer one. When the air

* Might not cork be employed instead of the cushion here proposed?

is to be changed, the spikes may be drawn out, and put into a box divided into small cells, in the same manner as printing types when distributed in the cases. The interior cylinder may then be made to revolve a little, in order to separate the holes in the cushion from those in the exterior cylinder, and a new air may then be set with as much facility as the former.

We shall not examine, with Diderot, all the advantages of such an instrument, because it must be allowed that it never can be of much utility, and will have no value in the eyes of the musician. It is however certain that it would be agreeable, for those who possess such instruments, to be able to give more variety to the airs they are capable of performing; and this end would be answered by the construction here described.

ARTICLE XVI.

Of some musical instruments or machines remarkable for their singularity or construction.

At the head of all these musical instruments, or machines, we ought doubtless to place the organ; the extent and variety of the tones of which would excite much more admiration, were it not so common in our churches; for, besides the artifice necessary to produce the tones by means of keys, what ingenuity must have been required to contrive mechanism for giving that variety of character to the tones, which is obtained by means of the different stops, such as those called the voice stop, flute stop, &c? A complete description therefore of an organ, and of its construction, would be sufficient to occupy a large volume.

The ancients had hydraulic organs, that is, organs the sound of which was occasioned by air produced by the motion of water. These machines were invented by Ctesibius of Alexandria, and his scholar Hero. From the description of these hydraulic organs, given by Vitruvius, in the tenth book of his Architecture, Perrault constructed

one which he deposited in the King's library, where the Royal Academy of Sciences held their sittings. This instrument indeed is not to be compared to the modern organs; but it is evident that the mechanism of it has served as a basis for that of ours. St. Jerome speaks with enthusiasm of an organ which had twelve pair of bellows, and which could be heard at the distance of a mile. It thence appears that the method employed by Ctesibius, to produce air to fill the wind-box, was soon laid aside, for one more simple; that is, for a pair of bellows.

The performer on the *tambour de basque*, and the automaton flute-player of Vaucanson, which were exhibited and seen with admiration in most parts of Europe, in the year 1749, may be classed among the most curious musical machines ever invented. We shall not however say any thing of the former of these machines, because the latter appears to have been far more complex.

The automaton flute-player performed several airs on the flute, with the precision and correctness of the most expert musician. It held the flute in the usual manner, and produced the tone by means of its mouth; while its fingers applied on the holes produced the different notes. It is well known how the fingers might be raised by spikes fixed in a cylinder, so as to produce these sounds; but it is difficult to conceive how that part could be executed which is performed by the tongue, and without which the music would be very defective. Vaucanson indeed confesses that this motion in his machine was that which cost him the greatest labour. Those desirous of farther information on this subject may consult a small work, in quarto, which Vaucanson published respecting these machines.

A very convenient instrument for composers was invented some years ago in Germany: it consists of a harpsichord which, by certain machinery added to it, notes down any air or piece of music, while a person is playing it. This is a great advantage to composers, as it enables

them, when hurried away by the fervour of their imagination, to preserve what has successively received from their fingers a fleeting existence, and what otherwise it would often be impossible for them to remember. A description of this machine may be found in the memoirs of the Academy of Berlin, for the year 1773.

ARTICLE XVII.

Of a new instrument called the Harmonica.

This new instrument was invented in America by Dr. Franklin, who gave a description of it to Father Beccaria, which the latter published in his works, printed in 1773.

It is well known that when the finger a little moistened is rubbed against the edge of a drinking glass, a sweet sound is produced; and that the tone varies according to the form, size, and thickness of the glass. The tone may be raised or lowered also by putting into the glass a greater or less quantity of water. Dr. Franklin says that an Irishman, named Puckeridge, first conceived the idea, about twenty years before that time, of constructing an instrument with several glasses of this kind, adjusted to the various tones, and fixed to a stand in such a manner, that different airs could be played upon them. Mr. Puckeridge having been afterwards burnt in his house along with this instrument, Mr. Delaval constructed another of the same kind, with glasses better chosen, which he applied to the like purpose. Dr. Franklin hearing this instrument, was so delighted with the sweetness of its tones, that he endeavoured to improve it; and the result of his researches was the instrument which we are now going to describe.

Cause to be blown on purpose glasses of different sizes, and of a form nearly hemispherical, having each in the middle an open neck. The thickness of the glass, near the edge, should be at most one tenth of an inch, and ought to increase gradually to the neck, which in the largest glasses should be an inch in length, and an inch and a half

in breadth in the inside. In regard to the dimensions of the glasses themselves, the largest may be about 9 inches in diameter at the mouth, and the least 3 inches, each glass decreasing in size a quarter of an inch. It will be proper to have five or six of the same diameter, in order that they may be more easily tuned to the proper tones; for a very slight difference will be sufficient to make them vary a tone, and even a third.

When these arrangements are made, try the different glasses, in order to form of them a series of three or four chromatic octaves. To elevate the tone, the edge towards the neck ought to be ground, trying them every moment, for if they be raised too high, it will afterwards be impossible to lower them.

When the glasses have been thus graduated, they must be arranged on a common axis. For this purpose, put a cork stopper very closely into the neck of each, so as to project from it about half an inch; then make a hole of a proper size in all these corks, and thrust into them an iron axis, but not with too much force, otherwise the necks might burst. Care must also be taken to place the glasses in such a manner, that their edges may be about an inch distant from each other, which is nearly the distance between the middle of the keys of a harpsichord.

To one of the extremities of the axis affix a wheel of about 18 inches in diameter, loaded with a weight of from 20 to 25 pounds, that it may retain for some time the motion communicated to it. This wheel, which must be turned by the same mechanism as that employed to turn a spinning wheel, communicates, as it revolves, its motion to the axis, which rests in two collars, one at the extremity and the other at some distance from the wheel. The whole may be fitted into a box of the proper form, placed on a frame supported by four feet. The glasses corresponding to the seven tones of the diatonic octave, may be painted of the seven prismatic colours in their natural

order, that the different tones to which they correspond may be more readily distinguished.

The person who plays on this instrument, is seated before the row of glasses, as if before the keys of a harpsichord; the glasses are slightly moistened, and the wheel being made to revolve, communicates the same motion to the glasses; the fingers are then applied to the edges of the glasses, and the different sounds are by these means produced. It may be easily seen that different parts can be executed with this instrument, as with the harpsichord.

About fourteen or fifteen years ago, an English lady at Paris performed, it is said, exceedingly well on this instrument. The sounds it emits are remarkably sweet, and would be very proper as an accompaniment to certain tender and pathetic airs. It is attended with one advantage, which is, that the sounds can be maintained or prolonged, and made to swell at pleasure; and the instrument, when once tuned, never requires to be altered. It afforded great satisfaction to many amateurs; but we have heard that the sound, on account of its great sweetness, became at last somewhat insipid, and for this reason perhaps it is now laid aside, and confined to cabinets, among other musical curiosities.

A few years ago Dr. Chladni, who has made various researches respecting the theory of sound, and the vibrations of sonorous bodies*, invented a new instrument of this kind, to which he gave the name of *euphon.* This instrument has some resemblance to a small writing desk, and contains in the inside 40 glass tubes of different colours, of the thickness of the barrel of a quill, and about 16 inches in length. They are wetted with water by means of a sponge, and stroked with the fingers in the direction of their length; so that the increase of the tone depends

* He published a work on this subject entitled, *Entdeckungen über die Theorie des Klanges.* Leipsic 1787. 4to.

merely on the stronger or weaker pressure, and the slower or quicker movement of the fingers. In the back part there is a perpendicular sounding board, through which the tubes pass. In sweetness of sound this instrument approaches near to the harmonica; but seems to be attended with advantages which the other does not possess.

1st. It is simpler, both in regard to its construction, and the movement necessary to produce the sound; as neither turning nor stopping is required, but merely the motion of the finger.—2d. It produces its sound speedier; so that as soon as touched the tone may be made as full as the instrument is capable of giving it: whereas in the harmonica the tones, and particularly the lower ones, must be made to increase gradually.—3d. It has more distinctness in quick passages, because the tones do not resound so long as in the harmonica, where the sound of one low tone is often heard when you wish only to hear the following one.—4th. The unison is purer than is generally the case in the harmonica; where it is difficult to have perfect glasses, which in every part give like tones with mathematical exactness. It is however as difficult to be tuned as the harmonica.—5th. It does not affect the nerves of the performer; for a person scarcely feels a weak agitation in the fingers; whereas in the harmonica, particularly in concords of the lower notes, the agitation extends to the arms, and even through the whole body of the performer.—6th. The expense of this instrument will be much less than that of the harmonica.—7th. When one of the tubes breaks, or any other part is deranged, it can be easily repaired: whereas when one of the glasses of the harmonica breaks, it requires much time, and is difficult to procure another capable of giving the same tone as the former, and which will correspond sufficiently with the rest.

For farther particulars respecting this instrument, and the history of its invention, see *The Philosophical Magazine*, n°. 8, or vol. 2, p. 391.

ARTICLE XVIII.

Of some singular ideas in regard to music.

1st. One perhaps would scarcely believe it possible for a person to compose an air, though entirely ignorant of music, or at least of composition. This secret however was published a few years ago, in a small work entitled *Le Jeu de Dez harmonique*, or *Ludus Melothedicus*, containing various calculations, by means of which any person, even ignorant of music, may compose minuets, with the accompaniment of a bass. 8vo. Paris 1757. In this work the author shows how a minuet and its bass may be composed, according to the points thrown with two dice, by means of certain tables.

This author gives a method also of performing the same thing by means of a pack of cards. We do not remember the title of this work; and we confess that we ought to attach no more importance to it than the author does himself.

We shall therefore content ourselves with having mentioned works to which the reader may have recourse, for information respecting this kind of amusement, the combination of which must have cost more labour than the subject deserved. We shall however observe, that this author published another work entitled, *Invention d'une Manufacture et Fabrique de Vers au petit metier*, &c. 8vo. 1759, in which he taught a method of answering, in Latin verse, by means of two dice and certain tables, any question proposed. This, it must be confessed, was expending much labour to little purpose.

2d. A physician of Lorrain some years ago, published a small treatise, in which he employed music in determining the state of the pulse. He represented the beats of a regular pulse by minuet time, and those of the other kinds of pulse by different measures, more or less accelerated. If this method of medical practice should be introduced, it

will be a curious spectacle to see a disciple of Hippocrates feeling the pulse of his patient by the sound of an instrument, and trying airs analogous by their time to the motion of his pulse, in order to discover its quality. If all other diseases should baffle the physician's skill, there is reason to believe that low spirits will not be able to withstand such a practice.

ARTICLE XIX.

On the Figures formed by Sand and other light Substances on Vibrating Surfaces.*

Dr. Chladni of Wittenberg, by his experiments on vibrating surfaces, published in 1787, opened a new field in this department of science, viz, the consideration of the curves formed by sand and other light bodies, on surfaces put into a state of motion. As this subject is curious, and seems worthy of further research, we shall present the reader with a few observations on the method of repeating these experiments, taken from Gren's Journal of Natural Philosophy, vol. 3.

Vibration figures, as they are called, are produced on vibrating surfaces, because some parts of these surfaces are at rest, and others in motion. The surfaces fittest for being made to vibrate, are panes of glass; though the experiments will succeed equally well with plates of metal, or pieces of board, a line or two in thickness. If the surface of any of these bodies be strewed over with substances easily put in motion; such for example as fine sand; these, during the vibration of the body, will remain on the parts at rest, and be thrown from the parts in motion, so as to form mathematical figures. To produce such figures, nothing is necessary but to know the method of bringing that part of the surface which you wish not to vibrate into a state of rest; and of putting in motion that which you

* See also Phil. Mag. No. 12.

wish to vibrate: on this depends the whole expertness of producing vibration figures.

Those who have never tried these experiments might imagine, that to produce fig. 2 pl. 19, it would be necessary to damp, in particular, every point of the part to be kept at rest, viz, the two concentric circles and the diameter, and to put in motion every part intended to vibrate. This however is not the case; for you need damp only the points *a* and *b*, and cause to vibrate one part *c*, at the edge of the plate; for the motion is soon communicated to the other parts, which you wish to vibrate, and the required figure will in this manner be produced.

The damping may be best effected by laying hold of the place to be damped between two fingers, or by supporting it only by one finger. This will be more clearly comprehended by turning to fig. 6, where the hand is represented in that position necessary to hold the plate. In order to produce fig. 3, you must hold the plate horizontally, placing the thumb above at *a*, with the second finger directly below it; and besides this, you must support the point *b* on the under side of the plate. If the bow of a violin be then rubbed against the plate at *c*, you will produce on the glass the figure which is delineated fig. 3. When the point to be supported or damped lies too near the centre of the plate, you may rest it on a cork, not too broad at the end, brought into contact with the glass in such a manner, as to supply the place of the finger. It is convenient also, when you wish to damp several points at the circumference of the glass, to place your thumb on the cork, and to use the rest of your fingers for touching the parts which you wish to keep at rest. For example, if you wish to produce fig. 4, on an elliptic plate, the larger axis of which is to the less as 4 to 3, you must place the cork under *e*, the centre of the plate; put your thumb upon this point, and then damp the two points of the edge *p* and *q*, as may be seen fig. 5, and make the plate to vibrate by

rubbing the violin bow against it at r. There is still another convenient method of damping several points at the edge, when large plates are employed. Fig. 1 represents a strong square bit of metal ab, a line in circumference, which is screwed to the edge of the table, or made fast in any other manner: and a notch, about as broad as the edge of the plate, is cut into one side of it with a file. You then hold the plate resting against this bit of metal, by two or more fingers when requisite, as at c and d; by which means the edge of the plate will be damped in three points $d\,c\,e$; and in this manner, by putting the plate in vibration at f, you can produce fig. 10. In cases of necessity you may use the edge of a table, instead of the bit of metal; but it will not answer the purpose so well.

To produce the vibration at any required place, a common violin bow, rubbed with rosin, is the most proper instrument to be employed. The hair must not be too slack, because it is sometimes necessary to press pretty hard on the plate, in order to produce the tone sooner.

When you wish to produce any particular figure, you must first form it in idea on the plate, in order that you may be able to determine where a line at rest, and where a vibrating part, will occur. The greatest rest will always be where two or more lines intersect each other, and such places must in particular be damped. For example, in fig. 7 you must damp the part n, and stroke with the bow in p. Fig. 11 may be produced with no less ease, if you hold the plate at r, and stroke with the bow at f. The strongest vibration seems always to be in that part of the edge which is bounded by a curve: for example, in fig. 8 and fig 9, at n. To produce these figures therefore, you must rub with the bow at n, and not at r.

You must however damp, not only those points where two lines intersect each other, but endeavour to support at least one which is suited to that figure, and to no other.

For example, when you support *a* and *b*, fig. 2, and rub with the bow at *c*, fig. 7 also may be produced; because both these figures have these two points at rest. To produce fig. 2, you must support with one finger the part *c*, and rub with the bow in *c*; but fig. 7 cannot be produced in this manner, because it has not the point *c* at rest.

One of the greatest difficulties in producing the figures, is to determine before-hand the vibrating and resting points which belong to a certain figure, and to no other. Hence, when one is not able to damp those points which distinguish one figure from another, if the violin bow be rubbed against the plate, several hollow tones are heard, without the sand forming itself as expected. You must therefore acquire by experience a readiness, in being able to search out among these tones, that which belongs to the required figure, and to produce it on the plate by rubbing the bow against it. When you have acquired sufficient expertness in this respect, you can determine before-hand, with a considerable degree of certainty, the figures to be produced, and even the most difficult. It may be easily conceived, that you must not forget what part of the plate, and in what manner you damped; and you may mark these points by making a scratch on the plate with a bit of flint.

When the plate has acquired the proper vibration, you must endeavour to keep it in that state for some seconds; which can be best done by rubbing the bow against it several times in succession. By these means the sand will be formed much more accurately.